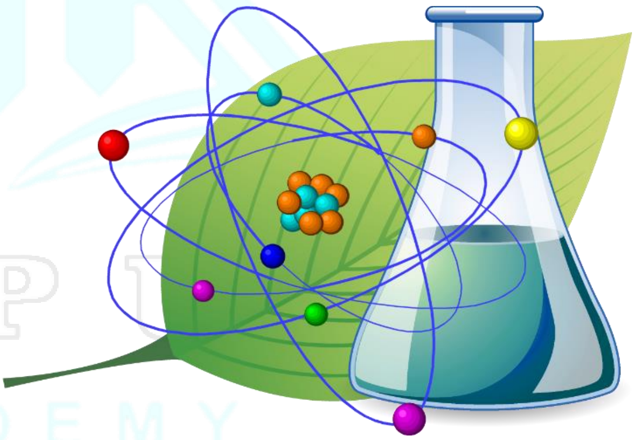
Physical World 1

Science

Science is a systematic attempt to understand natural phenomena in as much detail and depth as possible, and use the knowledge so gained to predict, modify and control phenomena.

Scientific method can be called as a method to acquire knowledge in a systematic and in-depth way. It is having:

- Systematic observations
- Controlled experiments
- Qualitative and Quantitative reasoning
- Mathematical modelling
- Prediction and verification (or falsification) of theories
- Speculation or Prediction

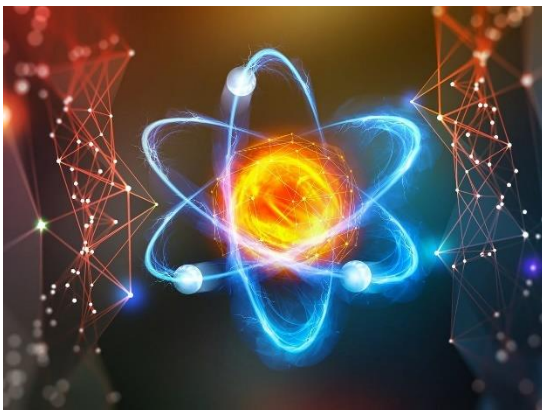


Science will be not having any final theory. The observations which are made using improved, accurate tools will be creating improved knowledge and perspective. Tycho Brahe's research on planetary motion has been used by Johannes Kepler for improving Nicolas Copernicus theory.

Quantum mechanics was developed in order to deal with atomic and nuclear phenomena. Work of Ernest Rutherford on nuclear model of atom made the basis of quantum theory suggested by Niels Bohr. The discovery of antielectron (positron) was led by the Antiparticle theory of Paul Dirac by Carl Anderson.

Natural Sciences: Natural science can be considered as a branch of science which is discussing about the description, prediction, and understanding of the natural phenomena which is on the basis of an observational and empirical evidence. It will be included of the disciplines mentioned below:

- Physics
- Chemistry
- Biology



Physics

Physics is a fundamental science concerned with understanding the natural phenomena that occur in our universe.

It has many branches such as Mechanics, Electromagnetism, Thermodynamics, Modem Physics, etc. Between 1600 and 1900, three broad areas were developed, which is together called

Classical Physics. These three areas of study are classical mechanics, thermodynamics and electromagnetism. But by 1905 it became apparent that classical ideas failed to explain several phenomena. Then some new theories were developed in what is called Modem Physics such as Special Relativity, Quantum Mechanics, etc.

Physical World

The physical world is referred to as the complexity in nature and solving its own complexities will give us new insights into this physical world. It is referred to as the analysis of nature conducted in order to understand how the world around us performs.

Top Concepts

- Physics is the branch of science that deals with nature and the natural phenomenon that occur.
- Physics has two domains - microscopic as well as macroscopic.

The microscopic domain contains atomic, molecular and nuclear phenomenon, whereas the macroscopic domain includes phenomena taking place at the laboratory, terrestrial and astronomical scales.

- Science has a great influence on technology. Some of the technological advancements which are governed by scientific concepts are given below.

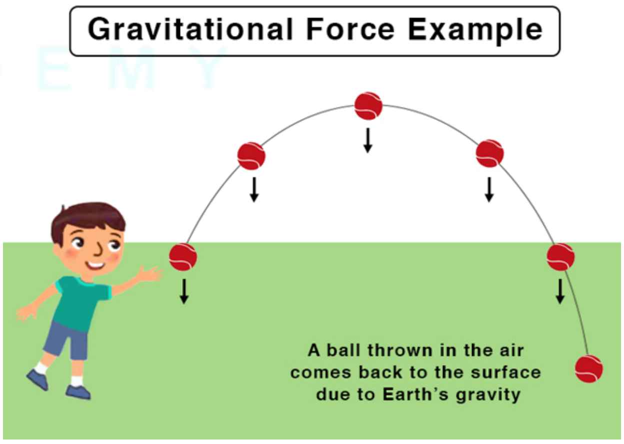
Technology	Scientific concept
Sonar	Reflection of ultrasonic waves
Rocket propulsion	Newton's laws of motion
Aeroplane	Bernoulli's principle in fluid dynamics
Steam engine	Law of thermodynamics
Optical fibres	Total internal reflection of light

The Fundamental Forces of nature

Gravitational Force: it is a universal force that exists which is of mutual attraction between any two objects by virtue of their masses.

The gravitational force has the following properties:

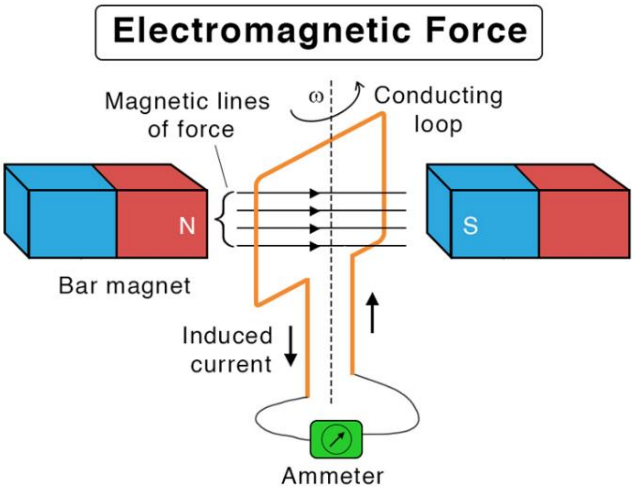
1. It obeys the inverse square law.
2. It is always attractive in nature.
3. It is a long range force and extends up to infinity.
4. The graviton is the field particle of gravitational force.
5. It is the weakest force operating in nature.
6. It is a central force and hence a conservative force.



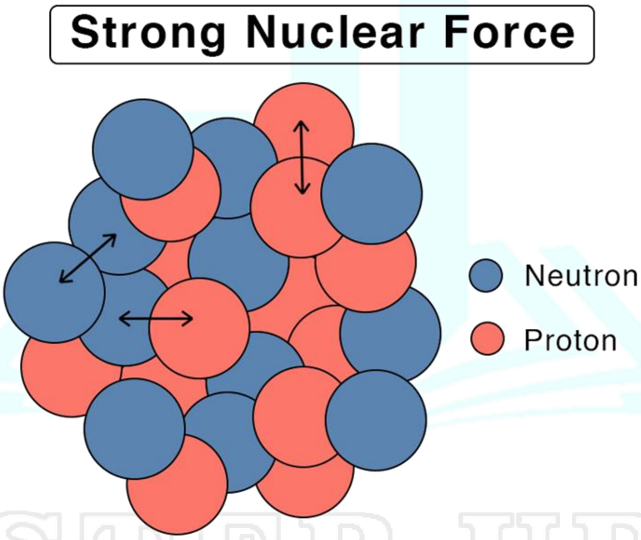
Electromagnetic Force: is the force between charged particles. If charges are in a state of rest, it is given by Coulumb's law whereas when they are in motion, they generate a magnetic field, hence the name electromagnetic forces as they are inseparable. They also act over a large distance as seen in the case of gravitational forces without the intervention of any medium.

The properties of electromagnetic force are as follows:

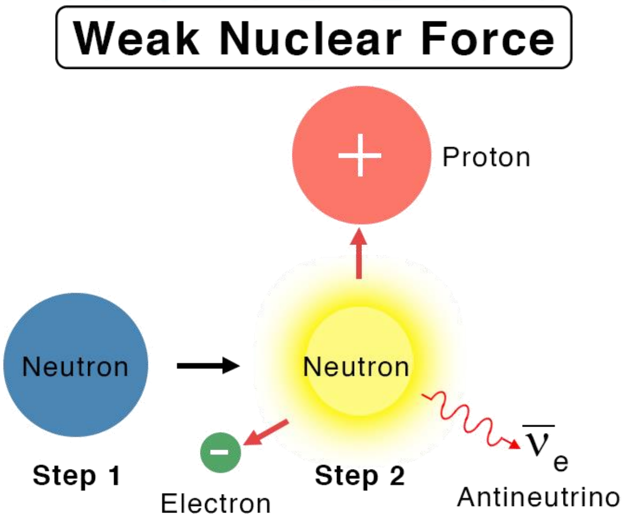
1. It obeys the inverse square law.
2. It may be attractive or repulsive in nature.
3. It is a long range force.
4. The photon is the field particle of electromagnetic force.
5. It is about 10^{36} times stronger than the gravitational force.
6. It is a central as well as a conservative force.



Strong Nuclear Force: in a nucleus it binds protons and neutrons. It is the strongest of all the fundamental forces and is charge-independent acting between proton-proton, proton-neutron, or neutron-neutron



Weak Nuclear Force: observed only in some nuclear processes. Example: β-decay of a nucleus. It is not as weak as the gravitational force but weaker than electromagnetic and strong nuclear force



The nuclear force has the following properties:

1. The strong nuclear force binds protons and neutrons in a nucleus. The weak nuclear force appears only in certain nuclear processes such as beta decay.
2. A strong nuclear force is the strongest force in nature. It is about 100 times stronger than the electromagnetic force. Weak nuclear force is stronger than the gravitational force but weaker than the electromagnetic or strong nuclear force.
3. It is a short-range force and is operative only over the size of nucleus.
4. A strong nuclear force is responsible for the stability of nuclei.

Name of force	Relative strength (w.r.t. strong nuclear force)	Rane	Operates among
Gravitational	10^{-39}	Infinite	All objects in universe
Weak nuclear	10^{-13}	Sub-nuclear size ($\approx 10^{-16}$ m)	Electron and neutrino
Electromagnetic	10^{-2}	Infinite	Charged particles
Strong nuclear	1	Nuclear size ($\approx 10^{-15}$ m)	Nucleons and heavier elementary particles

Unification of Forces

The unification of forces is the idea that it's possible to view all of nature's forces as manifestations of one single, all-encompassing force. Scientists have made great strides toward the goal of understanding how the forces can be combined

Conserved Quantities: In any physical phenomenon governed by different forces, several quantities change with time, while several quantities remain constant. Such quantities are called conserved quantities.

- **Law of conservation of linear momentum**: It states that if no external force acts on a system, the linear momentum of the system remains conserved. The law of gravitation is exactly identical on earth and moon even when the acceleration due to gravity at moon is $\frac{1}{6}th$ than that at earth.
- **Law of conservation of energy:** In accordance to the general Law of conservation of energy, the energies will be fixed over time and get transformed from one form to another. The law of conservation of energy will be applied to the whole universe and it has been considered that the total energy of the universe is fixed. The nature develops symmetric results at different time under similar conditions. It states that energy can neither be created nor destroyed; however it may change from one form to another.
- **Law of Conservation of Mass:** A chemical reaction can be defined as a rearrangement of atoms among various molecules. The difference will be formed as heat and the reaction is exothermic when the total binding energy of the reacting molecules will be less than the total binding energy of the product molecules. The opposite will be correct for energy-absorbing reactions such as endothermic reactions. As the atoms are not destroyed, only just rearranged, the summation of the mass of the reactants will be identified as the total mass of the products in a chemical reaction. Mass will be in relation to energy through Einstein theory, $E= mc^2$, where c will be the speed of light in vacuum.
- **Law of conservation of angular momentum:** It states that if no external torque acts on a system, then the total angular momentum of the system remains conserved.

Laws of Physics

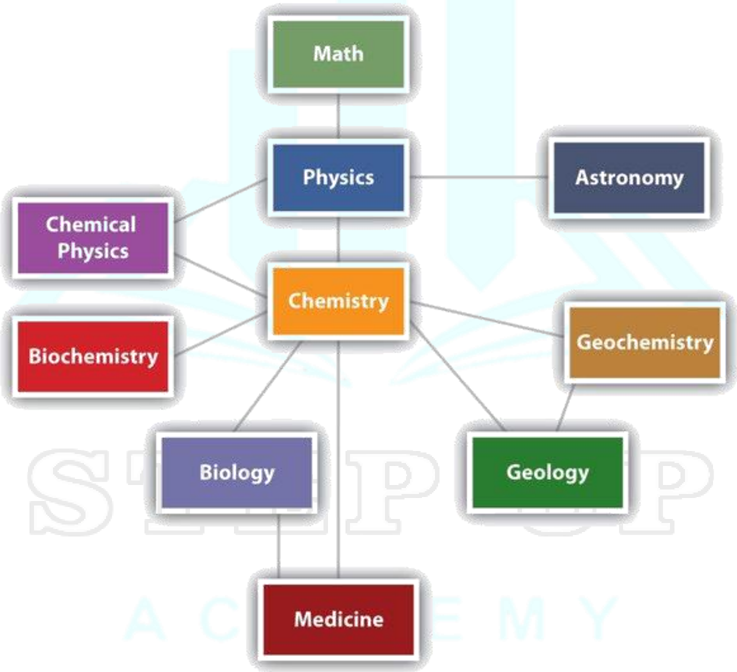
By nature, laws of Physics are stated facts which have been deduced and derived based on empirical observations. Simply put, the world around us works in a certain way, and physical laws are a way of classifying that “working.”

Physical laws are just conclusions drawn based on years (or however long it takes) of scientific observations and experiments which are repeated over and over under different conditions to reach inferences which can be accepted worldwide. These are continuously validated by the scientific community over time.

The different properties of laws of Physics which shed information about their nature are given below:

- True, under specified conditions
- Universal and do not deviate anywhere in the universe
- Simple in terms of representation
- Absolute and unaffected by external factors
- Stable and appear to be unchanging
- Omnipresent and everything in the universe is compliant (in terms of observations)
- Conservative in terms of quantity
- Homogeneous in terms of space and time
- Theoretically reversible in time

Physics in Relation to Other Sciences



Physics is a very significant branch of science which plays a crucial role in understanding the developments pertaining to the other branches of science such as Chemistry, Biology etc.

Relation to Biology: The conceptual study of pressure and its measurement has helped us to know blood pressure and hence the functioning of heart. Invention of X-rays developed the field of diagnosis. Electron and optical microscopic designs have revolutionized the study of medical.

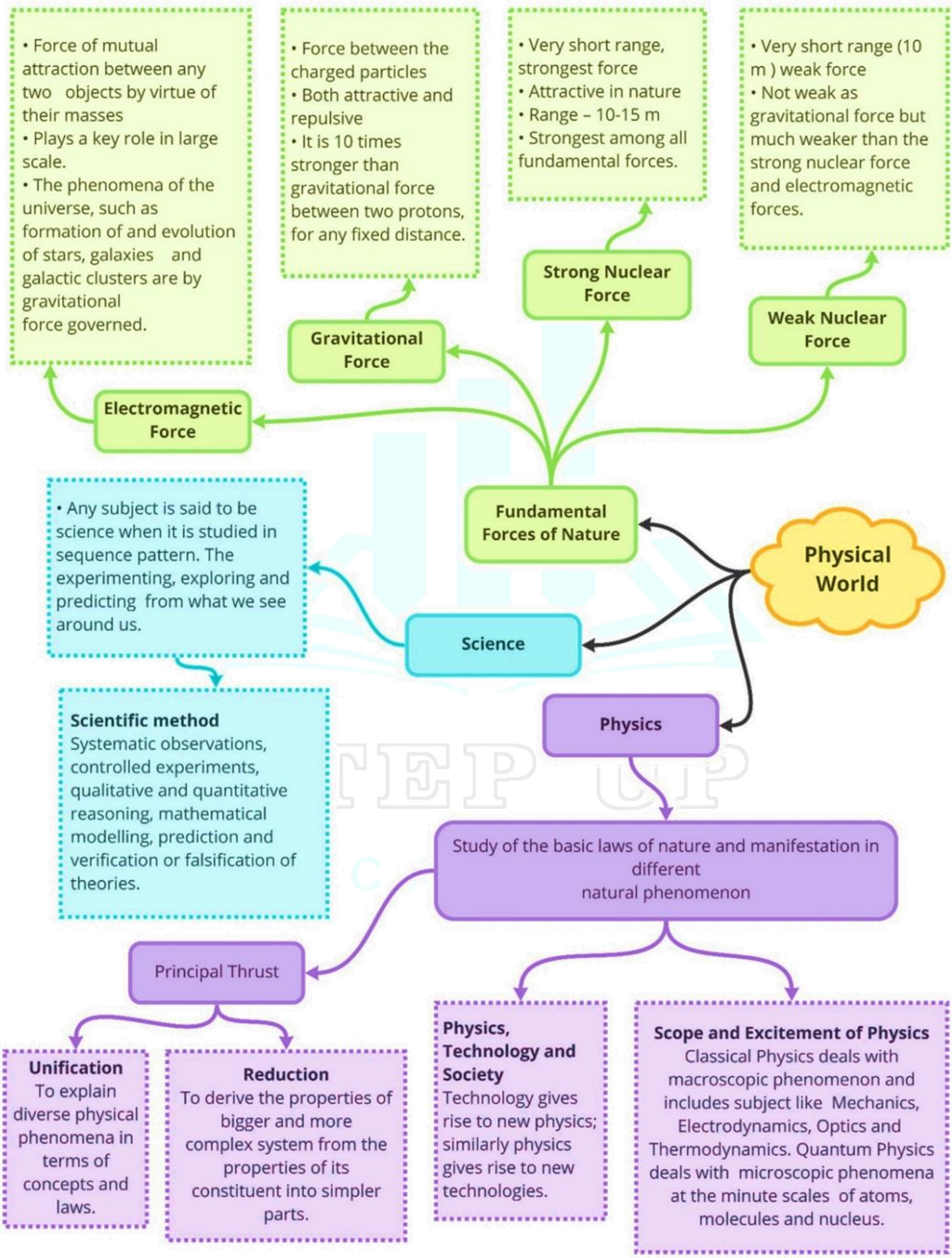
Relation to Chemistry: The concept of X-ray diffraction and radioactivity has helped to distinguish between the various solids and to modify the periodic table.

Understanding the bonding and the chemical structure of substances is easy with the help of the concept of interactions between various particles.

Relation to Mathematics: Study of physical variables led to the idea of differentiation, integration and differential equation. Meaningful interpretation of Mathematics becomes Physics.

Relation to Astronomy: Optical telescopes of reflecting and refracting type enabled man to explore the space around. Discoveries like radio telescopes have revolutionized the study of Astronomy.

Class : 11th Physics
Chapter- 1 : Physical World



Important Questions

Multiple Choice Questions

1. Atomic and molecular phenomena are dealt with by
   (a) Newtonian Mechanics
   (b) fluid Mechanics
   (c) applied Mechanics
   (d) Quantum Mechanics
2. Which of the following is a possible final step in applying the scientific method
   (a) Formulating a hypothesis
   (b) Building a theory
   (c) Analysis of test results
   (d) Formulation of a question
3. Which of the following is a possible first step in applying the scientific method
   (a) Conducting tests
   (b) Formulating a hypothesis
   (c) Formulation of a question
   (d) Building a theory
4. A scientific theory
   (a) cannot be changed but can be reformulated
   (b) is fixed once and for all because it is logical
   (c) is changed to suit new fashion among scientists
   (d) can be revised if required to fit new phenomenon or data
5. The scientific method is
   (a) a prescribed method for investigating phenomena, acquiring new knowledge.
   (b) A procedure for proposing new hypothesis
   (c) a body of techniques for investigating phenomena, acquiring new knowledge.
   (d) A method for proposing new theories.
6. Newtonian mechanics could not explain
   (a) fall of bodies on earth
   (b) Some of the most basic features of atomic phenomena.
   (c) movement of planets
   (d) flight of rockets
7. Heliocentric theory proposed by Nicolas Copernicus was
   (a) replaced by circular orbits to fit the data better
   (b) replaced by elliptical orbits to fit the data better
   (c) replaced by elliptical orbits to fit the taste of new rulers of Italy
   (d) replaced by parabolic orbits to fit the data better
8. Physics is a
   (a) Applied Science
   (b) Mathematical Science
   (c) Engineering Science
   (d) Natural Science
9. The word Science originates from the Latin verb Scientia meaning
   (a) to know
   (b) to see
   (c) to experience
   (d) to observe
10. Just as a new experiment may suggest an alternative theoretical model, a theoretical advance may suggest what to look for in some for in some experiments. Which of the following experiments can be considered to support this claim?
   (a) Davisson and Germier Experiment
   (b) experimental discovery of positron
   (c) scattering of alpha particle or the gold foil experiment
   (d) Michelson Morley experiment

Very Short:

1. Name that branch of science that deals with the study of Earth.
2. Name that branch of science that deals with the study of stars.
3. Name the scientist and the country of his origin whose field of work was elasticity.
4. The word “Physics” comes from a Greek word. Name the word.

5. The word science has come from a Latin verb. Name the verb.
6. What is the meaning of the verb 'Scientia'?
7. Name the scientist and the country of his origin who received the Nobel Prize for his work on molecular spectra.
8. What is the most incomprehensible thing about the world?
9. Name a great scientist who gave the following comment on science.

   "Science is not just a collection of laws, a catalogue of unrelated facts. It is a creation of the human mind, with its freely invented ideas and concepts."
10. Which famous philosopher gave the following comments on science?

    "We know very little and yet it is astonishing that we know so much, and still more astonishing that so little knowledge of science can give so much power."

Short Questions:

1. Differentiate between Biological and Physical sciences?
2. What is the relation between Physics and Technology?
3. What is the relation between Physics and society?
4. Is Science on speaking terms with humanities?
5. What is the relation between Physics and Technology?
6. Is Physics more of a philosophy or more of a mathematical science?
7. Define Biophysics.
8. Define Technology?

Long Questions:

1. How Physics is related to other sciences?
2. Write a short note on origin and Fundamental forces in nature.
3. Distinguish between the studies in the fields of science, engineering, and technology. Give an outline of the two or three industrial revolutions brought about by advancements in technology over the last twenty-five years or so.

Assertion Reason Questions:

1. Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below
   (a) Both A and R are true, and R is the correct explanation of A.
   (b) Both A and R are true, but R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false and R is also false.

   Assertion: The concept of energy is central to Physics and its expression can be written for every physical system.

   Reason: Law of conservation of energy is not valid for all forces and for any kind of transformation between different forms of energy.
2. Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below
   (a) Both A and R are true, and R is the correct explanation of A.
   (b) Both A and R are true, but R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false and R is also false.

   Assertion: Physics generates new technology.

   Reason: Technology give rise to new physics.

Answer Key

Multiple Choice Answers-

1. **Answer:** (d) Quantum Mechanics
2. **Answer:** (c) Analysis of test results
3. **Answer:** (c) Formulation of a question
4. **Answer:** (d) can be revised if required to fit new phenomenon or data
5. **Answer:** (c) a body of techniques for investigating phenomena, acquiring new knowledge.
6. **Answer:** (b) Some of the most basic features of atomic phenomena.

7. **Answer:** (b) replaced by elliptical orbits to fit the data better
8. **Answer:** (d) Natural Science
9. **Answer:** (a) to know
10. **Answer:** (b) experimental discovery of positron

Very Short Answers:

1. **Answer:** Geology.
2. **Answer:** Astronomy.
3. **Answer:** Robert Hook, England.
4. **Answer:** The word is 'fuses meaning 'Nature'.
5. **Answer:** The name of the Latin verb is 'Scientia'.
6. **Answer:** To 'know'
7. **Answer:** C.V. Raman, India.
8. **Answer:** It is comprehensible.
9. **Answer:** Albert Einstein.
10. **Answer:** Bertrand Russel.

Short Questions Answers:

1. Answer:

S.No.	Biological Science	Physical Science
(i)	They deal with living things.	They deal with non-living things.
(ii)	The study of the biological specimens are conducted at molecular level.	The study of matter are conducted at atomic of ionic levels, i.e., at much smaller level.

2. **Answer:** Broadly speaking, physics and technology both constitute science. Physics is the heart and technology is the body of science.

   The application of the principles of physics for practical purposes becomes technology, e.g.

   - Airplanes fly on the basis of Bernoulli's theorem.
   - Rockets propulsion is based on Newton's second and third laws of motion.
   - The generation of power from the nuclear reactor is based on the phenomenon of controlled nuclear fission.
   - Lasers are based on the population inversion of electrons and so on. Thus, we can say that to some extent technology is applied to Physics.

3. **Answer:** Most of the development made in Physics has a direct impact on society, e.g.

   - Exploration of new sources of energy is of great importance to society.
   - Rapid means of transport are no less important for society.
   - society has-been enriched due to the advances in electronics, lasers, and computers.
   - The development of T.V., radio, satellites, telephone, the telegraph has revolutionized the means of communications which have a direct impact on society and so on.

4. **Answer:** Yes, there is a deep relation between the development of humanity on account of science. Many socio-economic, political, and ethical problems are being tackled and solved by science. Science has greatly helped in developing art and culture. Many musical instruments have been developed due to the theories in Physics. The steam engine is inseparable from the industrial revolution which had a great impact on human civilization.

5. **Answer:** The interplay between physics and technology is the basic to the progress of science which is ever dynamic. Laws in waves and oscillation opened several technological fields which include telescopy, ultrasounds, microscopy, X-rays, and laser. Powerhouses, big cranes, healing devices, etc. work on the principle of electromagnetism. Atomic energy and nuclear weapons are on account of fission. Similarly, Radar, television, the internet, etc. are all based on simple laws of physics. So until there is no theory i.e. physics, there can be no experiment i.e. technology. Hence both are deeply related.

6. **Answer:** Physics is not a purely abstract science devoid of philosophy. Physicists are natural philosophers and Einstein is an example to quote. So Philosophy has provided the backbone to Physics.

7. **Answer:** It is defined as the understanding of biological processes based upon the principles of Physics. For example, spectroscopic techniques

are used to study the constitution of biological molecules and disorders in them. Laws of thermodynamics are used to explain various biological activities of predators and also the activities of molecules.

Hence the application of Physics to bioscience is now well known to all of us.

8. **Answer:** It is defined as the study of newer techniques of producing machines, gadgets, etc. by using scientific discoveries and advancements. It is largely dependent on Physics.

Long Questions Answers:

1. **Answer:** Physics is so important to a branch of science that without the knowledge of Physics, other branches of science cannot make any progress.

   This can be seen from the following:

   (a) **Physics in relation to Mathematics:** The theories and concepts of Physics lead to the development of various mathematical tools like differential equations, equations of motion, etc.

   (b) **Physics in nation to Chemistry:** The concept of interaction between various particles leads to understanding the bonding and the chemical structure of a substance. The concept of X-ray diffraction and radioactivity has helped to distinguish between the various solids and to modify the periodic table.

   (c) **Physics in relation to Biology:** The concept of pressure and its measurement has helped us to know the blood pressure of a human being, which in turn is helpful to know the working of the heart. The discovery of X-rays has made it possible to diagnose the various diseases in the body and fracture in bones.

   The optical and electron microscopes are helpful in the studies of various organisms. Skin diseases and cancer can be cured with the help of high-energy radiation like x-rays, ultraviolet rays.

   (d) **Physics in relation to Geology:** The internal structure of various rocks can be known with the study of the crystal structure. The age of rocks and fossils can be known easily with the help of radioactivity i.e., with the help of carbon dating.

   (e) **Physics in relation to Astronomy:** Optical telescope has made it possible to study the motion of various planets and satellites in our solar system.

   The radio telescope has helped to study the structure of our galaxy and to discover pulsars and quasars (heavenly bodies having star-like structures). Pulsars are rapidly rotating neutron stars. Doppler's effect predicted the expansion of the universe. Kepler's laws are responsible to understand the nature of the orbits of the planets around the sun.

   (f) **Physics in the relation to Meteorology:** The variation of pressure with temperature leads to the forecast of the weather.

   (g) **Physics in relation to Seismology:** The movement of the earth's crust and the types of waves produced help us in studying the earthquake and its effect.

2. **Answer:** These are the following four basic forces in nature:

   (a) Gravitational forces

   (b) Electromagnetic forces

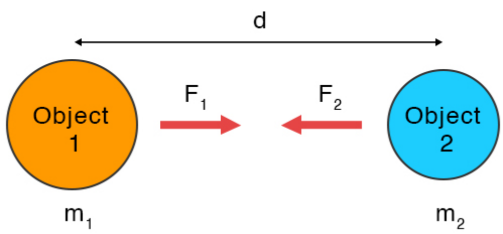
   (c) Weak forces

   (d) Strong force or nuclear forces.

   Some of the important features of these forces are discussed below:

   (a) **Gravitational forces:** These are the forces of attraction between any two bodies in the universe due to their masses separated by a definite distance. These are governed by Newton's law of gravitation given by

$$F = G\frac{m_1 m_2}{r_2}$$



where m_1, m_2 are the masses of two bodies

r = distance between them

G = universal gravitational constant

$= 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Characteristics of gravitational forces:

- They are always attractive. They are never repulsive. They exist between macroscopic as well as microscopic bodies.
- They are the weakest forces in nature.
- They are central forces in nature i.e., they set along the line joining the centres of two bodies.
- They are conservative forces.
- They obey inverse square law i.e., F ∝1r^2 they vary inversely as the square of the distance between the two bodies.
- They are long-range forces i.e., gravitational forces between any two bodies exist even when their distance of separation is quite large.
- The field particles of gravitational forces are called gravitons. The concept of the exchange of field particles between two bodies explains how the two bodies interact from a distance.

(b) **Electromagnetic forces:** They include the electrostatic and magnetic forces. The electrostatic forces are the forces between two static charges while magnetic forces are the forces between two magnetic poles. The moving charges give rise to the magnetic force. The combined action of these forces is called electromagnetic forces.

Characteristics of electromagnetic forces:

- These forces are both attractive as well as repulsive.
- They are central forces in nature.
- They obey inverse square law.
- They are conservative forces in nature.
- These forces are due to the exchange of particles known as photons which carry no charge and have zero rest mass.
- They are 1036 times stronger as compared to gravitational forces and 1011 times stronger than weak forces.

(c) **Strong forces:** They are the forces of nuclear origin. The particles inside the nucleus are charged particles (protons) and neutral particles (neutrons) which are bonded to each other by a strong interaction called nuclear force or strong force. Hence they may be defined as the forces binding the nucleons (protons and neutrons) together in a nucleus. They are responsible for the stability of the atomic nucleus.

They are of three types:

- n-n forces are the forces of attraction between two neutrons.
- p-p forces are the forces of attraction between two protons.
- n-p forces are the forces of attraction between a proton and a neutron.

Characteristics of Nuclear forces:

- They are basically attractive in nature and become repulsive when the distance between nucleons is less than O.S fermi.
- They obey inverse square law.

(a) and

(b) types are the forces that we encounter in the macroscopic world while

(c) and

(d) types are the forces that we encountered in the microscopic world.

(c) **Weak forces:** They are defined as the interactions which take, place between elementary particles during radioactive decay of a radioactive substance. In P-decay, the nucleus changes into a proton, an electron, and a particle called anti-neutrino (which is uncharged). The interaction between the electron and the anti-neutrino is known as weak interaction or weak force.

Characteristics of Weak forces:

- They are 1025 times stronger than the gravitational forces.
- They exist between leptons and leptons, leptons, and mesons. etc.

3. **Answer:**

Science is concerned with the unfolding of the basic aspects of nature. It formulates simple laws and finds the rhythm in nature, materials, and energy. Using basic principles of science, the ways to use them for the production of different kinds of articles is called technology, i.e., it is the application of science. The execution of the application of technology in engineering. The production of articles using machines and implements in engineering. This involves the design, development, and manufacturing of articles.

The most notable technology development in the last 25 years is in the field of information technology, computers, and electronic media. The revolution in information technology has opened up fields on the internet, satellite linking of information systems and services other peripheral developments in the industry.

Computers have changed the face of society and made life easy in several fields. It has improved work efficiency in many segments of the industry and public life. Computers have touched the lives of children playing video games and adults alike. It has helped big organizations like railways, banks, and financial institutions like the insurance sector.

India has become one of the biggest centres of software exports and a big foreign exchange earner. Advance scientific research and industrial designing are being done by computers. TV has entered most Indian houses and community centres-courtesy revolution in electronic media.

The younger generation is mad after the stereo music with CD facilities. The transistors and tape recorders are left far behind. Electronic media has changed the face of the entertainment industry as well as information dissemination. Quick transmission of news, views, and comments are accepted as natural ones by listeners and viewers.

Assertion Reason Answer:

1. (c) A is true but R is false.

   Explanation:

   Law of conservation of energy is always valid for all forces and for any kind of transformation between different forms of energy.

   Therefore, A is true, but R is false.

2. (b) Both A and R are true, but R is not the correct explanation of A.

   Explanation:

   Sometimes physics generates new technology and at others technology gives rise to new physics. Both have desired impact on society. Therefore, both A and R are true, but R is not the correct explanation of A.P

Units and Measurements 2

Units

A unit is an internationally accepted standard for measurements of quantities.

Measurement consists of a numeric quantity along with a relevant unit.

Units for Fundamental or base quantities (like length, time etc.) are called Fundamental units.

Units which are combination of fundamental units are called Derived units.

Fundamental and Derived units together form a System of Units.

Internationally accepted system of units is SystèmeInternationale d' Unites (French for International system of Units) or SI. It was developed and recommended by General Conference on Weights and Measures in 1971.

SI lists 7 base units as in the table below. Along with it, there are two units - radian or rad (unit for plane angle) and steradian or sr (unit for solid angle). They both are dimensionless.

Base Quantity	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermo dynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd

Measurement of Length

Length can be measured using metre scale (10^{-3}m to 102m), verniercallipers (10^{-4}m) and screw gauge and spherometer (10^{-5}m).

Range of Length

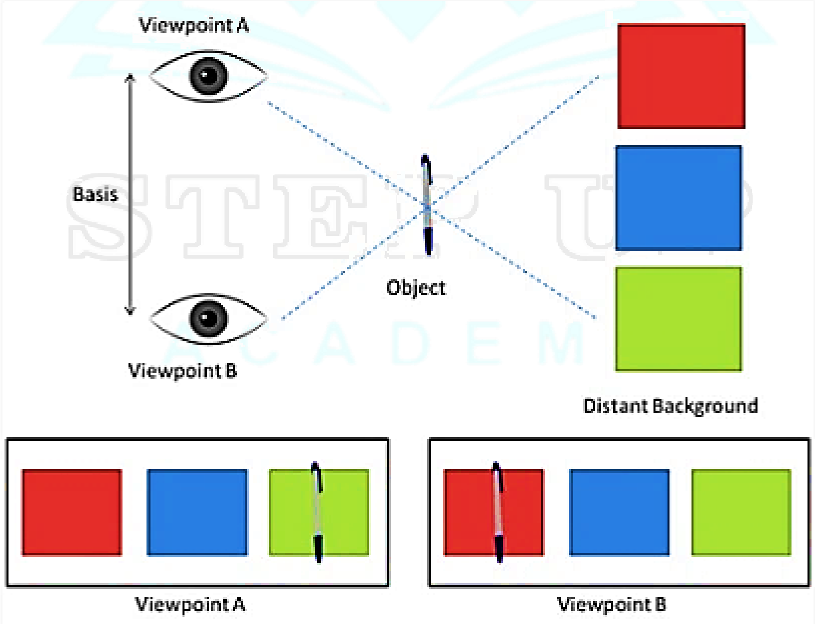
Size of object or distance	Length (m)
Size of proton	10^{-15}
Size of atomic nucleus	10^{-14}
Length of typical virus	10^{-8}
Wavelength of light	10^{-7}
Thickness of paper	10^{-4}
Height of Mount Everest above sea level	10^{4}

Radius of earth	10^7
Distance of moon from earth	10^8
Distance of sun from earth	10^{11}
Distance of pluto from sun	10^{13}
Size of our galaxy	10^{21}
Distance to Andromeda Galaxy	10^{22}
Distance to observable universe boundaries	10^{26}

Objects in the increasing order of their lengths

Measuring large Distances – Parallax Method

Parallax is a displacement or difference in the apparent position of an object viewed along two different lines of sight, and is measured by the angle or semi-angle of inclination between those two lines. Distance between the two viewpoints is called Basis.



Parallax. From viewpoint A the pen appears over green box while from viewpoint B the pen appears over red box.

Measuring distance of a planet using parallax method:

Similarly, $\alpha = d/D$

Where **α = angular size** of the planet (angle subtended by d at earth) and **d** is the **diameter** of the planet.$\alpha$is angle between the direction of the telescope when two diametrically opposite points of the planet are viewed.

What is Speed?

Speed is defined as

The rate of change of position of an object in any direction.

Speed is measured as the ratio of distance to the time in which the distance was covered. Speed is a scalar quantity as it has only direction and no magnitude.

Speed Formula

The formula of speed is given in the table below:

$$s = \frac{d}{t}$$

Where,

- s is the speed in $m.s^{-1}$
- d is the distance traveled in m
- t is the time taken in s

Speed Unit

Following are the units of speed are:

CGS system	$cm.s^{-1}$
SI system	ms^{-1}

Finding the Dimensional Formula of Speed.

The mathematical representation of speed is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Dimensional formula of Distance = $M^0L^1T^0$

Dimensional formula of time = $M^0L^0T^1$

Dividing the dimensional formula of distance by the dimensional formula of time, we get:

$$\frac{M^0L^1T^0}{M^0L^0T^1} = M^0L^0T^{-1}$$

Types of Speed

There are four types of speed and they are:

- Uniform speed
- Variable speed
- Average speed
- Instantaneous speed

Uniform speed: A object is said to be in uniform speed when the object covers equal distance in equal time intervals.

Variable speed: A object is said to be in variable speed when the object covers a different distance at equal intervals of times.

Average speed: Average speed is defined as the uniform speed which is given by the ratio of total distance travelled by an object to the total time taken by the object.

Instantaneous speed: When an object is moving with variable speed, then the speed of that object at any instant of time is known as instantaneous speed.

Measurement of Speed

For the measurement of speed in vehicles, speedometers are used. To measure the distance covered odometers are used. Speed can also be calculated with the help of a graph. The Distance-time graph helps in understanding the speed of an object.

Measurement Mass Weight

Mass is a basic characteristic property of matter. It exists self-sufficiently and is independent of all other parameters such as the temperature, pressure, and the location of the object in space. Atomic mass is the mass of an atom expressed in atomic mass units. The matter has mass and occupies space. These two things are taught to us as soon as we can grasp these concepts. A matter is anything you can touch physically, so everything you see and interact with around you has a mass. Mass is often confused with another parameter. This confusion occurs due to the fact that this parameter is mistakenly used around the globe instead of mass due to its convenience and also due to the fact that we weigh things to find out their mass. This parameter is called weight. Let's explore both of these essential parameters thoroughly.

What is Mass?

Mass by definition refers to the amount of matter in a particular object. This value of the amount of matter i.e. mass of an object is an intrinsic value of that body and it can help us find out various other parameters that are dependent on the mass. Mass determines the strength of its mutual gravitational attraction to other bodies, its resistance to acceleration due to a force, Inertia, and mass can also be used to derive the energy content of a sample through the theory of Relativity using Albert Einstein's $E = mc^2$.

Atomic Mass Unit

For tiny and larger objects we use other units;

- Tonne (Metric Ton) is equal to 1000kg
- The Atomic Mass Unit is used while dealing with atoms and molecules whose masses are so small that the kilogram becomes inconvenient. One atomic mass unit is defined as 1/12th the mass of a Carbon-12 atom. The value of 1 atomic mass unit is obtained as 1.66×10^{-27}

Measurement of mass

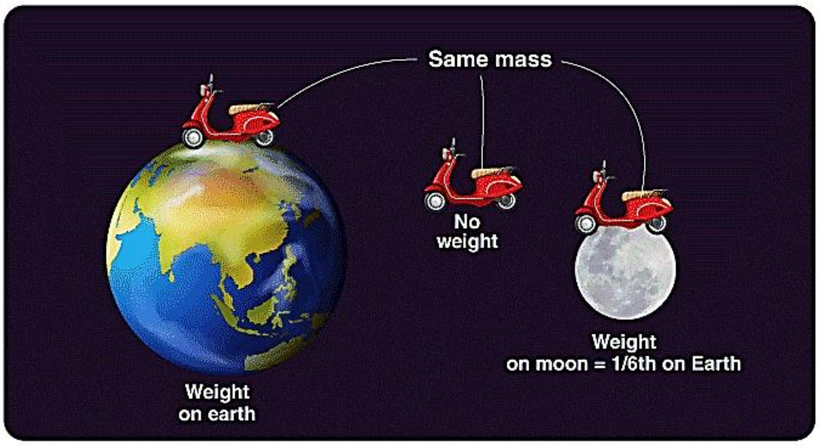
Measurement of mass is most commonly done by a Balance. The unknown mass of a body is compared with a known value of mass. We obtain the value of an unknown mass in terms of a known value of mass. A balance works in space and in places of no gravity as well since changes in gravity affect both the masses on the balance equally.

What is Weight?

Mass is not the same as weight. While mass is the intrinsic property of the body, weight is the measure of the force exerted on the mass of the body due to gravity. Mass refers to a universal value of the object whereas weight is a localized interpretation of the mass of the object. Weight is the effect of gravity and therefore we describe weight with the formula;

$$W = mg$$

Where m is the mass and g is the acceleration due to gravity at that particular location. The unit of measurement of weight is Force, the SI Unit of which is Newton. For example, an object that has a mass of 50 kg experiences a gravitational force i.e. weight which is equal to $50 \times 9.8 = 490$ Newton. So when you tell your friends you weight 50kg you are telling them about your mass and not your weight. The same object albeit with the same mass of 50 kg will weigh 1/6th on the moon what it did on Earth. Weight and mass mean the same thing on Earth since the effects of gravity are fairly constant throughout the Earth. It was upon our venture into space that it became necessary to create a distinction between mass and weight.

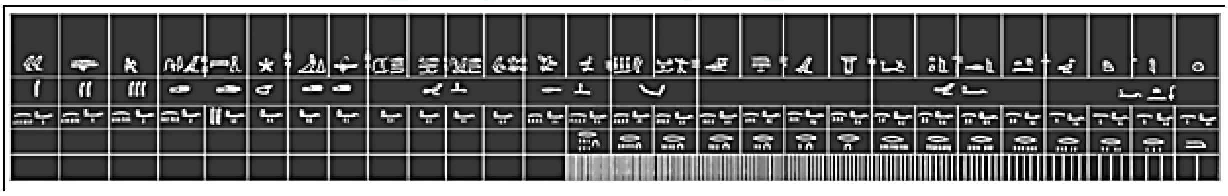


Here is a problem based on the weighing machine, the problem deals with an advanced question on how normal reaction and tension act in an accelerated pulley system when the support is provided by the man which is being measured.

Length

Historically, the human body was used to provide the basis for units of length

- **Inch:** Inch is the measure of the thumb, which was used to measure the length of items small, for example, the seam of a cloth, length of paper, etc.
- **Foot:** Foot is the measure of length typically defined as 15.3 % of the height of a human body with an average height of 160 cm. This unit differed from place to place and trade to trade. This unit was preferred by Roman and Greeks and was typically used to calculate the size of a piece of cloth, the height of human beings and cattle, the size of a building, etc.
- **Cubit:** Cubit is the unit of measurement of length based on the length of the forearm, typically the tip of the middle finger to the elbow bottom. This unit of measurement was preferred by Egyptians and Mesopotamians. Cubit rods have been discovered in the remains of the ancient Egyptian civilization. These rods are usually 20 inches in length, and are divided into seven palms; each palm is further divided into four fingers which are further subdivided.



- **Yard:** Yard is the unit of distance typically based on human paces. A yard is typically equivalent to two cubits or three feet, which is approximately 36 inches.

- **Miles:** A mile is equivalent to a thousand paces, where the pace is equal to two steps, such that the walker is back to the same foot.

A foot comprises 12 inches and three feet comprise a yard. With measurements such as these, it was easy to explain how far the next village was and to find out whether an object will get through a doorway. These measurements also helped the people exchange clothes and wood in a barter system.

Weight

- The grains of wheat were used as a measure of weight due to their approximate standard size. The number of grains of wheat was taken as a standard, which even now is used by some jewellers. One grain is equal to 64.79891 milligrams.

- A measured length of metal used to be kept in the town centre or the temples and copies of the same were distributed among the people of that community. This metal lump was considered as a standard of weight.

Time

- **Sundial:** The movement of the sun in the sky was one of the measures to estimate time, which was done on the basis of length and position of the shadow cast by a vertical stick. Later, the marks were made where the sun's shadow fell, which gave an approximate measure of time of the day consistently. The device came on to be called as a sundial.

- **Water Clock**: The water clock was used to measure time on the basis of the amount of water dripping from a tank. This method was not considered reliable because the flow of water is difficult to be controlled. The device was termed as Clepsydra.

- **Hour Glass**: The hourglass works on the same principle as a water clock, using sand instead of water. It is still found in some places, in a reduced form.

Dimensional formula:

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

Dimensional formula: The expression which shows how and which of the base quantities represent the dimensions of a physical quantity.

Dimensional Formula		
Physical quantity	**Expression**	**Dimensional formula**
Area	length × breadth	$[L^2]$
Volume	Area × height	$[L^3]$
Density	mass / volume	$[ML^{-3}]$
Velocity	displacement/time	$[LT^{-1}]$
Acceleration	velocity / time	$[LT^{-2}]$
Momentum	mass × velocity	$[MLT^{-1}]$
Force	mass × acceleration	$[MLT^{-2}]$
Work	force × distance	$[ML^2T^{-2}]$
Power	work / time	$[ML^2T^{-3}]$
Energy	Work	$[ML^2T^{-2}]$
Impulse	force × time	$[MLT^{-1}]$
Radius of gyration	Distance	$[L]$
Pressure (or) stress	force / area	$[ML^{-1}T^{-2}]$
Surface tension	force / length	$[MT^{-2}]$
Frequency	1 / time period	$[T^{-1}]$
Moment of Inertia	mass × (distance)^2	$[ML^2]$
Moment of force (or torque)	force × distance	$[ML^2T^{-2}]$
Angular velocity	angular displacement / time	$[T^{-1}]$
Angular acceleration	angular velocity / time	$[T^{-2}]$
Angular momentum	linear momentum × distance	$[ML^2T^{-1}]$
Co-efficient of Elasticity	stress/strain	$[ML^{-1}T^{-2}]$
Co-efficient of viscosity	(force × distance) / (area × velocity)	$[ML^{-1}T^{-1}]$
Surface energy	work / area	$[MT^{-2}]$
Heat capacity	heat energy / temperature	$[ML^2T^{-2}K^{-1}]$
Charge	current × time	$[AT]$
Magnetic induction	force / (current × length)	$[MT^{-2}A^{-1}]$
Force constant	force / displacement	$[MT^{-2}]$
Gravitational constant	[force × (distance)^2] / (mass)^2	$[M^{-1}L^3T^{-2}]$
Planck's constant	energy / frequency	$[ML^2T^{-1}]$
Faraday constant	avogadro constant × elementary charge	$[AT\ mol^{-1}]$
Boltzmann constant	energy / temperature	$[ML^2T^{-2}K^{-1}]$

Applications of dimensional analysis:

i. To derive a physical equation.

ii. To verify if the given equation is dimensionally correct.

iii. To find the dimensions of an unknown parameter in the equation.

Class : 11th Physics
Chapter- 2 : Units and Measurements

Units and Measurements

Unit

* Comparison with a certain internationally accepted reference standard is called unit
* Reference standard use to measure physical quantities.

System of units
Set of fundamental and derived units
CGS – Centimeter, Gram, Second
FPS – Foot, Pound, Second
MKS– Metre, Kilogram, Second S.I - System International (Internationally accepted)

Properties
Universal acceptance Non perishable
Well defined
Does not change with time

Types

Derived Units
* The units which are derived from fundamental units are called derived units.
* Physical quantities like speed acceleration have derived units like m/s, m/s^2

Fundamental Units
Mass, length, time, temperature, electric current, luminous intensity and amount of substance have fundamental units. Kilogram, metre, second, Kelvin, ampere, candela and mol respectively.

Supplementary Units
Quantities plane angle and solid angle have radian and steradian as units

Combination of Errors in

Multiplication or Division
If $Z = A/B$
$$\frac{\Delta Z}{Z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Sum of Difference
$$\pm \Delta Z = \pm \Delta A \pm \Delta B$$

Measure Quantity raised to the power
If $Z = A^p \times B^q / C^r$
$$\frac{\Delta Z}{Z} = p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)$$

Error = True Value – Measured Value

Types

Absolute Errror
$$\Delta a_n = a_n - a_{mean}$$
$$a_{mean} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}$$

Relative Errror
$$= \frac{\Delta a_{mean}}{a_{mean}}$$
$$\Delta a_{mean} = \frac{\left(|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|\right)}{n}$$

Percentage Errror
$$\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100$$

Measurement

Significant Figures
All non-zero digits
All zero between two non-zero digits
If the number is less than 1, the zero(s) on the right of decimal point but to the left of the non-digit zero are not significant.
Terminal or trailing zero (s) in a number without a decimal point are not significant
e.g In 020342.010
Here 20342.01 are significant

Dimensions of Physical Quantities
Expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called dimension formula.
e.g:- Force = $[MLT^{-2}]$

Precision
Limit of resolution of quantity measured.

Accuracy
Closeness of the measured value to the true value

Important Questions

Multiple Choice Questions

1. Electron volt is a unit of
   (a) charge
   (b) potential difference
   (c) energy
   (d) magnetic force
2. Light year is a unit of
   (a) time
   (b) distance
   (c) sunlight intensity
   (d) mass
3. Which of the following pairs has the same dimensions?
   (a) specific heat and latent heat
   (b) impulse and momentum
   (c) surface tension and force
   (d) moment of inertia and torque
4. Which of the following sets of quantities has the same dimensional formula?
   (a) Frequency, angular frequency and angular momentum
   (b) Surface tension, stress and spring constant
   (c) Acceleration, momentum and retardation
   (d) Work, energy and torque
5. If C and R denote capacitance and resistance respectively, what will be the dimensions of C x R?
   (a) $[M^0L^0TA^0]$
   (b) $[ML^0TA^{-2}]$
   (c) $[ML^0TA^2]$
   (d) $[MLTA^{-2}]$
6. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are (The path of the particle makes with the x-axis an angle of
   (a) 300
   (b) 450
   (c) 600
   (d) 0
7. Resolution is
   (a) a measure of the bias in the instrument
   (b) None of these
   (c) the smallest amount of input signal change that the instrument can detect reliably
   (d) a measure of the systematic errors
8. Fundamental or base quantities are arbitrary. In SI system these are
   (a) length, force, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity
   (b) length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity
   (c) as length, mass, time, electric charge, thermodynamic temperature, amount of substance, and luminous intensity
   (d) length, mass, force, electric current, thermodynamic temperature, amount of substance, and luminous intensity
9. Unit for a fundamental physical quantity is
   (a) defined as best of various reference standards
   (b) the smallest measurable value of the physical quantity
   (c) defined as average various reference standards
   (d) reference standard for the physical quantity
10. The volume of a cube in m^3 is equal to the surface area of the cube in m^2. The volume of the cube is
   (a) $64\ m^3$
   (b) $216\ m^3$
   (c) $512\ m^3$
   (d) $196\ m^3$

Very Short:

1. If the size of the atom were enlarged to the tip of the sharp pin, how large would the height of Mount Everest be?
2. What does the LASER mean?
3. If the Universe were shrunk to the size of the Earth, how large would the Earth be on this scale?

4. A research worker takes 100 careful readings in an experiment. If he repeats the same experiment by taking 400 readings, then by what factor will the probable error be reduced?
5. What is the number of significant figures in 0.06070?
6. The density of a cube is calculated by measuring the length of one side and its mass. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then what is the maximum possible error in the measurement of density?
7. The mass of a body as measured by two students is given as 1.2 kg and 1.23 kg. Which of the two is more accurate and why?
8. Do the inertial and gravitational masses of ordinary objects differ in magnitude?
9. Are S.I. units Coherent? Why?
10. Do A.U. And Å represents the same magnitudes of distance?

Short Questions:

1. If the size of a nucleus is scaled up to the tip of a sharp pin, what roughly is the size of an atom?
2. (a) What do you mean by physical quantity?
   (b) What do you understand by:
   (i) Fundamental physical quantities?
   (ii) Derived physical quantities?
3. (a) Define the unit of a physical quantity.
   (b) Define
   (i) Fundamental units.
   (ii) Derived units.
4. Define one Candela.
5. What is the advantage of choosing wavelength of light radiation as standard of length?
6. Which type of phenomenon can be used as a measure of time? Give two examples of it.
7. Find the number of times the heart of a human being beats in 10 years. Assume that the heartbeats once in 0.8s.
8. Why it is not possible to establish a physical relation involving more than three variables using the method of dimensions?

Long Questions:

1. State the rules for writing the units of physical quantities in the S.I. system.
2. Explain the Triangular method.
3. What are the uses of dimensional analysis? Explain each of them.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct

   Assertion: Dimensional constants are the quantities whose values are constant.

   Reason: Dimensional constants are dimensionless.
2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct

   Assertion: Parallax method cannot be used for measuring distances of stars more than 100 light years away.

   Reason: Because parallax angle reduces so much that it cannot be measured accurately.

Case Study Questions:

1. Measurement of Physical Quantity All engineering phenomena deal with definite and measured quantities and so depend on the making of the measurement. We must be clear and precise in making these measurements. To make a measurement, magnitude of the physical quantity (unknown) is compared. The record of a

measurement consists of three parts, i.e., the dimension of the quantity, the unit which represents a standard quantity and a number which is the ratio of the measured quantity to the standard quantity.

1. A device which is used for measurement of length to an accuracy of about 10^{-5m}, is
   (a) Screw gauge
   (b) Spherometer
   (c) Vernier callipers
   (d) Either (a) or (b)
2. Which of the technique is not used for measuring time intervals?
   (a) Electrical oscillator
   (b) Atomic clock
   (c) Spring oscillator
   (d) Decay of elementary particles
3. The mean length of an object is 5cm. Which of the following measurements is most accurate?
   (a) 4.9cm
   (b) 4.805cm
   (c) 5.25cm
   (d) 5.4cm
4. If the length of rectangle l = 105.cm, breadth b = 21.cm and minimum possible measurement by scale = 01.cm, then the area is
   (a) $22.0cm^2$
   (b) $21.0cm^2$
   (c) $22.5cm^2$
   (d) $21.5cm^2$
5. Age of the universe is about 1010 yr., whereas the mankind has existed for 106 yr. For how many seconds would the man have existed if age of universe were 1 day?
   (a) 9.2 s
   (b) 10.2 s
   (c) 8.6 s
   (d) 10.5 s

2. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus first digit that is uncertain. The digits that are known reliably plus the first uncertain digit are known as significant digits or significant figures.

e.g., When a measured distance is reported to be 374.5m, it has four significant figures 3, 7, 4 and 5. The figures 3, 7, 4 are certain and reliable, while the digit 5 is uncertain. Clearly, the digits beyond the significant digits reported in any result are superfluous.

i. In 4700m, significant digits are
   (a) 2
   (b) 3
   (c) 4
   (d) 5

ii. To determine the number of significant figures, scientific notation is
   (a) a^b
   (b) $ab \times 10^b$
   (c) $a \times 10^2$
   (d) $a \times 10^4$

iii. 5.74 g of a substance occupies $1.2cm^3$ Express its density by keeping the significant figures in view.
   (a) 4.9 g cm^{-3}
   (b) 5.2 g cm^{-3}
   (c) 4.8 g cm^{-3}
   (d) 4.4 g cm^{-3}

iv. Choose the correct option.
   (a) Change in unit does not change the significant figure.
   (b) 4 700. m=4700 mm, here there is a change of significant number from 4 to 2 due to change in unit.
   (c) 4700 4 700 103 m =′. m, here there is change in numbers of significant numbers.
   (d) Change in unit changes the number of significant figure.

v. Consider the following rules of significant figures.
   I. All the non-zero digits are significant.
   II. All the zeroes between two non-zero digits are significant.
   III. The terminal or trailing zero(s) in a number without a decimal point are significant.

Which of the above statement(s) is/are? correct?
(a) I and II
(b) II and III
(c) I and III
(d) All of these

Answer Key

Multiple Choice Answers

1. **Answer:** (c) energy
2. **Answer:** (b) distance
3. **Answer:** (b) impulse and momentum
4. **Answer:** (d) Work, energy, and torque
5. **Answer:** (a) $[M^0L^0TA^0]$
6. **Answer:** (c) 600
7. **Answer:** (d) a measure of the systematic errors
8. **Answer:** (b) length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity
9. **Answer:** (d) reference standard for the physical quantity
10. **Answer:** (b) 216 m^3

Very Short Answers:

1. **Answer:** 10^{10} m.
2. **Answer:** It stands for Light Amplification by Stimulated Emission of Radiation.
3. **Answer:** 10^{-11} m (size of an atom.).
4. **Answer:** By a factor of 4.
5. **Answer:** 4.
6. **Answer:** 3% + 3 × 2% = 9%.
7. **Answer:** The second measurement is more accurate as it has been made to the second decimal point.
8. **Answer:** No.
9. **Answer:** Yes, because all the derived units in this system can be obtained by multiplying or dividing a certain set of basic units.
10. **Answer:** No, 1 A.U. = 1.496×10^{11} m and 1 Å = 10^{10} m.

Short Questions Answers:

1. **Answer:** The size of a nucleus is in the range of 10^{-15} m to 10^{-14} m. The tip of a sharp pain may be taken to be in the range of 10^{-5} m to 10^{-4} m. Thus, we are scaling up the size of the nucleus by a factor of $10^{-5}/10^{-15} = 10^{10}$. An atom roughly of size 10-10 m will be scaled up to a rough size of $10^{-10} \times 10^{10} = 1$ m. Thus, nucleus in an atom is as small in size as the tip of a sharp pin placed at the center of a sphere of radius about a meter.
2. **Answer:** It is defined as a quantity that can be measured, e.g., mass, length, time, etc.

   (b)

   (i) They are defined as those quantities which cannot be expressed in terms of other quantities and are independent of each other, e.g., mass, length, time.

   (ii) They are defined as the quantities which can be expressed in terms of fundamental quantities, e.g., velocity, acceleration, density, pressure, etc.
3. **Answer:** It is defined as the reference standard used to measure a physical quantity.

   (b)

   (i) They are defined as the units of fundamental quantities. They are independent of each other and are expressed by writing the letter of the fundamental quantity in a parenthesis. e.g., Fundamental units of mass, length and time are [M], [L], [T] respectively.

   (ii) They are defined as those units which can be derived from fundamental units. They are expressed by writing the symbol of a derived quantity in a parenthesis.

   e.g., D.U. of velocity = [u]

   acceleration = [a]

   pressure = [P]

   work = [W] and so on.
4. **Answer:** It is defined as the luminous intensity in a perpendicular direction of a surface of $\frac{1}{600,000}$ square meter area of a black body at a temperature of freezing platinum (1773°C) under a pressure of 101,325 N/m^2.
5. **Answer:**
   - It can be easily made available in any standard laboratory as Krypton is available everywhere.
   - It is well defined and does not change with temperature, time, place or pressure, etc.
   - It is invariable.
   - It increases the accuracy of the measurement of length (1 part in 10^9).

6. **Answer:** Any phenomenon that repeats itself regularly at equal intervals of time can be used to measure time.

   The examples are:

   - Rotation of earth – the time interval for one complete rotation is called a day.
   - Oscillations of a pendulum.

7. **Answer:** In 0.8 s, the human heart makes one beat.

   $\therefore$ In 1 s, the human heart makes $= \frac{1}{0.8} = \frac{10}{8}$ beats.

   ∴ In 10 years, the human heart makes

   $= \frac{10}{8} \times 365 \times 24 \times 60 \times 60$ beats.

   $= 3.942 \times 10^8$ beats.

8. **Answer:** The dimensional analysis fails to derive a relation involving more than three unknown variables. The reason is that there will be more than three unknown factors in that case whose values cannot be determined from the three relations which we get by comparing the powers of M, L, and T.

Long Questions Answers:

1. **Answer:** While writing the units of physical quantities following rules are followed with S.L units:

   (1) The S.l. units are written in the form of symbols after the number i.e., number of time, the unit is contained in the physical quantity so that physical quantity = nu

   With symbols, certain rules are laid down:

   - Units in symbols are never written in plural i.e., meters is only m and not ms, years is y.
   - The units based on the name of the scientists are written beginning with small letters and with capital letters in symbolic form viz, weber (Wb), newton (N), etc.
   - No full stop is used at the end of the symbol.
   - Symbols of units not based on the name of scientists are written as small letters viz. kilogram (kg), second (s), etc.

   (2) Bigger and smaller number of units are represented with symbols corresponding to the power of 10 viz. 106 is mega (M), 10^{12} is Tera (T), 10^{-3} is milli (m), 10^{-9} is nano (n), etc.

   (3) All units are written in numerator viz. kg/m^3 is kg m, Nm^{2}c^2.

   (4) The units are written within parenthesis in graphs below the corresponding taxes viz. (ms^{-1}) and (s) in the velocity-time graph.

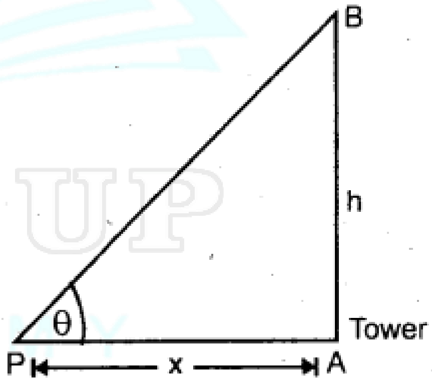
   (5) Units of a similar physical quantity can be added or subtracted.

2. **Answer:** It is used to measure the distance of an accessible or inaccessible hill or a tower by measuring the angle which the object makes at point P (say)

   Let x = distance y of point P from the foot of tower = PA .

   ∴ h = x tan θ

   It is also used to measure the distance of an inaccessible object eg. a tree on the other bank of a river.



Let h = height of the inaccessible object.

Let θ1, θ2 = be the angle made at P and Q by the object.

Let PA = d, PQ = x.

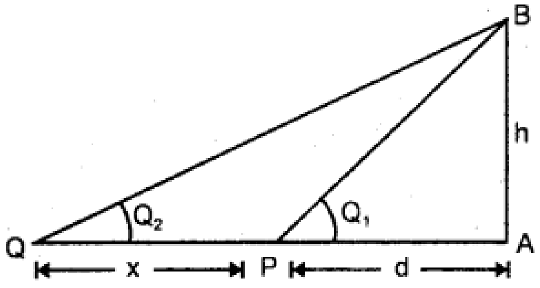
∴ In ΔPAB and ΔQAB,

$$d = h \cot \theta \quad ....(i)$$

and $$d + x = h \cot \theta_2 \quad ....(ii)$$

(ii) and (i) gives, $$x = h (\cot \theta_2 - \cot \theta_1)$$

∴ $$h = x/(\cot \theta_2 - \cot \theta_1)$$



3. **Answer:**

Dimensional analysis is used for:

(a) checking the dimensional correctness of the given physical equation or relation.

(b) converting one system of units to another system.

(c) deriving the relationship between various physical quantities.

(a) checking of the dimensional correctness of a physical relationship is done by using the principle of homogeneity of dimensions. If the dimensions of M, L, T of each term on R.H.S. are equal to the dimensions of M, L, T of each term on L.H.S., then the given-physical relation is dimensionally correct, otherwise wrong.

(b) conversion: It is based on the fact that the magnitude of a physical quantity remains the same whatever may be the system of units, i.e., $n_1u_1 = n_2u_2$.

or $$n_2 = n_1\frac{u_1}{u_2}$$

where $$u_1 = M_1^a L_1^b T_1^c$$

and $$u_2 = M_2^a L_2^b T_2^c$$

are the units of M, L, T in the first and second system of units of a physical quantity having dimensions of M, L, T, and a, b, c respectively.

$$\therefore \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c \quad \text{....(i)}$$

Thus, if fundamental units of both systems, dimensions of the quantity, and its numerical value n1 in one system, are known then we can easily calculate n2 in another system.

(c) Derivation of a relationship between various physical quantities is based on the principle of homogeneity of dimensions.

Following are the steps used:

- We must Know the physical quantities (say p, q, r) upon which a physical quantity say x depends.
- We must know the dimensions of p, q, r say a, b, c respectively.

Then we write $x \propto p^a$

$$\propto q^b$$

$$\propto r^c$$

or $$x \propto p^a q^b r^c$$

or $$x \propto kp^a q^b r^c \quad \text{....(i)}$$

- Now, write the dimensions of each physical quantity on both sides of the equation
- and compare the powers of M, L, T to find a, b, c. Putting values of a, b, c in the equation
- we get the required relation.

Assertion Reason Answer:

1. (c) Assertion is correct, reason is incorrect

Explanation:

Dimensional constants are not dimensionless.

2. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

As the distance of star increases, the parallax angle decreases, and great degree of accuracy is required for its measurement. Keeping in view the practical limitation in measuring the parallax angle, the maximum distance of a star we can measure is limited to 100 light years.

Case Study Answer:

1. i. (d) Either (a) or (b)

Explanation:

A screw gauge and a spherometer can be used to measure length accurately as less as

$10m^{-5}$

ii. (c) Spring oscillator

Explanation:

Spring oscillator cannot be used to measure time intervals.

iii. (a) 4.9 cm

Explanation:

Given, length, l = 5cm Now, checking the errors with each options one-by-one, we get

$\Delta l_1 = 5 - 4.9 = 0.1$ cm

$\Delta l_2 = 5 - 4.805 = 0.195$ cm

$\Delta l_3 = 5.25 - 5 = 0.25$ cm

$\Delta l_4 = 5.4 - 5 = 0.4$ cm

Error Δl_1 is least.

Hence, 4.9cm is most precise or accurate.

iv. (a) 22.0cm^2

Explanation:

Area of rectangle, A = Length´ Breadth

So, A lb = 10. × 5 = 22. 05cm^2

Minimum possible measurement of

scale = 01. cm.

So, area measured by scale = 22 0. cm^2

v. (c) 8.6 s

Explanation:

$$\text{Magnification in time} = \frac{\text{Age of mankind}}{\text{Age of universe}}$$

$$= \frac{10^6}{10^{10}} = 10^{-4}$$

Apparent age of mankind = 10–4 × 1 day

= 10–4 × 86400 s

= 8.64 s ≈ 8.6 s

2. i (a) 2

Explanation:

As, we know that the terminal or trailing zero(s) in a number without a decimal point are not significant. So, 4700m has two significant figures.

ii. (b) ab × 10^b

Explanation:

Every number is expressed as ab × 10^b, where a is a number between 1 & 10 and b is any positive or negative exponent (or power) of 10.

iii. (c) 4.8 g cm^{-3}

Explanation:

There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence, the density should be expressed to only 2 significant figures.

$$\text{Density} = \frac{5.74}{1.2} = 4.8 \text{ g cm}^{-3}$$

iv. (a) Change in unit does not change the significant figure.

Explanation:

There is no change in number of significant figures on changing the units. For it, the convention is that we write,

4700 m = 4700 × 10^{3}m

This convention ensures no change in number of significant numbers.

v. (a) I and II

Explanation:

Following rules of significant figures are

I. All the non-zero digits are significant.

II. All the zeroes between two non-zero digits are significant, no matter where the decimal point is, if at all.

III. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 123m =12300cm =123000 mm has three significant figures, the trailing zero(s) being not significant.

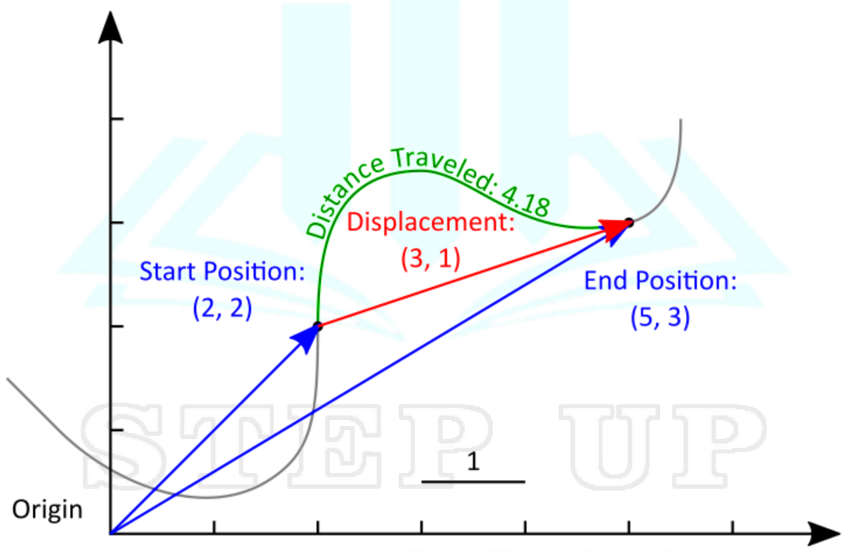
Motion in a Straight Line | 3

If an object changes its position with respect to its surroundings with time, then it is called in motion. It is a change in the position of an object over time. Motion in a straight line is nothing but linear motion. As the name suggests, it's in a particular straight line, thus it can be said that it uses only one dimension.

There are two branches in physics that examine the motion of an object.

- **Kinematics:** It describes the motion of objects, without looking at the cause of the motion.
- **Dynamics:** It relates the motion of objects to the forces which cause them.

Position, Distance, Displacement:



Position: Position of an object is always expressed with respect to some reference point which we generally account to as origin. To express the change in position, we consider two physical quantities.

Distance: It refers to the actual path traversed by the object during the course of motion.

Displacement: It refers to the difference between the final and initial positions of the object during the course of motion.

Distance	Displacement
It refers to the actual path traversed by the object during the course of motion.	It refers to the difference between the initial and the final positions $\Delta x = x_2 - x_1$, where, x_2 and x_1 are final and initial position respectively.
It is a scalar quantity.	It is a vector quantity.
The distance covered by an object during the course of motion can never be negative or zero. It is always positive.	The displacement of an object can be positive, negative or zero during the course of motion.

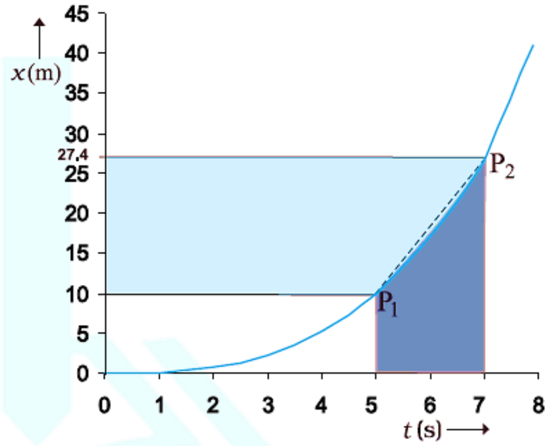
The distance travelled is either equal to or greater than displacement and is never less than magnitude of displacement.	The magnitude of displacement is less than or equal to the distance travelled during the course of motion.
The distance is dependent upon the path travelled by the object.	The magnitude of displacement is not dependent on the path taken by an object during the course of motion.

Average Velocity and Average Speed

Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs:

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_2 and x_1 are the positions of the object at time t_2 and t_1, respectively. Here the bar over the symbol for velocity is a standard notation used to indicate an average quantity. The SI unit for velocity is m/s or m s^{-1}, although km h^{-1} is used in many everyday applications.



The average velocity is the slope of line P1 P2

The portion of the x-t graph between t = 0 s and t = 8 s is blown up and shown in Fig. As seen from the plot, the average velocity of the car between time t = 5 s and t = 7 s i :

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(27.4 - 10.0)\text{m}}{(7-5)s} = 8.7m\,s^{-1}$$

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place:

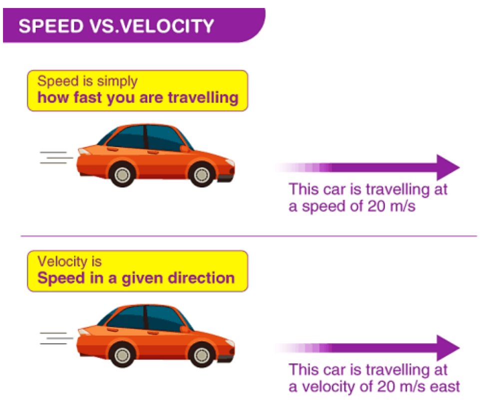
$$Average\ speed = \frac{Total\ path\ length}{Total\ time\ interval}$$

Average speed has obviously the same unit (m s^{-1}) as that of velocity. But it does not tell us in what direction an object is moving. Thus, it is always positive (in contrast to the average velocity which can be positive or negative). If the

motion of an object is along a straight line and in the same direction, the magnitude of displacement is equal to the total path length.In that case, the magnitude of average velocity is equal to the average speed.

However, this is not always the case. The average velocity gives an idea on how fast an object has been moving over a given interval but does give an idea on how fast it moves at different instants of time during that interval.

Difference Between Speed and Velocity:



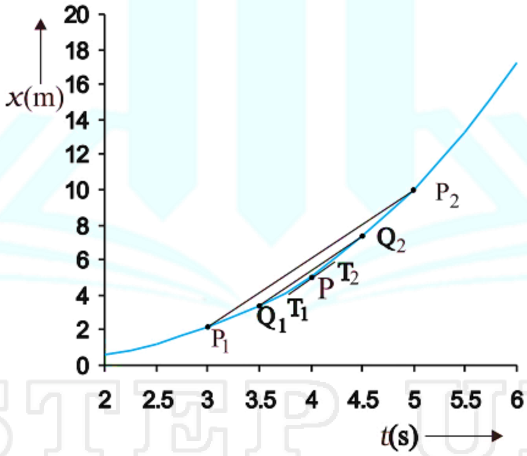
Speed	Velocity
It refers to the total path length travelled divided by the total time interval during which the motion has taken place.	It refers to the change in position or displacement divided by the time intervals, in which this displacement occurs.
It is a scalar quantity.	It is a vector quantity.
It is always positive during the course of the motion.	It may be positive, negative or zero during the course of the motion.
It is greater than or equal to the magnitude of velocity.	It is less than or equal to the speed.

Instantaneous Velocity and Instantaneous Speed:

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words,

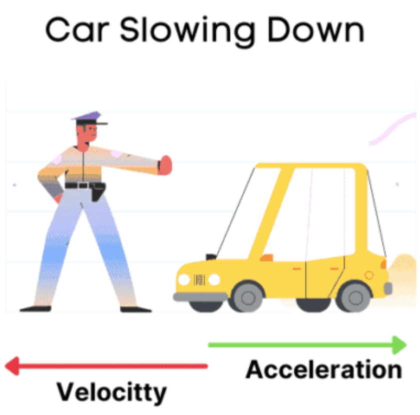
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

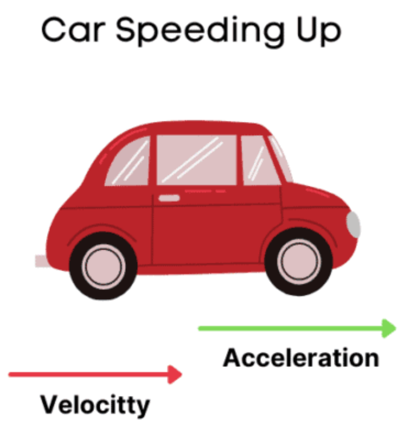
where the symbol $\lim\limits_{\Delta t \to 0}$ stands for the operation of taking limit as $\Delta t \to 0$ of the quantity on its right. In the language of calculus, the quantity on the right hand side is the differential coefficient of x with respect to t and is denoted by $\frac{dx}{dt}$. It is the rate of change of position with respect to time, at that instant.



Determining velocity from position-time graph. Velocity at t = 4 s is the slope of the tangent to the graph at that instant.

Acceleration





The **average acceleration** $\bar{a}$ over a time interval is defined as the change of velocity divided by the time interval:

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where v_2 and v_1 are the instantaneous velocities or simply velocities at time t_2 and t_1. It is the average change of velocity per unit time. The SI unit of acceleration is m s^{-2}.

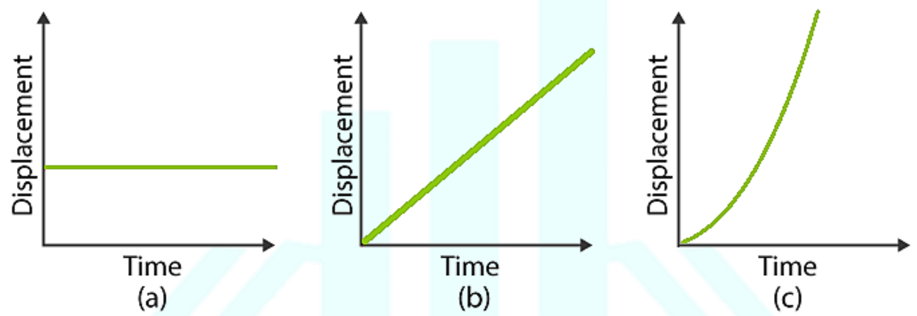
Instantaneous Acceleration: Mathematically, instantaneous acceleration can be expressed in the same way as the instantaneous velocity as follows:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

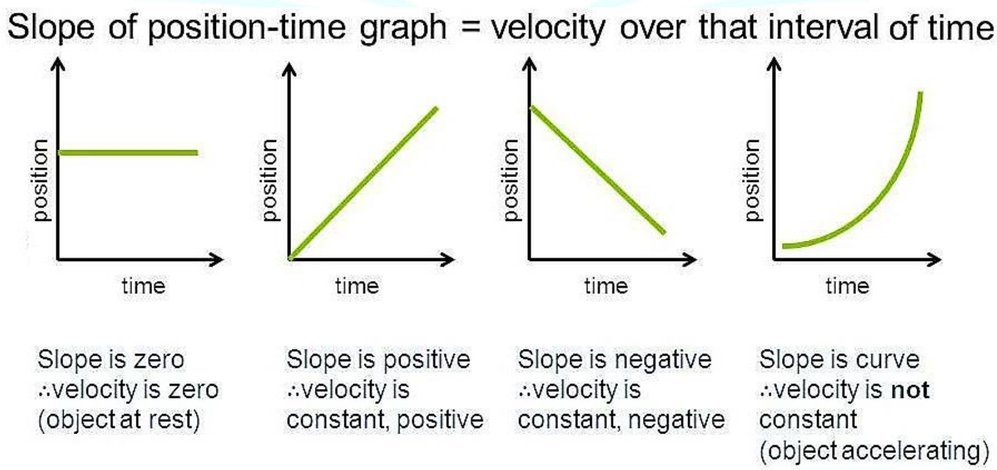
The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion, acceleration is zero and the x-t graph is a straight line inclined to the time axis and the v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola while the v-t graph is a straight line inclined to the time axis.

Different Graphs of Motion

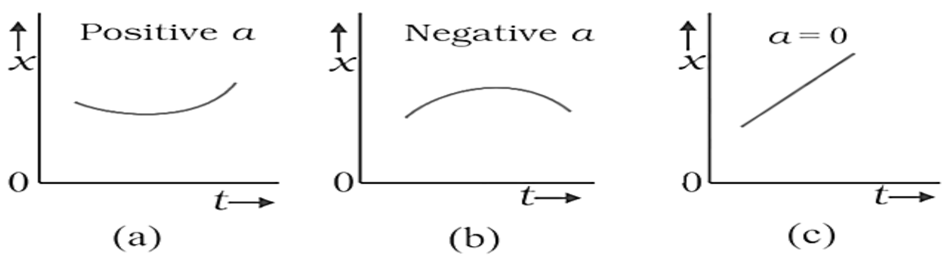
Displacement - Time Graph



Velocity - Time Graph



Acceleration - Time Graph



Position-time graph for motion with (a) positive acceleration; (b) negative acceleration, and (c) zero acceleration

Uniform motion:

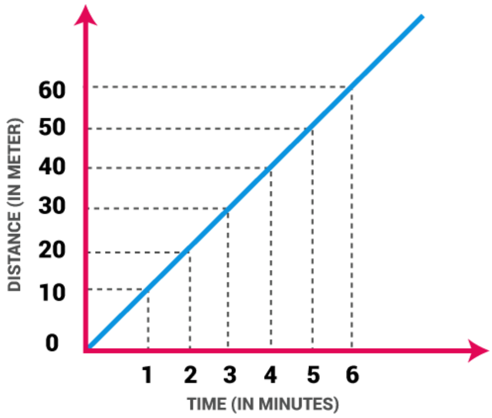
If a body is said to be in uniform motion, the body completes equal distances in equal intervals of time.

Here, velocity is constant during the course of motion. Also, acceleration is zero during the course of motion.

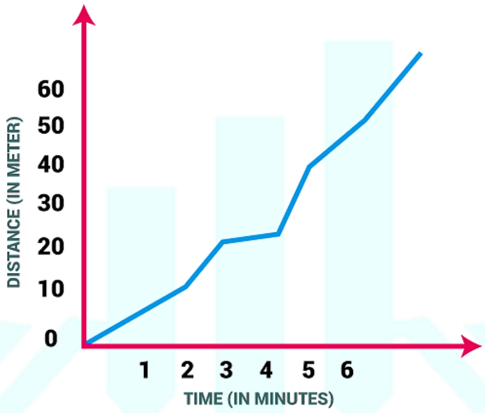
Non-Uniform motion:

If a body undergoes non-uniform motion, the body is said to be in uniformly accelerated motion. Here, the magnitude of velocity increases or decreases with the passage of time. Also, acceleration would not be zero as it undergoes accelerated motion.

UNIFORM MOTION GRAPH



NON-UNIFORM MOTION GRAPH



Top Formulae

Displacement	$\Delta x = x_2 = x_1$
Average velocity	$\bar{v} = \frac{\text{Displacement}}{\text{time interval}} = \frac{\Delta x}{\Delta t}$
Instantaneous velocity	$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
Average acceleration	$\bar{a} = \frac{\Delta v}{\Delta t}$
Instantaneous acceleration	$a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$
Kinematic equations of motion	$v = v_0 + at$ $x = v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^2 \quad 2ax$

Equation of Kinematics:

These are the various relations between u, v, a, t and s for the particle moving with uniform acceleration where the notations are used as :

u = Initial velocity of the particle at time t = 0 sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

s_n = Distance travelled by the body in n^{th} sec

(1) When particle moves with zero acceleration

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) v = u, s = ut [As a = 0]

(2) When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion (in scalar from) | Equation of motion (in vector from)

Equations of motion (in scalar from)	Equation of motion (in vector from)
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

Motion of Body Under Gravity (Free Fall):

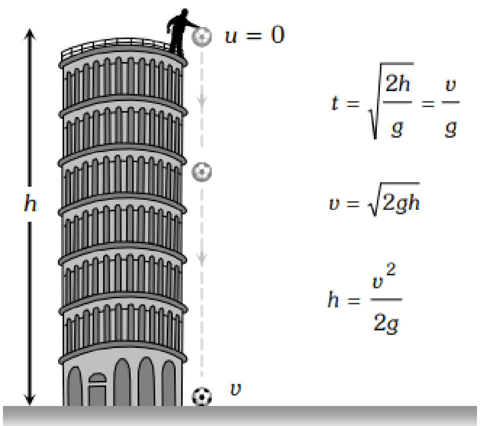
The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall.

An ideal example of one-dimensional motion is motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) If a body is dropped from some height (initial velocity zero)

(i) **Equations of motion :** Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have



$u = 0$ [As body starts from rest]

$a = +g$ [As acceleration is in the direction of motion]

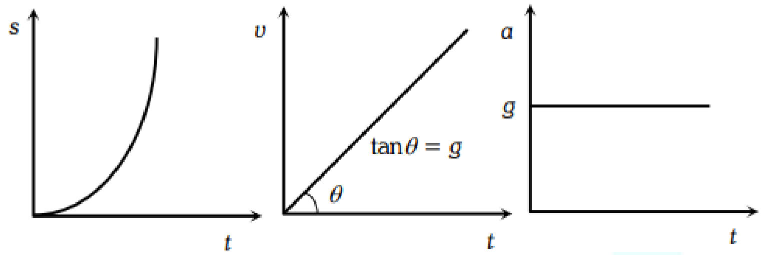
$v = gt$ ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

$v^2 = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)

(ii) Graph of distance, velocity and acceleration with respect to time :



(iii) As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

(iv) The distance covered in the nth sec, $h_n = \frac{g}{2}(2n-1)$

So distance covered in 1st, 2nd, 3rd sec, etc., will be in the ratio of 1 : 3 : 5, i.e., odd integers only.

(2) If a body is projected vertically downward with some initial velocity

Equation of motion : $v = u + gt$

$h = ut \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

(3) If a body is projected vertically upward

(i) **Equation of motion :** Taking initial position as origin and direction of motion (i.e., vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

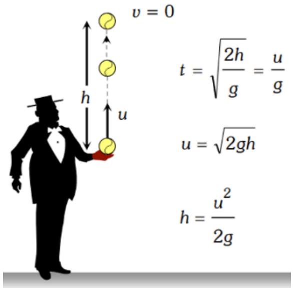
$v = u - gt$; $h = ut - \frac{1}{2}gt^2$; $v^2 = u^2 - 2gh$; $h_n = u + \frac{g}{2}(2n-1)$

(ii) For maximum height $v = 0$

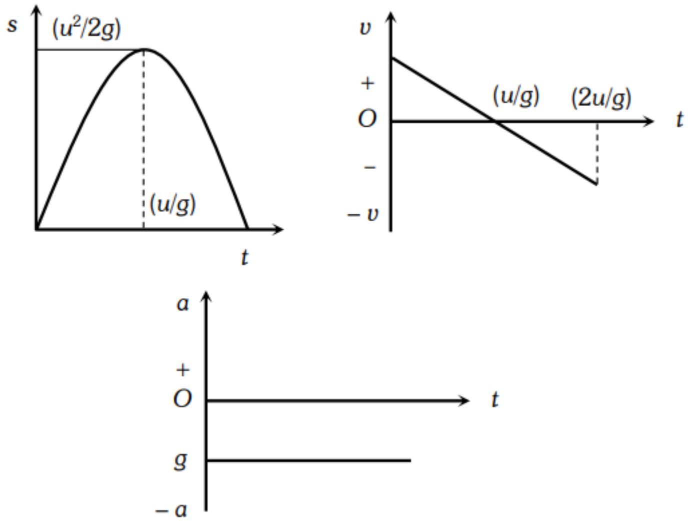
So from above equation $u = gt$,

$h = \frac{1}{2}gt^2$

and $u^2 - 2gh$



(iii) Graph of displacement, velocity and acceleration with respect to time (for maximum height) :



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t=\sqrt{(2h/g)}$ and $v=\sqrt{2gh}$.

(5) In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g

∴ Total time of flight T = $t_1 + t_2 = \frac{2u}{g}$

(6) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(7) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$.

Let u is the initial velocity of body then time of ascent $t_1=\frac{u}{g+a}$ and $h=\frac{u^2}{2(g+a)}$.

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

So $h=\frac{1}{2}(g-a)t_2^2$

$$\Rightarrow \frac{u^2}{2(g-a)}=\frac{1}{2}(g-a)t_2^2$$

$$\Rightarrow t_2=\frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t1 and t2 we can say that t2 > t1 since (g + a) > (g – a).

Motion with Variable Acceleration:

(i) If acceleration is a function of time

a = f(t) then $v=u+\int_0^t f(t)dt$

and $s=ut+\int_0^t\left(\int f(t)dt\right)dt$

(ii) If acceleration is a function of distance

$a = f(x)$ then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)dt$

(iii) If acceleration is a function of velocity

$a = f(t)$ then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{vdv}{f(v)}$

Motion In Two Dimension:

The motion of an object is called two dimensional, if two of the three co-ordinates required to specify the position of the object in space, change w.r.t time.

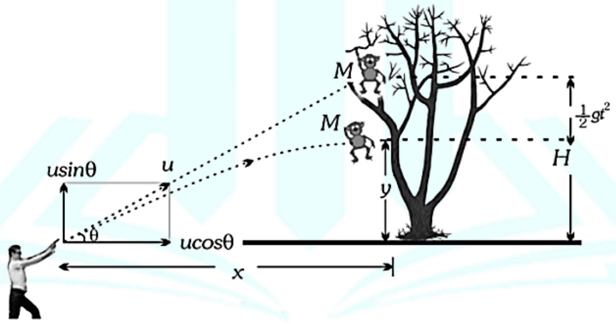
In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun etc.

Two special cases of motion in two dimension are

1. Projectile motion 2. Circular motion

Introduction of Projectile Motion:

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion.

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

Projectile:

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile.

Example:

(i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

Assumptions of Projectile Motion:

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

Principle of Physical Independence of Motions:

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.

(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

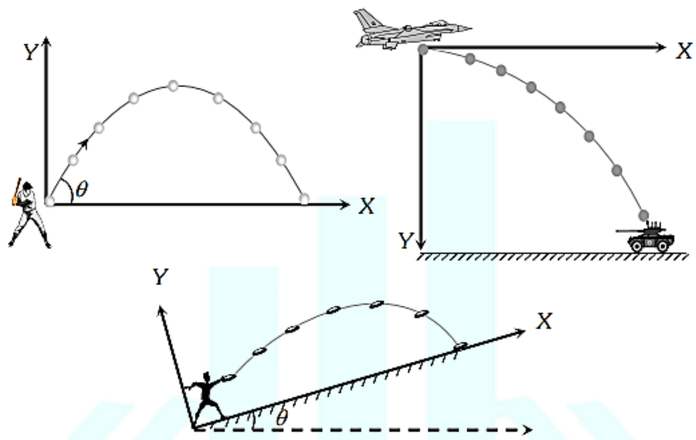
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated or retarded motion.

Types of Projectile Motion:

(1) Oblique projectile motion

(2) Horizontal projectile motion

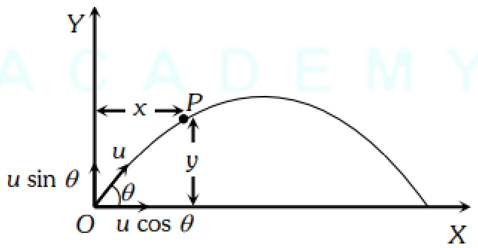
(3) Projectile motion on an inclined plane



Oblique Projectile:

In projectile motion, horizontal component of velocity (u cosθ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sinθ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(1) **Equation of trajectory :** A projectile is thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components.



v cos θ component along X–axis and u sin θ component along Y–axis.

For horizontal motion $x = u\cos\theta \times t \Rightarrow t = \dfrac{x}{u\cos\theta}$ ...(i)

For vertical motion $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ ...(ii)

From equation (i) and (ii)

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

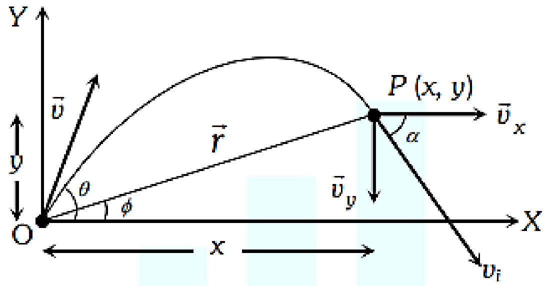
$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola $y = ax - bx^2$

Note : Equation of oblique projectile also can be written as

$$y = x\tan\theta - \left[1 - \frac{x}{R}\right] \text{ (where R = horizontal range = } \frac{u^2\sin 2\theta}{g}\text{)}$$

(2) **Displacement of projectile $(\vec{r})$:** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.



$$\vec{r} = x\hat{i} + y\hat{j} \qquad \text{...(i)}$$

The horizontal distance covered during time t is given as

$$x = v_x t \Rightarrow x = u\cos\theta t \qquad \text{...(ii)}$$

The vertical velocity of the particle at time t is given as

$$v_y = (v_0)_y - gt \qquad \text{...(iii)}$$

Now the vertical displacement y is given as

$$y = u\sin\theta t - 1/2\, gt^2 \qquad \text{...(iv)}$$

Putting the values of x and y from equation (ii) and equation (iv) in equation (i) we obtain the position vector at any time t as

$$\vec{r} = (u\cos\theta)t\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

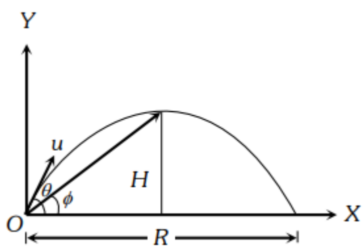
$$\Rightarrow r = \sqrt{(u\cos\theta)^2 + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)^2}$$

$$r = ut\sqrt{1 + \left(\frac{gt}{2u}\right)^2 - \frac{gt\sin\theta}{u}} \text{ and } \phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}\left(\frac{ut\sin\theta - \frac{1}{2}gt^2}{(ut\cos\theta)}\right) \text{ or } \phi = \tan^{-1}\left(\frac{2u\sin\theta - gt}{2u\cos\theta}\right)$$

Note : The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as

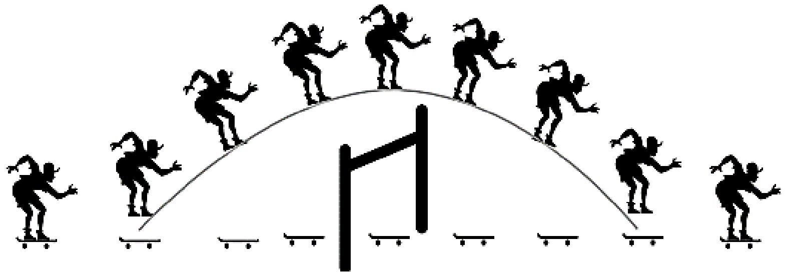
$$\tan\phi = \frac{1}{2}\tan\theta$$



(3) **Instantaneous velocity v :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example : When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.



Let v_i be the instantaneous velocity of projectile at time t, direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v}_i = v_x i + v_y \hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$v_i = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

Direction of instantaneous velocity $\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{u\sin\theta - gt}{u\cos\theta}$

or $\alpha = \tan^{-1}\left[\tan\theta - \dfrac{gt}{u}\sec\theta\right]$

(4) **Change in velocity :** Initial velocity (at projection point) $\vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Final velocity (at highest point) $\vec{u}_f = u\cos\theta\hat{i} + 0\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -u\sin\theta\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta\vec{u} = u_f - u_i = 2u\sin\theta\hat{i}$

(5) **Change in momentum :** Simply by the multiplication of mass in the above expression of velocity (Article-4).

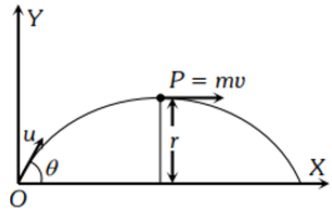
(i) Change in momentum (Between projection point and highest point) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -m\sin\theta\hat{j}$

(ii) Change in momentum (For the complete projectile $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\hat{j}$

(6) **Angular momentum :** Angular momentum of projectile at highest point of trajectory about the point of projection is given by

L = mvr $\left[\text{Here } r = H = \dfrac{u^2\sin^2\theta}{2g}\right]$

$$\therefore \quad L = m\,u\cos\theta\frac{u^2\sin^2\theta}{2g} = \frac{mu^3\cos\theta\sin^2\theta}{2g}$$



(7) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion 0 = u sin θ – gt

$\Rightarrow t = (u\sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \dfrac{2u\sin\theta}{g}$

(i) Time of flight can also be expressed as : $T = \dfrac{2.u_y}{g}$ (where u_y is the vertical component of initial velocity).

(ii) For complementary angles of projection θ and 90° – θ.

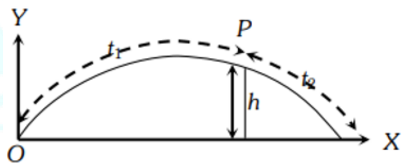
(a) Ratio of time of flight = $\dfrac{T_1}{T_2} = \dfrac{2u\sin\theta/g}{2u\sin(90-\theta)/g}$

$= \tan\theta \Rightarrow \dfrac{T_1}{T_2} = \tan\theta$

(b) Multiplication of time of flight = $T_1T_2 = \dfrac{2u\sin\theta}{g} - \dfrac{2u\cos\theta}{g}$

$\Rightarrow T_1T_2 = \dfrac{2R}{g}$

(iii) If t_1 is the time taken by projectile to rise upto point p and t_2 is the time taken in falling from point p to ground level then $T_1 + T_2 = \dfrac{2u\sin\theta}{g}$ time of flight or $u\sin\theta = \dfrac{g(t_1+t_2)}{2}$ and height of the point p is given by $h = u\sin\theta\, t_1 - \dfrac{1}{2}gt_1^2$

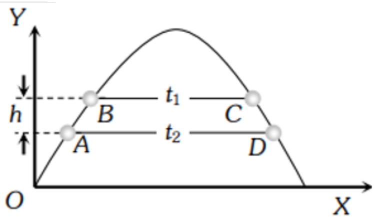


$h = g\dfrac{(t_1+t_2)}{2}t_1 - \dfrac{1}{2}gt_1^2$

by solving $h = \dfrac{gt_1t_2}{2}$

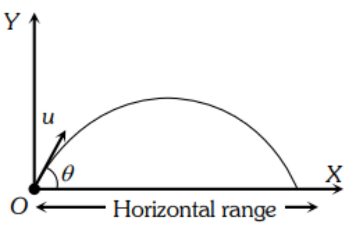
(iv) If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then

$t_2^2 - t_1^2 = \dfrac{8h}{g}$



(8) **Horizontal range :** It is the horizontal distance travelled by a body during the time of flight.

So by using second equation of motion in x-direction R = u cosθ × T



$= u\cos\theta \times (2u\sin\theta/g)$

$= \dfrac{u^2\sin 2\theta}{g}$

$R = \dfrac{u^2\sin 2\theta}{g}$

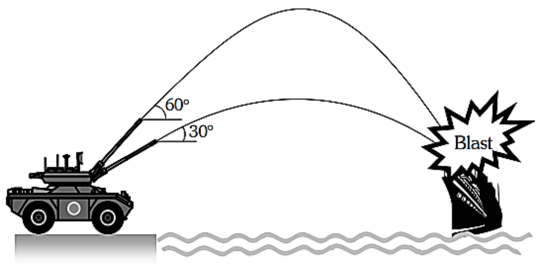
(i) Range of projectile can also be expressed as :

$R = u\cos\theta \times T = u\cos\theta\dfrac{2u\sin\theta}{g}$

$\therefore\ R = \dfrac{2u_xu_x}{g}$ (where ux and uy are the horizontal and vertical component of initial velocity)

(ii) If angle of projection is changed from θ to θ' = (90 – θ) then range remains unchanged.

$$R' = \frac{u^2 \sin 2\theta'}{g} = \frac{u^2 \sin[2(90° - \theta)]}{g} = \frac{u^2 \sin 2\theta}{g} = R$$



So a projectile has same range at angles of projection θ and (90 – θ), though time of flight, maximum height and trajectories are different.

These angles θ and 90° – θ are called complementary angles of projection and for complementary angles of projection, ratio of range $\frac{R_1}{R_2} = \frac{u^2 \sin 2\theta / g}{u^2 \sin[2(90° - \theta)]/g} = 1 \Rightarrow \frac{R_1}{R_2} = 1$

(iii) For angle of projection $\theta_1 = (45 - \theta)$ and $\theta_2 = (45 + \theta)$, range will be same and equal to $u^2 \cos 2\alpha / g$.

θ_1 and θ_2 are also the complementary angles.

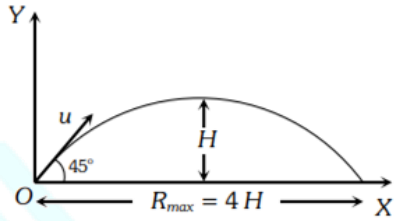
(iv) Maximum range : For range to be maximum

$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right] = 0$$

$$\Rightarrow \cos 2\theta = 0 \text{ i.e. } 2\theta = 90° \Rightarrow \theta = 45°$$

i.e., a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile.



$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}. The maximum height during the flight to which it will rise is $\left(\frac{R_{max}}{4}\right)$.

(v) Relation between horizontal range and maximum height : $R = \frac{u^2 \sin 2\theta}{2g}$ and $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\therefore \frac{R}{H} = \frac{u^2 \sin 2\theta / g}{u^2 \sin^2 \theta / 2g} = 4\cot\theta \Rightarrow R = 4H\cot\theta$$

(vi) If in case of projectile motion range R is n times the maximum height H

$$\text{i.e. } R = nH \Rightarrow \frac{u^2 \sin 2\theta}{g} = n\frac{u^2 \sin^2 \theta}{2g}$$

⇒ tan θ = [4/n] or θ = $\tan^{-1}$[4/n] o

The angle of projection is given by θ = $\tan^{-1}$[4/n].

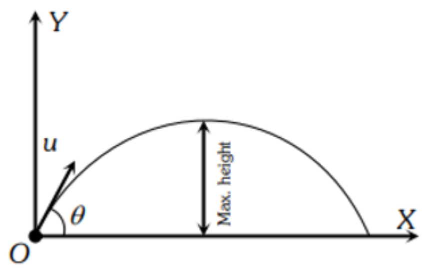
Note : If R = H then θ = $\tan^{-1}$[4/n] or θ = 76°.

(9) **Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$0 = (u \sin\theta)^2 - 2gH$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$



(i) Maximum height can also be expressed as

$H = \frac{u_y^2}{2g}$ where u_y is the vertical component of initial velocity).

(ii) $H_{max} = \frac{u^2}{2g}$ (when $\sin^2\theta$ = max = 1, i.e., $\theta = 90°$)

i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

(iii) For complementary angles of projection θ and $90° - \theta$

Ratio of maximum height

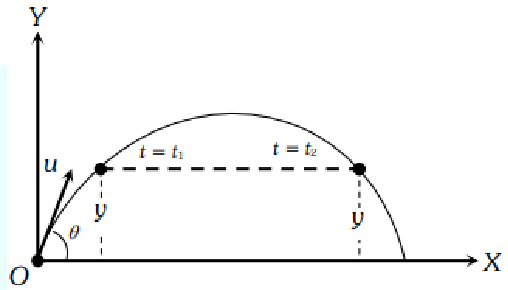
$$= \frac{H_1}{H_2} = \frac{u^2\sin^2\theta/2g}{u^2\sin^2(90°-\theta)/2g} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

$$\therefore \frac{H_1}{H_2} = \tan^2\theta$$

(10) **Projectile passing through two different points on same height at time t_1 and t_2 :** If the particle passes two points situated at equal height y at $t = t_1$ and $t = t_2$, then

(i) **Height (y):** $y = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$ ...(i)

and $y = (u\sin\theta)t_2 - \frac{1}{2}gt_2^2$ ...(ii)



Comparing equation (i) with equation (ii)

$$u\sin\theta = \frac{g(t_1+t_2)}{2}$$

Substituting this value in equation (i)

$$y = g\left(\frac{t_1+t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2 \Rightarrow y = \frac{gt_1t_2}{2}$$

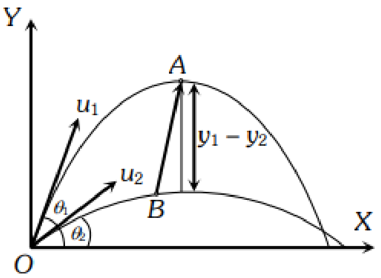
(ii) **Time (t_1 and t_2):** $y = u\sin\theta t - \frac{1}{2}gt^2$

$$t^2 - \frac{2u\sin\theta}{g} + \frac{2y}{g} = 0 \Rightarrow t = \frac{u\sin\theta}{g}\left[1 \pm \sqrt{1-\left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1-\left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

and $t_2 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1-\left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$

(11) **Motion of a projectile as observed from another projectile :** Suppose two balls A and B are projected simultaneously from the origin, with initial velocities u_1 and u_2 at angle θ_1 and θ_2, respectively with the horizontal.



The instantaneous positions of the two balls are given by

Ball $A: x_1 = (u_1\cos\theta_1)t,\ y_1 = (u_1\sin\theta_1)t - \frac{1}{2}gt^2$

Ball $B: x_2 = (u_2\cos\theta_2)t,\ y_2 = (u_2\sin\theta_2)t - \frac{1}{2}gt^2$

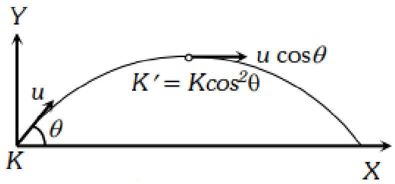
The position of the ball A with respect to ball B is given by

$$x = x_1 - x_2 = (u_1\cos\theta_1 - u_2\cos\theta_2)t$$

$$y = y_1 - y_2 = (u_1\sin\theta_1 - u_2\sin\theta_2)t$$

Now $\frac{y}{x}=\left(\frac{u_1\sin\theta_1-u_2\sin\theta_2}{u_1\cos\theta_1-u_2\cos\theta_2}\right)$ = constant

Thus motion of a projectile relative to another projectile is a straight line.



(12) **Energy of projectile :** When a projectile moves upward its kinetic energy decreases, potential energy increases but the total energy always remain constant.

If a body is projected with initial kinetic energy K(=1/2 mu²), with angle of projection θ with the horizontal then at the highest point of trajectory.

(i) **Kinetic energy** = $\frac{1}{2}m(u\cos\theta)^2=\frac{1}{2}mu^2\cos^2\theta$

$\therefore$ K' = K $\cos^2\theta$

(ii) **Potential energy** = $mgH=mg\frac{u^2\sin^2\theta}{2g}$

$=\frac{1}{2}mu^2\sin^2\theta \qquad \left(\text{As } H=\frac{u^2\sin^2\theta}{2g}\right)$

(iii) **Total energy** = Kinetic energy + Potential energy

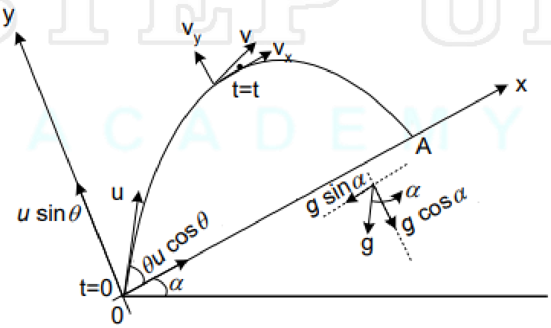
$=\frac{1}{2}mu^2\cos^2\theta+\frac{1}{2}mu^2\sin^2\theta$

$=\frac{1}{2}mu^2$ = Energy at the point of projection.

This is in accordance with the law of conservation of energy.

TIME OF FLIGHT & HORIZONTAL RANGE OF PROJECTILE MOTION ON INCLINED PLANE:

Projectile Motion on Inclined Plane: Figure shows an inclined plane at an angle α and a particle at an angle α with the direction of plane with initial velocity u. In such cases we take our reference x- and y-axes in the direction along and perpendicular to the inclined as shown.



Unlike to the simple projectile motion, here the x-component of the velocity of the projectile will also be retarded by a gsinα. Now y-component of the velocity is retarded by gcosα instead of g. As shown here g is resolved in two directions.

As here y-direction component is retarded by gcosα, to find the time of flight and maximum height, we can use equations $T=\frac{2u\sin\theta}{g}$ and $R=\frac{u^2\sin 2\theta}{g}$, replacing g by gcosα,

Time of flight on inclined plane projectile is

$$T=\frac{2u\sin\theta}{g\cos\alpha}$$

Maximum height of the projectile with respect to inclined plane is

$$H = \frac{u^2 \sin^2\theta}{2g\cos\alpha}$$

For evaluation of range on inclined plane we cannot use equation $R = \frac{u^2 \sin 2\theta}{g}$, just by replacing g by gcosα, as here we also have

Acceleration in x-axis $a_x = -g\sin\alpha$

Now we again find the distance traveled by the particle along x-direction in the duration time of flight is

$$R = u\sin\theta.T - \frac{1}{2}g\sin\alpha.T^2$$

On substituting the value of time of flight T, we get

$$R = \frac{u^2\sin 2\theta}{g\cos\alpha} - \frac{2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$$

Students are advised not to apply the above expression of range on inclined plane, as a standard result, it should be processed and evaluated according to the numerical problem. Above results we've derived for the projectile thrown up an inclined plane. If projectile is thrown down an inclined plane, the acceleration along the plane gsinα will increase the velocity of the particle along the plane, thus in the expression for range we should use +ve sing as

$$R = \frac{u^2\sin 2\theta}{g\cos\alpha} + \frac{2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$$

To find the maximum range on incline plane. One can use maxima-minima as $\frac{dR}{d\theta}$. The range on inclined plane has a maximum value given as

$$R = \frac{u^2}{g(1 \pm \sin\alpha)}$$

In above equation +ve sign is used for projectile up the plane and –ve sign is used for projectile down the plane.

Example. A ball is dropped from a height h above a point on an inclined plane, with angle of inclination θ. The ball makes an elastic collision with the surface and rebounds off the plane. Determine the distance from the point of first impact to the point where ball hit the plane second time.

Key concept: Take the point of first impact as the origin. Direction along the plane will be the x-axis and the direction perpendicular to the plane will be the y-axis. It is given that the ball rebounds elastically and implies that no change in kinetic energy of the ball before and after the collision. The ball rebounds with the same speed with which it will strike the plane after falling a distance h, which is $u = \sqrt{2gh}$. After rebound, the horizontal component of velocity u sinθ will be accelerated by g sinθ and the vertical component of the velocity ucosθ will be retarded by gcosθ.

Solution: Here time of flight from first impact to the second impact is given as

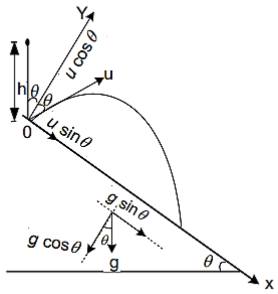
$$T = \frac{2u_y}{a_y} = \frac{2u\cos\theta}{g\cos\theta} = \frac{2u}{g}$$

In this duration the distance travelled by the horizontal component is

$$R = R = u\sin\theta.\frac{2u}{g} + \frac{1}{2}g\sin\theta\left(\frac{2u}{g}\right)^2 = \frac{4u^2\sin\theta}{g}$$

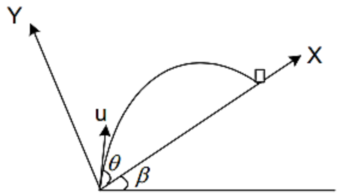
$$8h\sin\theta \qquad \left(\text{since } u = \sqrt{2gh}\right)$$

Example. A projectile is thrown with a speed u, at an angle θ to an inclined plane of inclination β. Find the angle θ at which the projectile is thrown such that it strikes the inclined plane normally.



Key concept: At the time of striking, x-component of velocity is zero (v_x=0) is zero.

Solution:



Here we have time of flight of particle is

$$T = \frac{2u\sin\theta}{g\cos\beta}$$

Thus from speed equation in x-direction, we have

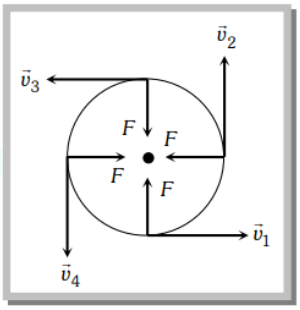
$$0 = u\cos\theta - g\sin\beta\left(\frac{2u\sin\theta}{g\cos\beta}\right)$$

or $\cot\theta = \tan\beta$

or $\theta = \cot^{-1}(\tan\beta)$

CIRCULAR MOTION:

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.



Since this force is always at right angles to the displacement due to the initial velocity therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle. Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

Variables of Circular Motion:

(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. i.e., $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) **Displacement :** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Rightarrow \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1| \qquad \Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta}$$

Putting $r_1 = r_2 = r$ we obtain

$$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$$

$$\Delta r = \sqrt{2r^2(1-\cos\theta)} = \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$

(ii) **Distance :** The distanced covered by the particle during the time t is given as

d = length of the arc AB = r θ

(iii) Ratio of distance and displacement : $\frac{d}{\Delta r}=\frac{r\theta}{2r\sin\theta/2}=\frac{\theta}{2}\text{cosec}(\theta/2)$

(2) **Angular displacement (θ) :** The angle turned by a body moving on a circle from some reference line is called angular displacement.

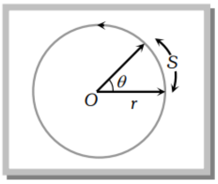
(i) Dimension = $[M^0L^0T^0]$ (as θ =arc / radius).

(ii) Units = Radian or Degree. It is sometimes also specified in terms of fraction or multiple of revolution.

(iii) 2 π rad=360° = 1 Revolution

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.



(v) Relation between linear displacement and angular displacement $\vec{s}=\vec{\theta}\times\vec{r}$

or $s = r\theta$

(3) **Angular velocity (ω) :** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity ω = $\frac{\text{angle traced}}{\text{time taken}}=\underset{\Delta t\to 0}{\text{Lt}}\frac{\Delta\theta}{\Delta t}=\frac{\Delta\theta}{dt}$

$\therefore \qquad \omega=\frac{d\theta}{dt}$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second (rad.s^{-1}) or Degree per second.

(iv) Angular velocity is an axial vector.

(v) Relation between angular velocity and linear velocity $\vec{v}=\vec{\omega}\times\vec{r}$

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

Note: It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

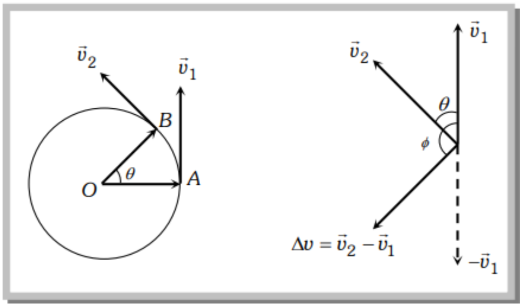
(4) **Change in velocity :** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vector is given as

$\Delta\vec{v}=\vec{v}_2-\vec{v}_1$

or $|\Delta\vec{v}|=|\vec{v}_2-\vec{v}_1| \Rightarrow \Delta v=\sqrt{v_1^2+v_2^2-2v_1v_2\cos\theta}$

For uniform circular motion $v_1 = v_2 = v$

So $\Delta v=\sqrt{2v^2(1-\cos\theta)}=2v\sin\frac{\theta}{2}$



The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$$\phi = \frac{180° - \theta}{2} = (90° - \theta/2)$$

Note: Relation between linear velocity and angular velocity.

In vector form $\vec{v} = \vec{\omega} \times \vec{r}$

(5) **Time period (T) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second.

(ii) Dimension : $[M^0L^0T]$

(iii) Time period of second's hand of watch = 60 second.

(iv) Time period of minute's hand of watch = 60 minute

(v) Time period of hour's hand of watch = 12 hour

(6) **Frequency (n) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz).

(ii) Dimension : $[M^0L^0T^{-1}]$

Note: Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in 1/n second.

$\therefore T = 1/n$

Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time t = T, θ = 2π radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $(\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$\omega_{rel} = \omega_B - \omega_A$

So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (i.e., time in which B completes one more or less revolution around O than A)

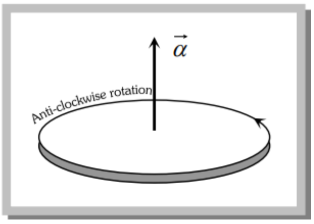
$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2} \quad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If $\omega_B = \omega_B$, $\omega_{rel} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If Δω be the change in angular velocity of the object in time interval t and t + Δt, while moving on a circular path, then angular acceleration of the object will be

$$\alpha = \underset{\Delta t \to 0}{\text{Lt}} \frac{d\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



(ii) Units : rad. s^{-2}

(iii) Dimension : $[M^0L^0T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{\Delta t} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Note: Relation between linear (tangential) acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

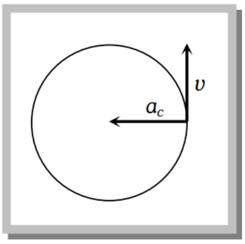
For uniform circular motion angular acceleration is zero, so tangential acceleration also is equal to zero.

For non-uniform circular motion $a \neq 0$ (because $\alpha \neq 0$).

Centripetal Acceleration:

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

(2) It always acts on the object along the radius towards the centre of the circular path.

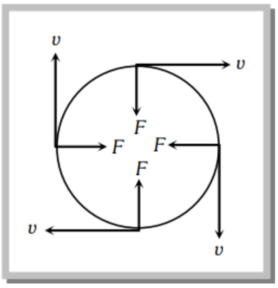


(3) Magnitude of centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = 4\pi n^2 r = \frac{4\pi^2}{T^2} r$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta \vec{v}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta \vec{v}$ becomes more and more perpendicular to $\vec{v}$. When $\Delta t \to 0$, $\Delta \vec{v}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta \vec{v}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path

Centripetal Force:

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously i.e. velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.



Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must acts towards the centre. This centre-seeking force is called the centripetal force.

Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formulae for centripetal force : $F = \frac{mv^2}{r} = m\omega^2 r = m4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle	Tension in the string

Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons in the nucleus
A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the agent that sets up the magnetic field

Centrifugal Force:

It is an imaginary force due to incorporated effects of inertia. When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangential at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body with the same velocity), the body appears to be stationary before it is released. When the body is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. In reality no force is actually seen to act on the body. In absence of any real force the body tends to continue its motion in a straight line due to its inertia. The observer A easily relates this events to be due to inertia but since the inertia of both the observer B and the body is same, the observer B can not relate the above happening to inertia. When the centripetal force ceases to act on the body, the body leaves its circular path and continues to moves in its straight-line motion but to observer B it appears that a real force has actually acted on the body and is responsible for throwing the body radially out-words. This imaginary force is given a name to explain the effects on inertia to the observer who is sharing the circular motion of the body. This inertial force is called centrifugal force. Thus centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

Class : 11th Physics
Chapter- 3 : Motion in a Straight Line

Motion in a Straight Line

Acceleration

Rate of change velocity, $\vec{a} = \frac{\vec{v}}{t}$

Average Acceleration
$\vec{a}_{avg} = \frac{\text{Total change in velocity}}{\text{Total time interval}}$

Instantaneous Acceleration
$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

Types

Uniform Motion
Equal distances are traversed in equal amount of time.

Non - Uniform Motion
When the magnitude or the direction of velocity changes w.r.t. time.

Displacement

* The shotest distance traversed by any object in called displacement
* It in a vector quantity.

Distance

Total length of the path traversed by any object is called its distance It is scalar quantity.

Speed

Distance traversed/time taken
Scalar quantity

Average Speed
When object traversed different distances with different speeds.
$\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + ... \frac{s_n}{v_n}$
When object traversed different speeds in different time of intervals.
$\frac{v_1 t_1 + v_2 t_2 + ... + v_n t_n}{t_1 + t_2 + ... + t_n}$

Free Fall

An object falling because of Earth's gravity (g = 9.8m/s On neglecting air resistance, it is a case of motion with uniform acceleration. e.g Apple falling from a tree.

Kinematic Equations of uniformly

(i) $v = v_0 + at$

(ii) $x = v_0 t + at$

(iii) $v^2 = v_0^2 + 2ax$

(iv) $x_{nth} = v_0 + \frac{a}{2}(2n-1)$

Velocity

Distance traversed/time taken
Scalar quantity

Relative Velocity
(i) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
(ii) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$
In opposite direction, it will be Difference In same direction, it will be Sum

Average Velocity
$\vec{v}_{avg} = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{\Delta x}{\Delta t}$

Instantaneous Velocity
$\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$

Important Questions

Multiple Choice Questions

1. A boy starts from a point A, travels to a point B at a distance of 3 km from A and returns to A. If he takes two hours to do so, his speed is

   (a) 3 km/h

   (b) zero

   (c) 2 km/h

   (d) 1.5 km/h

2. A body starts from rest and travels with uniform acceleration a to make a displacement of 6 m. If its velocity after making the displacement is 6 m/s, then its uniform acceleration a is

   (a) 6 m/s^2

   (b) 2 m/s^2

   (c) 3 m/s^2

   (d) 4 m/s^2

3. Which one of the following is the unit of acceleration?

   (a) m/s

   (b) m/s^2

   (c) km/hr

   (d) cm/s

4. The dimensional formula for speed is

   (a) T^{-1}

   (b) LT^{-1}

   (c) $L^{-1}T^{-1}$

   (d) $L^{-1}T$

5. A body starts from rest and travels for t second with uniform acceleration of 2 m/s^2. If the displacement made by it is 16 m, the time of travel t is

   (a) 4 s

   (b) 3 s

   (c) 6 s

   (d) 8 s

6. The dimensional formula for acceleration is

   (a) $[LT^2]$

   (b) $[LT^{-2}]$

   (c) $[L^2T]$

   (d) $[L^2T^2]$

7. A body starts from rest and travels for five seconds to make a displacement of 25 m. if it has travelled the distance with uniform acceleration a then a is

   (a) 3 m/s^2

   (b) 4 m/s^2

   (c) 2 m/s^2

   (d) 1 m/s^2

8. A 180 meter long train is moving due north at a speed of 25 m/s. A small bird is flying due south, a little above the train, with a speed of 5 m/s. The time taken by the bird to cross the train is

   (a) 10 s

   (b) 12 s

   (c) 9 s

   (d) 6 s

9. The dimensional formula for velocity is

   (a) [LT]

   (b) $[LT^{-1}]$

   (c) $[L^2T]$

   (d) $[L^{-1}T]$

10. A body starts from rest and travels with an acceleration of 2 m/s^2. After t seconds its velocity is 10 m/s. Then t is

    (a) 10 s

    (b) 5 s

    (c) 20 s

    (d) 6 s

Very Short:

1. Can a moving body have relative velocity zero with respect to another body? Give an example.
2. Can there be motion in two dimensions with acceleration in only one dimension?
3. Is it true that a body is always at rest in a frame that is fixed to the body itself?
4. Tell under what condition a body moving with uniform velocity can be in equilibrium?
5. What does the speedometer records: the average speed or the instantaneous speed?

6. Can an object be accelerated without speeding up or slowing down? Give examples,
7. Is it possible to have the rate of change of velocity constant while the velocity itself changes both in magnitude and direction? Give an example.
8. Which motion is exactly represented by $\Delta s = v\Delta t$?
9. In which frame of reference is the body always at rest?
10. What is common between the two graphs shown in figs, (a) and (b)?

Short Questions:

1. Prove that the average velocity of a particle over an interval of time is either smaller than or equal to the average speed of the particle over the same interval.
2. Two trains each of the length 109 m and 91 m are moving in opposite directions with velocities 34 km h-1 and 38 km h^{-1} respectively. At what time the two trains will completely cross each other?
3. Ambala is at a distance of 200 km from Delhi. Ram sets out from Ambala at a speed of 60 km h-1 and Sham set out at the same time from Delhi at a speed of 40 km h^{-1}. When will they meet?
4. A car travelling at a speed of 60 km h^{-1} on a straight road is ahead of a scooter travelling at a speed of 40 km h^{-1}. How would the relative velocity be altered if the scooter is ahead of the car?
5. Draw the position-time graphs for two objects initially occupying different positions but having zero relative velocity.
6. A ball is thrown vertically upward with a velocity of 20 ms^{-1}. It takes 4 seconds to return to its original position. Draw a velocity-time graph for the motion of the ball and answer the following questions:

   At which point P, Q, R, the stone has:

   (a) reached its maximum height.

   (b) stopped moving?
7. "It is the velocity and not the acceleration which decides the direction of motion of a body." Justify this statement with the help of a suitable example.
8. Two buses A and B starting from the same point move in a mutually perpendicular direction with speeds uA km h^{-1} and uB km h^{-1} respectively. Calculate the relative velocity of A w.r.t B.

Long Questions:

1. Define the following terms:

   (a) speed
   (b) uniform speed
   (c) variable speed
   (d) average speed
   (e) instantaneous speed
   (f) velocity
   (g) uniform velocity
   (h) variable velocity
   (i) uniform motion
   (j) average velocity in uniform
   (k) relative velocity motion
   (l) instantaneous velocity
   (m) acceleration
   (n) retardation
   (o) variable acceleration
   (p) average acceleration
   (q) uniform acceleration
   (r) instantaneous acceleration.
2. Explain the importance of the position-time graph.
3. Derive relations:

   (i) $v = u + at$

   (ii) $v_2 - u_2 = 2as$

   (iii) $s = ut + \frac{1}{2}at^2$.

Assertion Reason Questions:

1. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: A body may be accelerated even when it is moving uniformly.

   Reason: When direction of motion of the body is changing, the body must have acceleration.

2. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

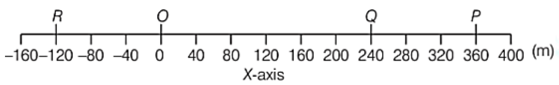
   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: Displacement of a body may be zero when distance travelled by it is not zero. **Reason:** The displacement is the longest distance between initial and final position.

Case Study Questions:

1. If the position of an object is continuously changing w.r.t. its surrounding, then it is said to be in the state of motion. Thus, motion can be defined as a change in position of an object with time. It is common to everything in the universe. In the given figure, let P, Q and R represent the position of a car at different instants of time.



   i. With reference to the given figure, the position coordinates of points P and R are

   (a) P = (+ 360, 0, 0); R = (- 120, 0, 0)

   (b) P = (- 360, 0, 0); R = (+ 120, 0, 0)

   (c) P = (0, + 360, 0); R = (- 120, 0, 0)

   (d) P = (0, 0 + 360) R = (0, 0, - 120)

   ii. Displacement of an object can be

   (a) Positive

   (b) Negative

   (c) Zero

   (d) All of the above

   iii. The displacement of a car in moving from O to P and its displacement in moving from P to Q are

   (a) + 360m and -120m

   (b) -120m and + 360m

   (c) + 360m and + 120m

   (d) + 360m and - 600m

   iv. If the car goes from O to P and returns back to O, the displacement of the journey is

   (a) Zero

   (b) 720m

   (c) 420m

   (d) 340m

   v. The path length of journey from O to P and back to O is

   (a) 0 m

   (b) 720m

   (c) 360m

   (d) 480m

2. When an object is in motion, its position changes with time. So, the quantity that describes how fast is the position changing w.r.t. time and in what direction is given by average velocity. It is defined as the change in position or displacement (Dx) divided by the time interval (Dt) in which that displacement occurs. However, the quantity used to describe the rate of motion over the actual path, is average speed. It defined as the total distance travelled by the object divided by the total time taken.

   i. A 250m long train is moving with a uniform velocity of 4.5 kmh^{-1}The time taken by the train to cross a bridge of length 750m is

   (a) 56 s

   (b) 68 s

   (c) 80 s

   (d) 92 s

   ii. A truck requires 3 hr to complete a journey of 150 km. What is average speed?

   (a) 50 km/h

   (b) 25 km/h

   (c) 15 km/h

   (d) 10 km/h

   iii. Average speed of a car between points A and B is 20 m/s, between B and C is 15 m/s and between C and D is 10 m/s. What is the average speed between A and D, if the time taken in the? mentioned sections is 20s, 10s and 5s, respectively?

   (a) 17.14 m/s

   (b) 15 m/s

   (c) 10 m/s

   (d) 45 m/s

iv. A cyclist is moving on a circular track of radius 40 m completes half a revolution in 40 s. Its average velocity is

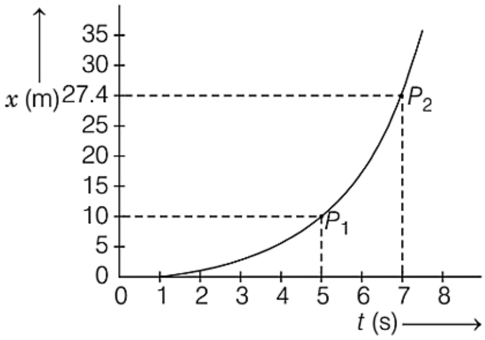
(a) zero

(b) 2 ms $^{-1}$

(c) 4π ms^{-1}

(d) 8π ms^{-1}

v. In the following graph, average velocity is geometrically represented by



(a) Length of the line $P_1 P_2$

(b) Slope of the straight-line $P_1 P_2$

(c) Slope of the tangent to the curve at P_1

(d) Slope of the tangent to the curve at P_2

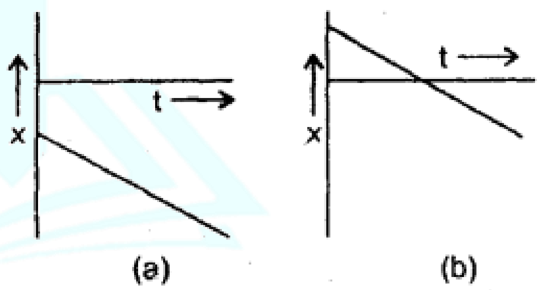
Answer Key

Multiple Choice Answers-

1. **Answer:** (a) 3 km/h
2. **Answer:** (c) 3 m/s^2
3. **Answer:** (b) m/s^2
4. **Answer:** (b) LT^{-1}
5. **Answer:** (b) 3 s
6. **Answer:** (b) [LT^{-2}]
7. **Answer:** (c) 2 m/s^2
8. **Answer:** (d) 6 s
9. **Answer:** (b) [LT^{-1}]
10. **Answer:** (b) 5 s

Very Short Answers:

1. **Answer:** Yes, two trains running on two parallel tracks with the same velocity in the same direction.
2. **Answer:** Yes, projectile motion.
3. **Answer:** Yes.
4. **Answer:** When the net force on the body is zero.
5. **Answer:** It records (or measures) the instantaneous speed.
6. **Answer:** Yes, circular motion.
7. **Answer:** Yes, in projectile motion.
8. **Answer:** It Represents motion with uniform velocity.
9. **Answer:** The body is always at rest in the frame attached to it i. e. inertial frame of reference.
10. **Answer:**



Both these graphs represent that the velocity is negative.

Short Questions Answers:

1. **Answer:** Average velocity is defined as the ratio of the total displacement to the total time. Average speed is defined as the ratio of the total distance to the total time. Since displacement is less than or equal to the distance, therefore the average velocity is less than or equal to the average speed.
2. **Answer:**

Let l_1, l_2 be the lengths of the two trains.

v_1, v_2 be their velocities respectively.

$\therefore l_1 = 109$m, $l_2 = 91$ m, $v_1 = 34$kmh^{-1}, $v_2 = 38$kmh^{-1}.

As the trains are moving in opposite directions so relative velocity of the trains is given by

$v_1 - (- v_2) = v_1 + v_2$

$= 34 + 38 = 72$ kmh^{-1}

$= 72 \times \frac{5}{18} = 20\ ms^{-1}$

Total distance to be covered by the two trains in crossing each other

$= l_1 + l_2 = 109 + 91 = 200$ m

If t be the time taken in crossing, then t can be calculated using the relation

x = vt

or

$t = \frac{200}{20} = 10s$

3. **Answer:**

S = 200 km. Let VR and vs be the speeds of Ram and Sham respectively moving in opposite directions.

$\therefore v_R = 60\ kmh^{-1}, v_S = 40\ kmh^{-1}$.

∴ Relative velocity of Ram *w.r.t.* Sham is

$V_{RS} = V_R - (- V_S)$

$= V_R + V_S$

$= 60 + 40 = 100\ kmh^{-1}$

If t = time after which they will meet, then

t = time taken in covering 200 km distance with VRS

i.e. $t = \frac{200}{\upsilon_{RS}} = \frac{200\ km}{100\ kmh^{-1}} = 2h$

∴ Time after which they meet = 2h.

4. **Answer:**

v_c = speed of car = 60 kmh^{-1}

v_s = speed of scooter = 40 kmh^{-1}

v_{cs} = relative velocity of car w.r.t. scooter

$= v_c - vs$

$= 60 - 40$

$= 20\ kmh^{-1}$

Similarly, vsc = relative velocity of scooter w.r.t. car

$= v_s - v_c$

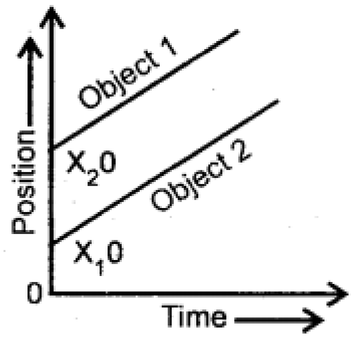
$= 40 - 60$

$= -20\ kmh^{-1}$

Thus, we conclude that the magnitude of the relative velocity is the same in both cases but the direction of relative velocity is reversed if the scooter is ahead of the car.

5. **Answer:**

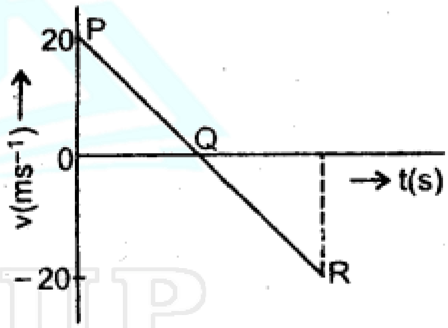
The positive T time graphs for two objects initially occupying different positions but having zero relative velocity are parallel to each other as shown in Fig.



6. **Answer:**

- Let P represent the initial position at the time when the ball is thrown vertically upward.
- Q represents the highest point reached by the ball.
- R represents the original position of the ball after 4 seconds.

Thus, the velocity-time graph for the motion of the ball is as shown in Fig.



(a) We know that at the highest point, the velocity of the object is zero. So, stone will reach its maximum height corresponding to point Q.

(b) The stone has stopped moving at point Q because at Q, v = 0.

7. **Answer:**

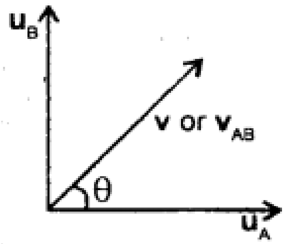
The direction of velocity is always in the direction of motion of the body whereas the direction of acceleration may or may not be in the direction of motion of the body. Thus we conclude that it is the velocity that decides the direction of motion of the body.

Example: When a ball is thrown vertically upwards, the direction of motion of the ball and velocity is the same i.e. vertically upwards. On the other hand, the acceleration due to gravity on the ball acts vertically downwards i.e. opposite to the direction of motion of the ball.

8. **Answer:**

Since uA and uB are in mutually perpendicular directions, they will cover uA and uB km in one hour respectively. Thus if v km be the separation between them in one hour,

Then $v = \sqrt{u_A^2 + u_B^2}$



Thus, if v_{AB} be the relative speed of A *w.r.t.* B, then

$$v_{AB} = \sqrt{u_A^2 + u_B^2} \text{ kmh} \quad \text{....(1)}$$

If θ be the direction of v_{AB} *w.r.t.* u_A, Then

$$\tan\theta = \frac{u_A}{\upsilon_{AB}} = \frac{u_A}{\sqrt{u_A^2 + u_B^2}} \quad \text{....(2)}$$

Thus, equations (1) and (2) give the magnitude and direction of relative velocity of A w.r.t. B.

Long Questions Answers:

1. **Answer:**

(a) **Speed:** It is defined as the time rate of change of position i. e. distance of an object.

i.e.

$$\text{Speed} = \frac{\text{Distance travelled by the object}}{\text{Time taken}}$$

(b) **Uniform Speed:** An object is said to be moving with uniform speed if it covers equal distances in equal small intervals of time.

(c) **Variable Speed:** An object is said to be moving with variable speed if it covers equal distances in unequal small intervals of time.

(d) **Average Speed:** It is used to measure the variable speed of an object.

It is defined as the ratio of the total distance travelled by the object to the total time taken.

$$\therefore \text{ vav} = \frac{\text{Total}}{\text{Total time taken}}$$

(e) **Instantaneous Speed:** It is defined as the speed of an object at a given instant of time. It is denoted by vins.

∴ If Δs be the distance covered by an object in a small-time interval Δt s.t. Δt → 0,

Then

$$v_{ins} = \underset{\Delta t \to 0}{Lt} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Thus in the case of the uniform motion of an object, the instantaneous speed is equal to its uniform speed.

(f) **Velocity:** It is defined as the time rate of change of displacement of an object.

(g) **Uniform Velocity:** An object is said to be moving with uniform velocity if it undergoes equal displacements in equal intervals of time however small these intervals may be.

(h) **Variable Velocity:** An object is said to be moving with variable velocity if either its magnitude (i.e. speed) or its direction or both change with time.

(i) **Uniform Motion:** An object is said to be in uniform motion if it undergoes equal displacements in equal intervals of time which may be very small.

(j) **Average Velocity in Uniform Motion:** The velocity of an object in uniform motion may be defined as the ratio of the. displacement of the object to the total time interval for which the motion takes place.

$$i.e.\ v = \frac{x_2 - x_1}{t_2 - t_1}$$

(k) **Relative Velocity:** The relative velocity of a moving object with respect to another object is defined as the rate of change of relative position of one object w.r.i. another object.

Or

It is the velocity with which one object moves with respect to another object.

(l) **The instantaneous velocity of an object:** It is defined as the velocity of an object at any instant of time or any point on its path.

Or

It is defined as the limiting value of the average velocity of the object as $\Delta t \to 0$.

i.e. $$v_{ins} = \underset{\Delta t \to 0}{Lt}\, v_{av}$$

$$= \underset{\Delta t \to 0}{Lt} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

(m) **Acceleration:** It is defined as the time rate of change of velocity of an object. It is a vector quantity.

(n) **Retardation:** It is defined as the negative acceleration produced in the object.

(o) **Variable Acceleration:** An object is said to be moving with variable acceleration if its velocity changes by unequal magnitudes in equal intervals of time.

(p) **Average Acceleration:** It is defined as the ratio of change in velocity in a given time interval to the total time taken.

(q) **Uniform Acceleration:** An object is said to be moving with uniform acceleration if it undergoes equal changes in velocity in equal intervals of time.

(r) **Instantaneous Acceleration:** It is defined as the acceleration of an object at a particular instant of time or at a particular point on its path.

Or

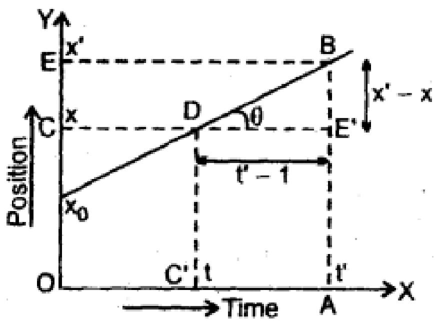
It may be defined as the limiting value of the average acceleration in a small time interval around that instant when the time-interval tends to zero.

i.e. $$a_{ins} = \underset{\Delta t \to 0}{Lt} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$= \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

2. **Answer:**

1. The importance of a position-time graph is that its slope gives the velocity of the object in uniform motion.



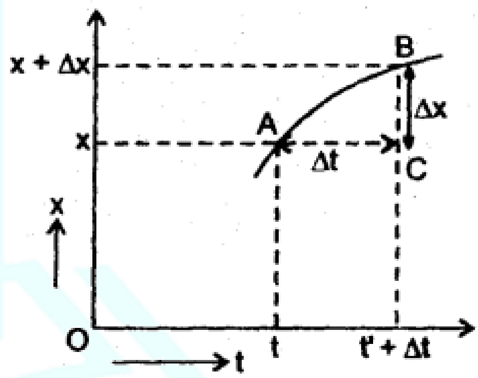
Let us consider the position-time graph of an object moving with uniform velocity represented by the line DB making angle 0 with the time axis. Let the coordinates of D and B be (x, t) and (x', t') respectively. Let BA and DC' be perpendiculars drawn from B and D respectively on the time axis and BE and DC be perpendiculars on the y-axis from B and D.

Now BE' = CE = x' – x

and C'A = t' – t = DE'

$$\text{Then velocity} = \frac{x'-x}{t'-t} = \frac{BE'}{DE'} = \tan\theta$$

So, velocity v = slope of position-time graph.



2. The position-time graph for a stationary object is a straight line parallel to the time axis. Here the slope of the curve is zero, which means the object is stationary as v = 0.

3. In the case of variable velocity the position time curve is not a straight line. In this case, the slope of the curve gives the average velocity

$$v_{average} = \frac{(x+\Delta x)-x}{(t+\Delta t)-t} = \frac{\Delta x}{\Delta t}$$

= slope or chord AB when $\Delta t \to 0$,

then the slope of curve gives the instantaneous velocity.

Instantaneous velocity = $\underset{\Delta t \to 0}{Lt} \frac{\Delta x}{\Delta t}$

Thus, the position-time graph gives information about velocity.

3. **Answer:**

$v = u + at$:

Derivation: By def. of acceleration, we know that

$$a=\frac{v_2-v_1}{t_2-t_1}$$

or $$v_2 - v_1 = a(t_2 - t_1)$$

or $$v_2 = v_1 + a(t_2 - t_1) \quad \text{....(1)}$$

where v_1 and v_2 are the velocities of an object at times t_1 and t_2 respectively.

If v_1 = u (initial velocity of the object) at $t_1 = 0$

v_2 = v (final velocity of the object) at t_2 = t

Then (1) reduces to v = u + at

Hence derived.

ii) $v_2 - u_2 = 2as$

Derivation: We know that acceleration is given by $a = \frac{v_2-v_1}{t_2-t_1}$, where v1 and v2, t1 and t2 are as in (1).

or $$t_2-t_1=\frac{v_2-v_1}{a} \quad \text{....(1)}$$

Also we know that

$$x_2 - x_1 = v_1(t_1-t_2)+\frac{1}{2}a(t_2-t_1)^2 \quad \text{...(2)}$$

∴ From (1) and (2), we get

$$x_2-x_1=v_1\frac{(v_2-v_1)}{a}+\frac{1}{2}a\left(\frac{v_2-v_1}{a}\right)^2$$

$$=\frac{v_1v_2-v_1^2}{a}+\frac{v_2^2+v_1^2-2v_1v_2}{2a}$$

$$=\frac{2v_1v_2-2v_1^2+v_1^2+v_2^2-2v_1v_2}{2a}$$

$$=\frac{v_2^2-v_1^2}{2a}$$

or $$v_2^2-v_1^2=2a(x_2-x_1) \quad \text{....(3)}$$

Now if $$\left.\begin{aligned} v_1&=u \text{ at } t_1=0\\ v_2&=v \text{ at } t_2=t\end{aligned}\right\} \quad \text{....(4)}$$

$$x_2 - x_1 = s$$

Then from (3) and (4), we get

$$v^2 - u^2 = 2as \quad \text{....(5)}$$

$$s=ut+\frac{1}{2}at^2$$

(iii) $s = ut + \frac{1}{2}at^2$.

Derivation:

Let x_1, V_1 = position and velocity of the object at time t_1.

x_2, v_2 = position and velocity of the object at time t_2.

a = uniform acceleration of the object.

Also Let vav = average velocity in $t_2 - t_1$ interval

∴ By definition

$$v_{av}=\frac{x_2-x_1}{t_2-t_1}$$

or $$x_2-x_1=v_{av}(t_2-t_1) \quad \text{....(1)}$$

Also we know that

$$v_{av}=\frac{v_1+v_2}{2} \quad \text{....(2)}$$

∴ From (1) and (2), we get

$$x_2-x_1=\frac{v_1+v_2}{2}(t_2-t_1) \quad \text{....(3)}$$

Also we know that

$$v_2=v_1+a(t_2-t_1) \quad \text{....(4)}$$

∴ From (3) and (4), we get

$$v_2-x_1=\frac{1}{2}[v_1+v_1+a(t_2-t_1)](t_2-t_1)$$

$$=v_1(t_2-t_1)+\frac{1}{2}a(t_2-t_1)^2 \quad \text{....(5)}$$

Now if $$\left.\begin{aligned} x_1&=x_0 \text{ at } t_1=0\\ x_2&=x \text{ at } t_2=t\\ v_1&=u \text{ at } t_1=0\\ v_2&=v \text{ at } t_2=t\end{aligned}\right\} \quad \text{....(6)}$$

∴ From (1) and (2), we get

$$x-x_0=\text{ut}+\frac{1}{2}at^2$$

if $x - x_0 = S$, then

$$s=ut+\frac{1}{2}at^2$$

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

In uniform circular motion, there is acceleration of constant magnitude.

2. (c) Assertion is correct, reason is incorrect

Explanation:

The displacement is the shortest distance between initial and final position. When final position of a body coincides with its initial position, displacement is zero, but the distance travelled is not zero.

Case Study Answer:

1. i (a) P = (+ 360, 0, 0); R = (- 120, 0, 0)

Explanation:

The position coordinates of point P = (+ 360, 0, 0) and point R = (- 120, 0, 0)

ii (d) All of the above

Explanation:

Displacement is a vector quantity, it can be positive, negative and zero.

iii (a) +360 m and -120 m

Explanation:

Displacement, $\Delta x = x_2 - x_1$

For journey of car in moving from *O* to *P*,

$x_2 = +360$ m

$x_1 = 0$

$\Rightarrow \quad \Delta x = x_2 - x_1 = 360 - 0 = +360$ m

For journey, of car in moving from P to Q,

$x_2 = +240$ m

$x_1 = +360$ m

$\Rightarrow \quad \Delta x = x_2 - x_1 = 240 - 360 = -120$ m

Here, –ve sign implies that the displacement is in –ve direction, *i.e.* towards left.

iv (a) zero

Explanation:

$Dx = x_2 - x_1 = 0 - 0 = 0$

v (b) 720m

Explanation:

Path length of the journey

= OP + PO = + 360m + (+360) m = 720m

2. i. (c) 80 s

Explanation:

Total time taken $= \frac{\text{The distance}}{\text{Speed}}$

$$t = \frac{250+750}{45\times\frac{5}{18}} = 80\,s$$

ii. (a) 50 km/h

Explanation:

Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{150}{3} = 50$ km/h

iii. (a) 17.14 m/s

Explanation:

Total distance ($d = vt$)

= 20 × 20 + 15 × 10 + 10 × 5 = 600 m

Total time = 20 + 10 + 5 = 35 s

Therefore, average speed

= 600 / 34 = 17.14 m/s

iv (b) 2 ms^{-1}

Explanation:

Given, R = 40 m and t = 40 s

$$Average\ velocity = \frac{Displacement}{Time\ taken}$$

$$= \frac{2R}{t} = \frac{2\times 40}{40} = 2\,ms^{-1}$$

v (b) Slope of the straight line $P_1 P_2$

Explanation:

From the position-time graph, average velocity is geometrically represented by the slope of curve, *i.e.*, slope of straight line $P_1 P_2$

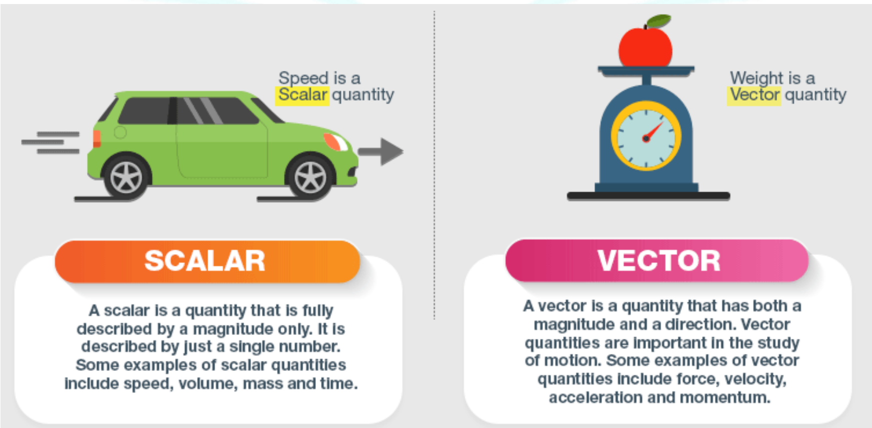
Motion in a Plane 4

Scalars Vs. Vectors

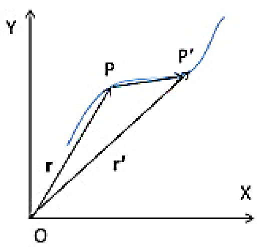
Criteria	Scalar	Vector
Definition	A scalar is a quantity with magnitude only.	A vector is a quantity with magnitude and direction.
Direction	No	Yes
Specified by	A number (magnitude) and a unit	A number (magnitude), direction and a unit
Represented by	quantity's symbol	quantity's symbol in bold or an arrow sign above
Example	mass, temperature	velocity, acceleration

Position and Displacement Vectors

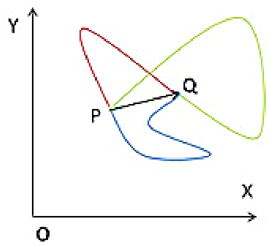
Position Vector: Position vector of an object at time t is the position of the object relative to the origin. It is represented by a straight line between the origin and the position at time t.



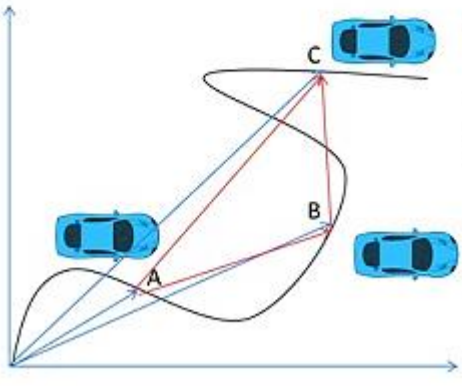
Displacement Vector: Displacement vector of an object between two points is the straight line between the two points irrespective of the path followed. The path length is always equal or greater than the displacement.



OP and OP' are position vectors represented by r and r'.
PP' is the displacement vector.



PQ is the displacement vector for any path followed (represented by green, blue and red paths).

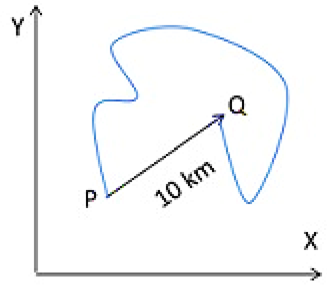


Example: Path taken by a car.

Path is represented by black line.

Blue lines represents Position vectors at point A, B and C.

Red lines represent Displacement vectors



If P to Q is a straight road of 10 km between station and hotel but the route taken is as shown in the figure of 23 km in 28 mins, then the average speed and average velocity of the car will be unequal.

Avg. Speed = Total Distance travelled/Total time
Avg. Speed = 23/(28/60) = 49.29 km/h

Avg. Velocity = Displacement/Total Time
Avg. Velocity = 10/(28/60) = 21.43 km/h

Free and Localized Vectors

A **free vector**(or non-localized vector) is a vector of which only the magnitude and direction are specified, not the position or line of action. Displacing it parallel to itself leaves it unchanged.

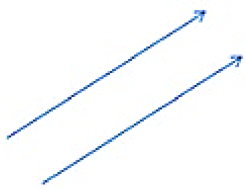
A **localized vector** is a vector where line of action and position are as important as magnitude and direction. These vectors change with change in position and direction.

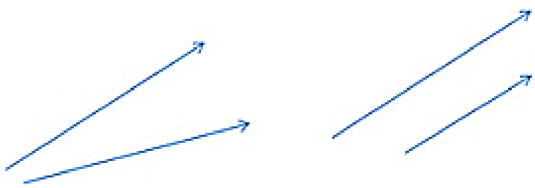
Velocity vector of a car moving in a straight line is a free vector

Force vector is a localized vector as it depends upon position as well

Equality of Vectors

Two vectors are said to be equal only when they have same direction and magnitude. For example, two cars travelling with same speed in same direction. If they are travelling in opposite directions with same speed, then the vectors are unequal.





Equal Vectors

Unequal Vectors

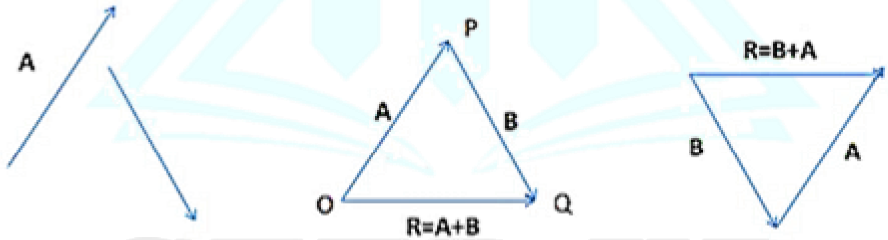
Multiplication of Vectors with real numbers

Multiplication Factor	Original vector	Magnitude of vector after multiplication	Direction of vector after multiplication
λ (>0)	A	λA	Same as that of A
-λ (<0)	A	λA	Opposite to that of A
λ (=0)	A	0 (null vector)	None. The initial and final positions coincide.

Multiplication of vector with +2 and -2

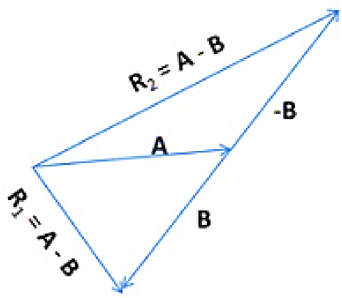
Addition and Subtraction of Vectors – Triangle Method

The method of adding vectors graphically is by arranging them so that head of first is touching the tail of second vector and making a triangle by joining the open sides. This method is called head-to-tail method or triangle method of vector addition



Head-to-Tail or Triangle Method of vector addition

- Vector addition is:
  - Commutative: A + B = B + A
  - Associative: (A + B) + C = A + (B+ C)
- Adding two vectors with equal magnitudes and opposite directions results in null vector.
  - Null Vector: A + (-A) = 0
- Subtraction is adding a negative vector (opposite direction) to a positive vector.
  - A – B = A + (-B)

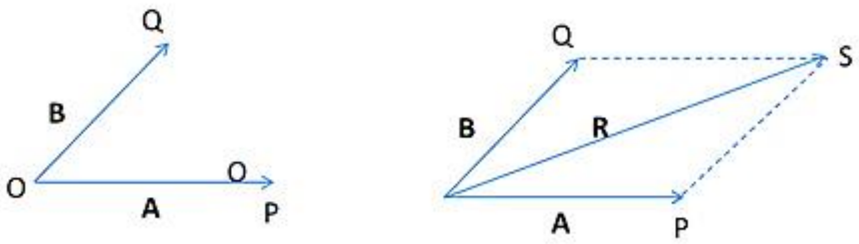


Addition and Subtraction of vectors using triangle method

Addition of Vectors – Parallelogram Method

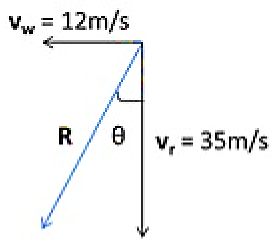
The method of adding vectors by parallelogram method is by:

- Touching the tail of the two vectors
- Complete a parallelogram by drawing lines from the heads of the two vectors.
- Vector resulting from the origin to the point of intersection of above lines gives the addition.



Parallelogram Method of vector addition

Example



If rain is falling vertically at a speed of 35 m/s and wind is blowing at 12 m/s (east to west), then the resultant vector **R** would be the actual path of rain.

R = $\sqrt{v_r^2 + v_w^2} = \sqrt{35^2 + 12^2}$ = 37m/s

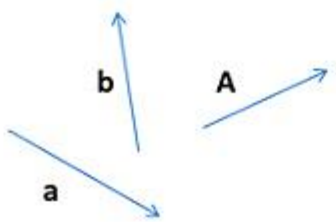
tanθ = v_w/v_r = 12/35 = 0.343

θ = 19°

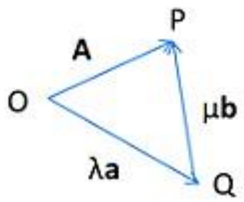
A person standing on ground must hold umbrella at an angle of 19° with vertical towards east to avoid rain.

Resolution of Vectors

A vector can be expressed in terms of other vectors in the same plane. If there are 3 vectors A, a and b, then A can be expressed as sum of a and b after multiplying them with some real numbers.



3 vectors in a plane



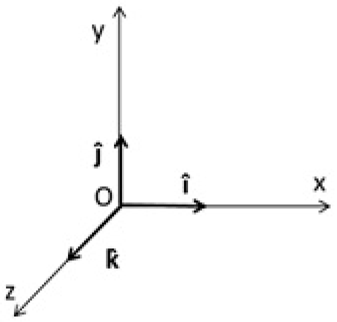
Joining the three to make a triangle

A can be resolved into two component vectors λa and μb. Hence, A = λa + μb. Here λ and μ are real numbers.

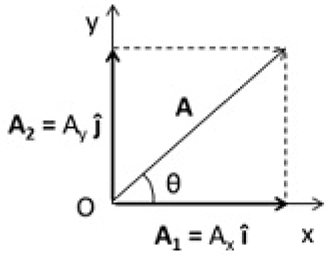
Unit Vectors

A unit vector is a vector of unit magnitude and a particular direction.

- They specify only direction. They do not have any dimension and unit.
- In a rectangular coordinate system, the x, y and z axes are represented by unit vectors, $\hat{i}$,$\hat{j}$ and $\hat{k}$
- These unit vectors are perpendicular to each other.
- $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$



Unit vectors in the coordinate system along the three axes.



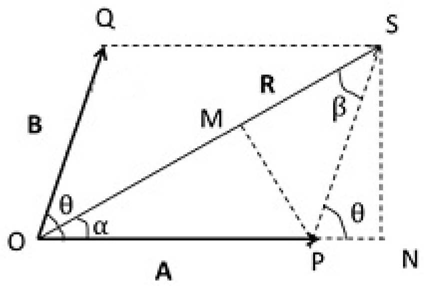
Vector **A** as combination of $\mathbf{A_1}$ and $\mathbf{A_2}$ which are expressed in terms of unit vectors.

In a 2-dimensional plane, a vector thus can be expressed as:

1. A = Ax î +Ay ĵ where, Ax = A cosθ and Ay = A sinθ
2. $A = \sqrt{A_x^2 + A_y^2}$

Analytical Method of Vector Addition

Vectors	Sum of the vectors	Subtraction of the vectors
A = Ax î +Ay ĵ and B = Bx î +By ĵ	R = A + B R = Rx î +Ry ĵwhere Rx = Ax + Bx and Ry = Ay + By	R = A - B R = Rx î +Ry ĵ where Rx = Ax – Bx and Ry = Ay - By
A = Ax î +Ay ĵ+Azk̂ B = Bx î +By ĵ+Bzk̂	R = A + B R = Rx î +Ry ĵ+Rzk̂ where Rx = Ax + Bx and Ry = Ay + By andRz = Az + Bz	R = A - B R = Rx î +Ry ĵ+Rzk̂ where Rx = Ax – Bx and Ry = Ay - By and Rz = Az - Bz



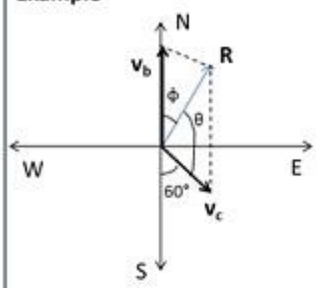
Law of Cosines

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Law of Sines

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

Example

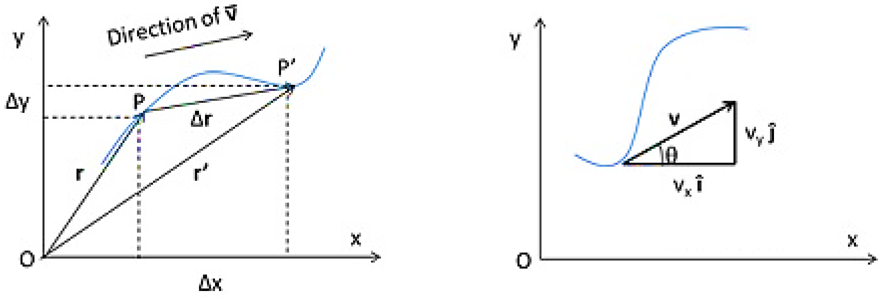


If a motorboat is travelling at a speed of 25 km/h north and water current is 10 km/h at an angle of 60° (east to south), then the resultant velocity and direction of boat can be obtained using law of sines and cosines.

Magnitude of R using law of cosines,
$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120^\circ} = 22$ km/h

Direction of R using law of sines,
$R/\sin\theta = v_c/\sin\phi$, $\sin\phi = v_c \sin\theta / R$
$\phi = 23.4^\circ$

Quantities related to motion of an object in a plane



Particle moving in a plane from P to P' in time t to t' and Velocity calculation of the particle in terms of unit vectors

Quantity	Value	Value in component form
Displacement Δr (Change in position)	r' - r	îΔx + ĵΔy
Average Velocity $\bar{v}$ (ratio of displacement and corresponding time interval)	Δr/Δt	vxî + vy ĵ vx= Δx/Δt, vy= Δy/Δt
Instantaneous velocity v (limiting value of average velocity as the time interval approached zero)	dr/dt	vxî + vy ĵ vx= dx/dt, vy= dy/dt
Magnitude of v		
Direction of v, θ (direction of velocity at any point on the path is tangential to the path at that point and is in the direction of motion)	tan-1(vy/vx)	
Average Acceleration ā (change in velocity divided by the time interval)	Δv/Δt	ax î + ay ĵ ax= Δvx/Δt, ay= Δvy/Δt
Instantaneous acceleration a (limiting value of the average acceleration as the time interval approaches zero)	dv/dt	ax î + ay ĵ ax= dvx/dt, ay= dvy/dt ax= d2x/dt2, ay= d2y/dt2

Motion in a plane with constant acceleration

Motion in a plane (two dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. X and Y directions are hence independent of each other.

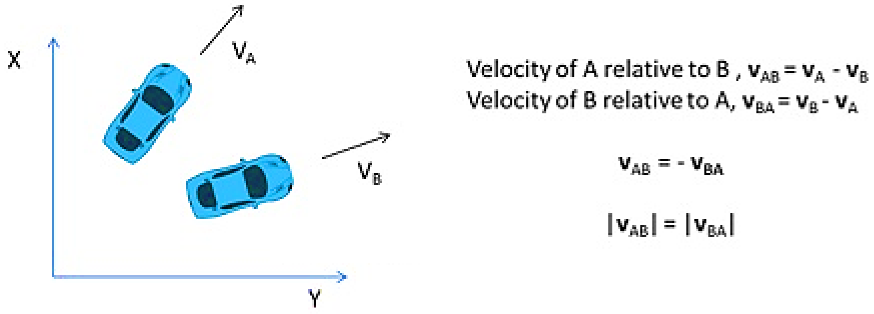
If v0 being the velocity at time 0, the displacement can be written as:

x = x0 + v0xt+ ½ axt2 and y = y0 + v0yt+ ½ ayt2

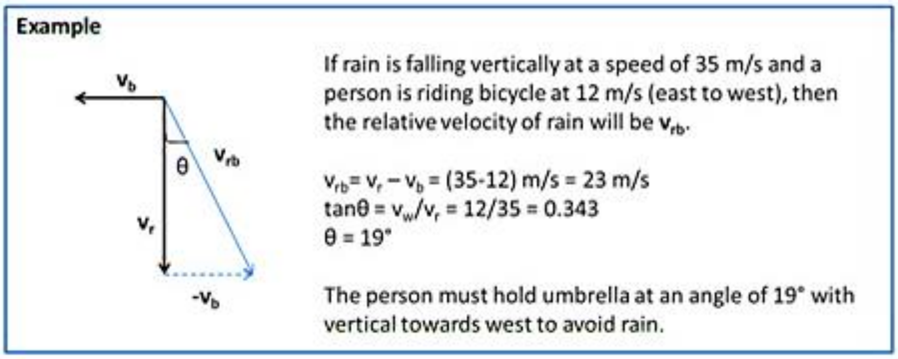
Motion of an object in a plane with constant acceleration		
Velocity	**Velocity in terms of components**	**Displacement**
v = v0+ at	vx = v0x + axt vy = v0y + ayt	r = r0+ v0t+ ½ at2

Relative velocity in two dimensions

The concept of relative velocity in a plane is similar to the concept of relative velocity in a straight line.

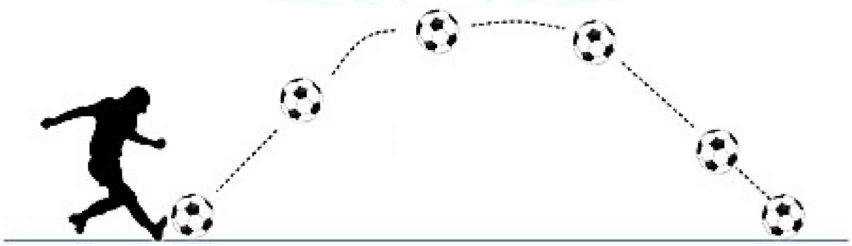


Relative Velocity in a plane



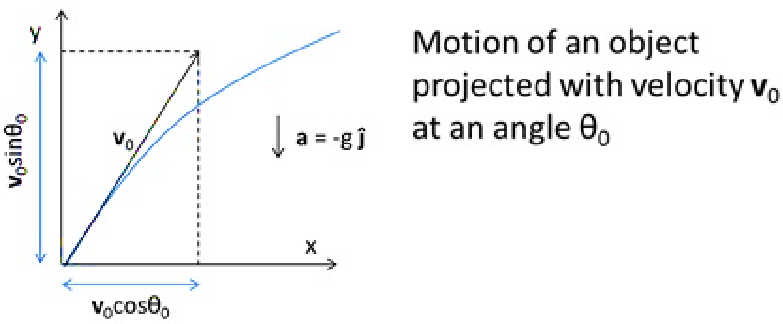
Projectile Motion

An object that becomes airborne after it is thrown or projected is called projectile. Example, football, javelin throw, etc.



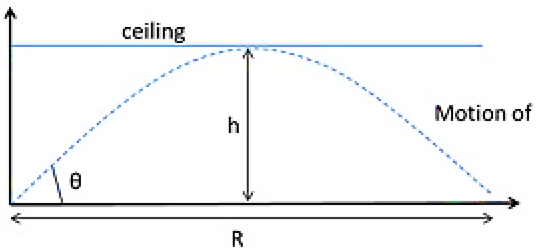
Projectile Motion

- Projectile motion comprises of two parts – horizontal motion of no acceleration and vertical motion of constant acceleration due to gravity.
- Projectile motion is in the form of a parabola, y = ax + bx2.
- Projectile motion is usually calculated by neglecting air resistance to simplify calculations.



Quantity	Value
Components of velocity at time t	vx = v0 cosθ0 vy = v0 sinθ0–gt
Position at time t	x = (v0 cosθ0)t y = (v0 sinθ0)t – ½ gt2
Equation of path of projectile motion	y = (tan θ0)x – gx2/2(v0 cosθ0)2
Time of maximum height	tm = v0 sinθ0 /g
Time of flight	2 tm = 2 (v0 sinθ0 /g)
Maximum height of projectile	hm = (v0 sinθ0)2/2g
Horizontal range of projectile	R = v02 sin 2θ0/g
Maximum horizontal range (θ0=45°)	Rm = v02/g

Example



h = 25 m
V = 40 m/s

If a ball is projected at a speed of 40 m/s and the maximum height it can achieve is 25 m, then the angle 'θ' and maximum distance 'R' should be:

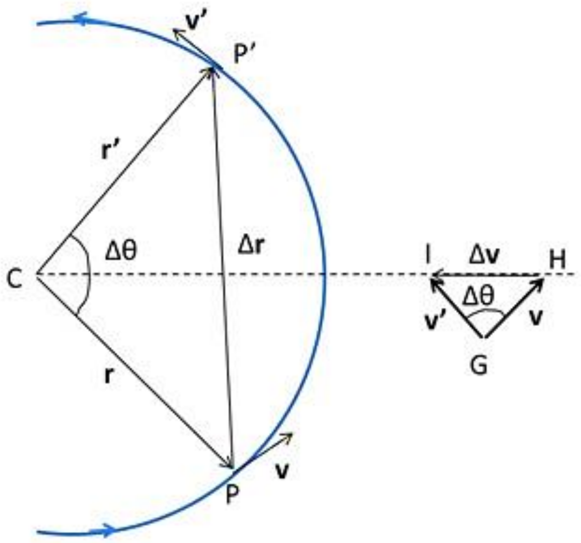
$h = (v_0 \sin\theta_0)^2/2g = (40 \sin\theta)^2/(2 \times 9.8)$
$\sin^2\theta = 0.30625$
$\sin\theta = 0.5534$
$\theta = 33.60°$

$R = v_0^2 \sin 2\theta_0/g$
$R = (40)^2 \times \sin(2 \times 33.6)/9.8$
$R = 1600 \times 0.922 / 9.8$
$R = 150.53$ m

Uniform circular motion

When an object follows a circular path at a constant speed, the motion is called uniform circular motion.

- Velocity at any point is along the tangent at that point in the direction of motion.
- Average velocity between two points is always perpendicular to Average displacement. Also, average acceleration is perpendicular to average displacement.
- For an infinitely small time interval, Δ tà 0, the average acceleration becomes instantaneous acceleration which means that in uniform circular motion the acceleration of an object is always directed towards the center. This is called centripetal acceleration.

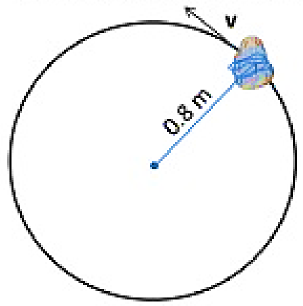


Velocity and Acceleration of an object in uniform circular motion

Quantity	Values
Centripetal Acceleration	ac = v2/R, R – radius of the circle ac = ω2R, ω – angular speed ac = 4π2ν2R, ν – frequency
Angular Distance	Δθ = ω Δt
Speed	v = Rω

Example

A stone tied to one end of a string is whirled at constant speed in a circle. If it makes 14 revolutions in 25 sec, the magnitude and direction of acceleration can be calculated as:



Frequency, ν = revolutions/time taken
$\nu = 14/25$ Hz

Angular frequency, $\omega = 2\pi\nu$
$\omega = 2 \times 22/7 \times 14/25 = 88/25$ rad/sec

Centripetal acceleration, $a_c = \omega^2 r$
$a_c = (88/25)^2 \times 0.8$
$a_c = 9.91\ m/s^2$

The direction of centripetal acceleration is always directed towards the center at all points.

Class : 11th Physics
Chapter- 4 : Motion in a plane

Motion in a plane

Vectors Types

Unit Vector
It has magnitude as one or unity

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Negative Vector
Vectors having same magnitude but opposite direction
eg. $\vec{A}$ is a negative of $\vec{B}$
If, $\vec{A} = -\vec{B}$

Equal Vectors
Vectors having same direction and magnitude

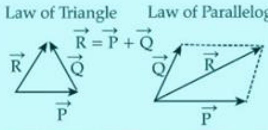
Co-initial vector
Vectors having common starting point.

Zero or Null vector
It has zero magnitude and orbitary direction

Acceleration vector
$\vec{a} = a_x i + a_y \hat{j}$; $a_x = dv_x/dt$ & $a_y = dv_y/dt$
$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

Position of displacement vector
Position vector , $\vec{r} = x\hat{i} + y\hat{j}$
Displacement vector , $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$

Additional vector
Law of Triangle
Law of Parallelogram
$\vec{R} = \vec{P} + \vec{Q}$



Velocity vector
$\vec{v} = v_x\hat{i} + v_y\hat{j}$,
magnitude $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

Relative velocity
$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$
$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$
$\vec{V}_{AB} = -\vec{V}_{BA}$ and
$|\vec{V}_{AB}| = |\vec{V}_{BA}|$

Multiplication of a vector by a real number
$\lambda \times \vec{A} = \lambda\vec{A}$

Substraction of two vectors
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Motion in a plane

Motion of Body under 2 dimensional frame.

Projectile motion
Motion of an object that is in flight after being thrown or projected.

Total time of flight, $T_f = \frac{2u\sin\theta}{g}$
Maximum height, $H_{max} = \frac{u^2\sin^2\theta}{2g}$
Horizontal range, $R = \frac{u^2\sin2\theta}{g}$

Components of velocity (in velocity)
$u_x = u\cos\theta, u_y = u\sin\theta$

Components of acceleration
at any instant
$a_x = 0, a_y = g$

Position after time t
$x = (u\cos\theta)t$;
$y = (u\sin\theta)t - \frac{1}{2}gt^2$

Equation of path of projectile
$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$

Uniform circular motion
When an object follows a circular path at a constant speed , the motion of the object is called uniform circular motion.
Angular velocity ,ω= θ/t Angular acceleration,d=Δω/Δt eg- merry go around.

Centripetal Acceleration
A body in a circular motion acted upon by an acceleration directed towards center of the circular motion.
$a_c = v^2/r = rw^2 = 4\pi^2 rv^2$

Important Questions

Multiple Choice Questions

1. A body of mass 500 gram is rotating in a vertical circle of radius 1 m. What is the difference in its kinetic energies at the top and the bottom of the circle?

   (a) 4.9 J

   (b) 19.8 J

   (c) 2.8 J

   (d) 9.8 J

2. A particle has a displacement of 2 units along the x -axis, 1 unit along the y – axis and 2 units along the z – axis. Then the resultant displacement of the particle is

   (a) 3 units

   (b) 5 units

   (c) 4 units

   (d) 1 units

3. A car is moving on a circular path and takes a turn. If R^1 and R^2 are the reactions on the inner and outer wheels respectively, then

   (a) $R^1 = >R^2$

   (b) $R^1 = R^2$

   (c) $R^1 < R^2$

   (d) $R^1 > R^2$

4. The angle between centripetal acceleration and tangential acceleration is?

   (a) 180°

   (b) 0°

   (c) 90°

   (d) 45°

5. Large angle produces?

   (a) high trajectory

   (b) curve trajectory

   (c) flat trajectory

   (d) straight trajectory

6. He dimensional formula for normal acceleration is

   (a) LT^{-1}

   (b) L^2T^2

   (c) L^3T^{-2}

   (d) LT^{-2}

7. A book is pushed with an initial horizontal velocity of 5.0 meters per second off the top of a desk. What is the initial vertical velocity of the book?

   (a) 10. m/s

   (b) 0 m/s

   (c) 50 m/s

   (d) 2.5 m/s

8. One radian is defined to be the angle subtended where the arc length S is exactly equal to the?

   (a) radius of the circle.

   (b) diameter of the circle.

   (c) circumference of the circle.

   (d) half of radius of the circle.

9. A body travels along the circumference of a circle of radius 2 m with a linear velocity of 6 m/s. Then its angular velocity is

   (a) 6 rad /s

   (b) 3 rad /s

   (c) 2 rad / s

   (d) 4 rad / s

10. One° (1°) is equal to?

    (a) 0.1745 rad

    (b) 0.01745 rad

    (c) 0.001745 rad

    (d) 7.1745 rad

Very Short:

1. Under what condition $|a + b| = |a| + |b|$ holds good?

2. Under what condition $|a - b| = |a| - |b|$ holds good?

3. The sum and difference of the two vectors are equal in magnitude

   i.e. $|a + b|=|a - b|$. What conclusion do you draw from this?

4. What is the angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$?

5. What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant?

6. When $a - b = a + b$ condition holds good than what can you say about b?

7. What is the magnitude of the component of the $9\hat{i} - 9\hat{j} + 19\hat{k}$ vector along the x-axis?
8. Can displacement vector be added to force vector?
9. What is the effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar?
10. (a) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i}$ vectors?

    (b) What is the angle between $\hat{i} - \hat{j}$ and the x-axis?

    (c) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?ss

Short Questions:

1. Name two quantities that are the largest when the maximum height attained by the projectile is largest.
2. A stone dropped from the window of a stationary railway carriage takes 2 seconds to reach the ground. At what time the stone will reach the ground when the carriage is moving with

   (a) the constant velocity of $80 kmh^{-1}$

   (b) constant acceleration of $2 ms^{-2}$?
3. Can a particle accelerate when its speed is constant? Explain.
4. (a) Is circular motion possible at a constant speed or at constant velocity? Explain.

   (b) Define frequency and time period.
5. When the component of a vector A along the direction of vector B is zero, what can you conclude about the two vectors?
6. Comment on the statement whether it is true or false "Displacement vector is fundamentally a position vector." Why?
7. Does the nature of a vector changes when it is multiplied by a scalar?
8. Can the walk of a man be an example of the resolution of vectors? Explain.

Long Questions:

1. Discuss the problem of a swimmer who wants to cross the river in the shortest time.
2. State and prove parallelogram law of vector addition. Discuss some special cases.
3. Derive the relation between linear velocity and angular velocity. Also, deduce its direction.

Assertion Reason Questions:

1. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180°.

   Reason: At the highest point, velocity of projectile will be in horizontal direction only.
2. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.

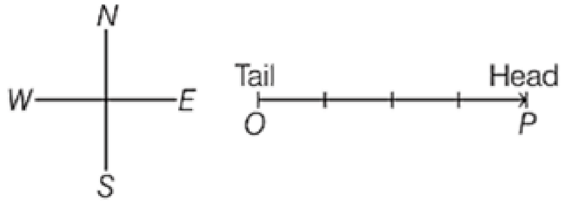
   Reason: The maximum height of projectile is independent of particle mass.

Case Study Questions:

1. Vectors are the physical quantities which have both magnitudes and directions and obey the triangle/parallelogram laws of addition and subtraction. It is specified by giving its magnitude by a number and its direction. e.g., Displacement, acceleration, velocity, momentum, force, etc. A vector is represented by a bold face type and also by an arrow placed over a letter, i.e.

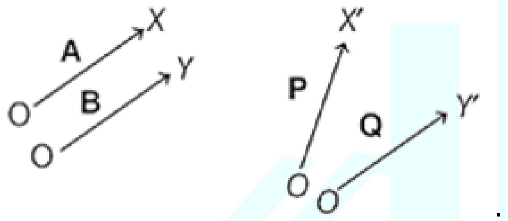
$$F, a, b \text{ or } \vec{F}, \vec{a}, \vec{b}$$

The length of the line gives the magnitude, and the arrowhead gives the direction. The point P is called head or terminal point and point O is called tail or initial point of the vector **OP**.



i. Amongst the following quantities, which is not a vector quantity?
(a) Force
(b) Acceleration
(c) Temperature
(d) Velocity

ii. Set of vectors A and B, P and Q are as shown below



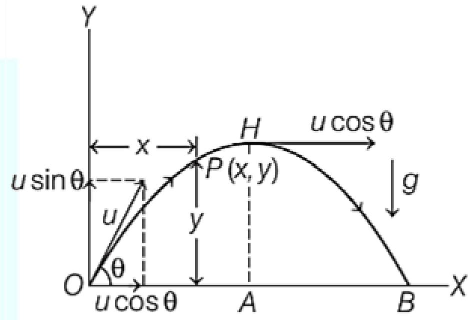
Length of A and B is equal, similarly length of P and Q is equal. Then, the vectors which are equal, are
(a) A and P
(b) P and Q
(c) A and B
(d) B and Q

iii. $|\lambda A| = \lambda |A|$, if
(a) $\lambda > 0$
(b) $\lambda < 0$
(c) $\lambda = 0$
(d) $\lambda \neq 0$

iv. Among the following properties regarding null vector which is incorrect?
(a) A + 0 = A
(b) $\lambda 0 = \lambda$
(c) 0A = 0
(d) A – A = 0

v. The x and y components of a position vector P have numerical values 5 and 6, respectively. Direction and magnitude of vector P are
(a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$
(b) $\tan^{-1}\left(\frac{5}{6}\right)$ and $\sqrt{61}$
(c) 60° and 8
(d) 30° and 9

2. Projectile motion is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface, and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its trajectory, which is shown below. When a projectile is projected obliquely, then its trajectory is as shown in the figure below



Here velocity u is resolved into two components, we get (a) u cos θ along OX and (b) u sin θ along OY

i. The example of such type of motion is
(a) Motion of car on a banked road
(b) Motion of boat in sea
(c) A javelin thrown by an athlete
(d) Motion of ball thrown vertically upward

ii. The acceleration of the object in horizontal direction is
(a) Constant
(b) Decreasing
(c) Increasing
(d) Zero

iii. The vertical component of velocity at point H is
(a) Maximum
(b) Zero
(c) Double to that at O
(d) Equal to horizontal component

iv. A cricket ball is thrown at a speed of 28 m/s in a direction 30° with the horizontal. The time taken by the ball to return to the same level will be

(a) 2.0 s

(b) 3.0 s

(c) 4.0 s

(d) 2.9 s

v. In above case, the distance from the thrower to the point where the ball returns to the same level will be

(a) 39 m

(b) 69 m

(c) 68 m

(d) 72 m

Answer Key

Multiple Choice Answers-

1. **Answer:** (d) 9.8 J
2. **Answer:** (a) 3 units
3. **Answer:** (c) $R^1 < R^2$
4. **Answer:** (c) 90°
5. **Answer:** (a) high trajectory
6. **Answer:** (d) LT^{-2}
7. **Answer:** (b) 0 m/s
8. **Answer:** (a) radius of the circle.
9. **Answer:** (b) 3 rad /s
10. **Answer:** (b) 0.01745 rad

Very Short Answers:

1. **Answer:** When a and b act in the same direction i. e. when 0 = 0 between • them, then |a + b|=|a| + |b|.
2. **Answer:** The condition |a – b|=|a| – |b| holds goods when a and b act in the opposite direction.
3. **Answer:** The two vectors are equal in magnitude and are perpendicular to each other.
4. **Answer:** The given vectors act along two parallel lines in opposite directions i.e. they are anti-parallel, so the angle between them is 180°.
5. **Answer:** 3, If three vectors can be represented completely by the three sides of a triangle taken in the same order, then their resultant is zero.
6. **Answer:** For a – b = a + b condition to hold good, b must be a null vector.
7. **Answer:** 9.
8. **Answer:** No.
9. **Answer:** There is no effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar.
10. **Answer:**
    (a) 45°
    (b) 45°
    (c) 90°

Short Questions Answers:

1. **Answer:** Time of flight and the vertical component of velocity are the two quantities that are the largest when the maximum height attained by the projectile is the largest.
2. **Answer:** The time taken by the freely falling stone to reach the ground is given by

$$t = \sqrt{\frac{2h}{g}}$$

In both cases, the stone will fall through the same height as it is falling when the railway carriage is stationary. Hence the stone will reach the ground after 2 seconds.

3. **Answer:** Yes. A particle can be accelerated if its velocity changes. A particle having uniform circular motion has constant speed but its direction of motion changes continuously. Due to this, there is a change in velocity and hence the particle is moving with variable velocity. Thus, particle is accelerating.
4. **Answer:**
   (a) Circular motion is possible at a constant speed because, in a circular motion, the magnitude of the velocity i.e. speed remains constant while the direction of motion changes continuously.
   (b) Frequency is defined as the number of rotations completed by a body in one second and the time period is defined as the time taken by an object to complete one rotation.
5. **Answer:**
   The two vectors A and B are perpendicular to each other.

Explanation: Let θ = angle between the two vectors A and B component of vector A along the direction of B is obtained by resolving A i.e. A cos θ.

Now according to the statement

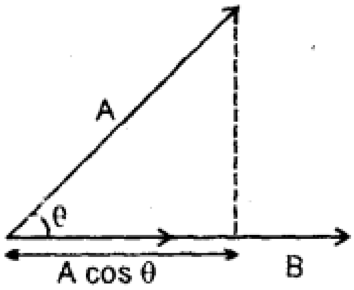
$A \cos \theta = 0$

or

$\cos \theta = 0 = \cos 90°$

$\theta = 90°$

i.e. $A \perp B$



Hence proved.

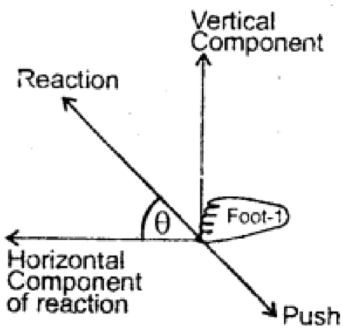
6. **Answer:** The given statement is true. The displacement vector gives the position of a point just like the position vector. The only difference between the displacement and the position vector is that the displacement vector gives the position of a point with reference to a point other than the origin, while the position vector gives the position of a point with reference to the origin. Since the choice of origin is quite arbitrary, so the given statement.

7. **Answer:** The nature of a vector may or may not be changed when it is multiplied.

   For example, when a vector is multiplied by a pure number like 1, 2, 3, ... etc., then the nature of the vector does not change. On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes.

   For example, when acceleration (vector) is multiplied by a mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration.

8. **Answer:** Yes, when a man walks, he pushes the ground with his foot. In return, an equal and opposite reaction acts on his foot. The reaction is resolved into two components: horizontal and vertical components. The horizontal component of the reaction helps the man to move forward while the vertical component balances the weight of the man.

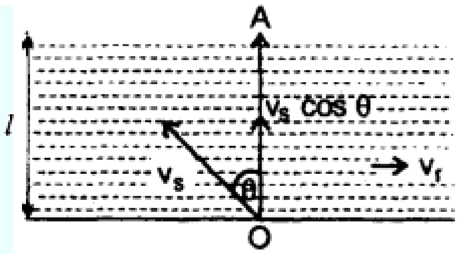


Long Questions Answers:

1. **Answer:**

   Let v_s and v_r be the velocities of swimmer and river respectively.

   Let v = resultant velocity of vs and v_r



1. Let the swimmer begins to swim at an angle θ with the line OA where OA is ⊥ to the flow of the river.

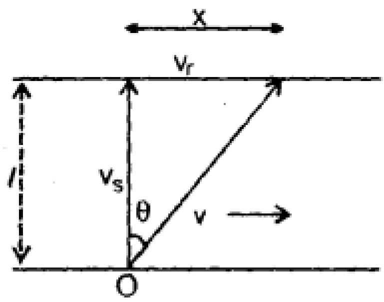
   If t = time taken to cross the river, then

   $$t = \frac{1}{\upsilon_s \cos \theta} \quad \text{....(i)}$$

   where l = breadth of the river

   For t to be minimum, cos 0 should be maximum.

   i.e., $\cos \theta = 1$



   This is possible if θ = 0

   Thus, we conclude that the swimmer should swim in a direction perpendicular to the direction of the flow of the river.

2. $v = \sqrt{\upsilon_s^2 + \upsilon_r^2}$

   where $v\vec{\upsilon}$ is the resultant velocity of v_s and v_r.

3. $\tan\theta = \dfrac{\upsilon_r}{v_s} = \dfrac{x}{l}$

or

$$x = l\frac{v_r}{V_s}$$

4. $t = \dfrac{l}{V_s}$

2. Answer:

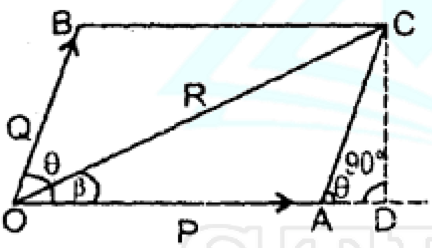
It states that if two vectors can be represented completely (i.e. both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point.

Proof: Let P and Q be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB s.t.

$$\overrightarrow{OA} = P,\ \overrightarrow{OB} = Q$$

or

$$|\overrightarrow{OA}| = |P|,\ |\overrightarrow{OB}| = |Q|$$



θ = angle between them = ∠AOB

If R be their resultant, then it will be represented completely by the diagonal OC through point O s.t. OC = R

The magnitude of R: Draw CD ⊥ to OA produced,

$$\therefore \quad \angle DAC = \angle AOB = \theta$$

Now it right angled triangle ODC,

$OC^2 = OD^2 + DC^2$

$(OA + AD)^2 + DC^2$

$OA^2 + AD^2 + 2.OA.AD + DC^2$

$OA^2 + (AD^2 + DC^2) + 2OA.AD$ ....(i)

Also in $r.t.\ \angle D\,\Delta ADC$,

$AC^2 = AD^2 + DC^2$ ....(ii)

Also $\dfrac{AD}{AC} = \cos\theta$

or AD = AC cos θ ....(iii)

and $\dfrac{DC}{AC} = \sin\theta$

or DC = AC sin θ ....(iv)

∴ from equation (i), (ii), (iii), we get

$OC^2 = OA^2 + AC^2 + 1.OA.AC\cos\theta$

or $OC = \sqrt{OA^2 + AC^2 + 2.OA.AC\cos\theta}$ ....(v)

As OC = R, OA = P, AC = OB = Q ....(vi)

∴ from equation (i), (ii), (iii), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad ....(vii)$$

eqn. (vii) gives the magnitude of R.

The direction of R: Let β be the angle made by R with P

∴ in rt. ∠d ΔODC,

$$\tan\beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC\sin\theta}{OA + AC\cos\theta}$$

[by using (iii) and (iv)]

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \quad ....(viii)$$

Special cases: (a) When two vectors are acting in the same direction:

Then θ = 0°

$$\therefore \quad R = \sqrt{(P+Q)^2} = P + Q$$

and $\tan\beta = \dfrac{Q.0}{P+Q} = 0$ or β = 0°

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction, and their resultant acts in the direction of P and Q.

(b) When two vectors act in the opposite directions:

Then θ = 180°

∴ cos θ = –1 and sin θ = 0

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ(-1)}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2}$$

$$= (P-Q) \text{ or } (Q-P)$$

and $\tan\beta = \dfrac{Q\times 0}{P+Q(-1)} = 0$

$= \tan 0°$ or $\tan 180°$

$\therefore$ $\beta = 0°$ or $180°$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of the bigger vector.

(c) If $\theta = 90°$ *i.e.* if $P \perp Q$,

then $\cos 90° = 0$

and

$\sin 90° = 1$

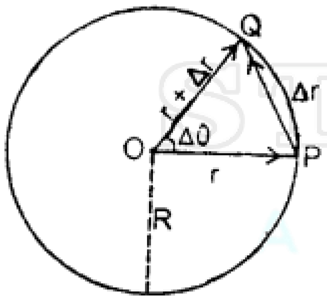
$R = \sqrt{P^2 + Q^2}$

and

$\tan\beta = \dfrac{O}{P}$

3. Answer:

Let R be the radius of the circular path of centre O on which an object is moving with uniform angular velocity co. Let v = its linear velocity. Let the object move from point P at time t to point Q at time t + Δt. If r and r + Δr be its position vectors at point P and Q respectively, then



$OP = r$

and $OQ = r + \Delta r$

Also $|r| = |r + \Delta r| = R$

=radius of circle.

$\therefore$ Linear displacement of the particle from P to Q in small time interval Δt = Δr.

Let Δθ = its angular displacement

$\therefore$ $\omega = \frac{\Delta\theta}{\Delta t}$

or

$\Delta\theta = \omega\Delta t$ ....(1)

Also we know that $\Delta\theta = \frac{\widehat{PQ}}{R}$ ....(2)

$\therefore$ from (1) and (2), we get

$$\omega\Delta t = \frac{PQ}{R} \text{ or } \frac{PQ}{\Delta t} = \omega R \quad ....(3)$$

Now when $\Delta t \to 0$, then from eqn. (1) $\Delta\theta \to 0$

so arc PQ = $\widehat{PQ}$ = chord PQ

Thus eqn. (3) reduces to

$$\frac{PQ}{\Delta t} = \omega R$$

or $v = R\omega$

where v = $\frac{PQ}{\Delta t}$ is the linear velocity of the object.

Direction of velocity vector: In isosceles ΔOPQ,

$$\angle PQO + \angle OPQ + \angle QOP = 180°$$

As $\angle Q = \angle P$

$\therefore$ $2\angle P + \Delta\theta = \pi$

or $\angle QPO = \dfrac{\pi}{2} - \dfrac{\Delta\theta}{2}$

$= \dfrac{\pi}{2} - \dfrac{\omega\Delta t}{2}$ ....(4)

when $\Delta t \to 0$, $\angle QPO \to \frac{\pi}{2}$

i.e. $\overrightarrow{OP}$ tends to become $\perp$ to $\overrightarrow{OP}$

or

$\overrightarrow{OP}$ tends to lie along the tangent at P. Hence velocity vector at P is directed along the tangent to the circle in the direction of motion

Assertion Reason Answer:

1. (d) Assertion is incorrect, reason is correct.
2. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Case Study Answer:

1. i. (c) Temperature

Explanation:

Temperature is not a vector quantity because it has magnitude only. However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

ii. (c) A and B

Explanation:

Two vectors are said to be equal, if and only if they have the same magnitude and direction. Among the given vectors A and B are equal vectors as they have same magnitude (length) and direction. However, P and Q are not equal even though they are of same magnitude because their directions are different.

iii. (a) $\lambda > 0$

Explanation:

$|\lambda A| = \lambda |A|$, if $\lambda > 0$ as multiplication of vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is same as that of A.

iv. (b) $\lambda 0 = \lambda$

Explanation:

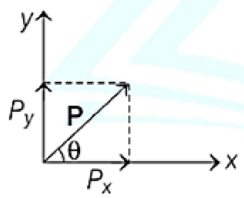
Null vector 0 is a vector, whose magnitude is zero and its direction cannot be specified. So, it means, $|0| = 0$. Thus, $\lambda 0 = 0$. Hence, property given in option (b) is incorrect.

v. (a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$

Explanation:

Let **P** be as shown in the figure, then according to the given information

$P_x = 5,\ P_y = 6$



$$\therefore \quad |\boldsymbol{P}| = \sqrt{P_x^2 + P_y^2}$$

$$= \sqrt{25 + 36}$$

$$\Rightarrow \quad |\boldsymbol{P}| = \sqrt{61}$$

and $\quad \tan\theta = \dfrac{P_y}{P_x} = \dfrac{6}{5}$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{6}{5}\right)$$

2. i. (c) a javelin thrown by an athlete

Explanation:

A javelin thrown by an athlete is an example of projectile motion.

ii. (d) zero

Explanation:

The horizontal component of velocity (u cos θ) is constant throughout the motion, so there will be no acceleration in horizontal direction.

iii. (b) zero

Explanation:

As the vertical components of velocity (u sin θ) decreases continuously with height, from O to H, due to downward force of gravity and becomes zero at H

iv. (d) 2.9 s

Explanation:

The time taken by the ball to return to the same level,

$$T = \frac{2\upsilon_0 \sin\theta}{g} = \frac{2 \times 28 \times \sin 30^\circ}{9.8} \approx 2.9\, s$$

v. (b) 69 m

Explanation:

The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{\upsilon_0^2 \sin 2\theta}{g} = \frac{28 \times 28 \times \sin 60^\circ}{9.8} \approx 69\, m$$

Laws of Motion 5

Point Mass

(1) An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

(2) Object with zero dimension considered as a point mass.

(3) Point mass is a mathematical concept to simplify the problems.

Inertia

(1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

(2) Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

(3) Inertia has no units and no dimensions

(4) Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

Linear Momentum

(1) Linear momentum of a body is the quantity of motion contained in the body.

(2) It is measured in terms of the force required to stop the body in unit time.

(3) It is measured as the product of the mass of the body and its velocity i.e., Momentum = mass × velocity. If a body of mass m is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$

(4) It is a vector quantity and it's direction is the same as the direction of velocity of the body.

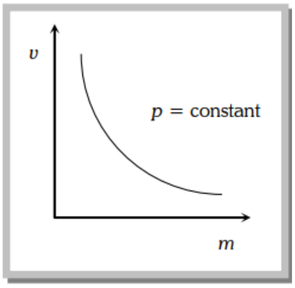
(5) Units : kg-m/sec [S.I.], g-cm/sec [C.G.S.]

(6) Dimension : $[MLT^{-1}]$

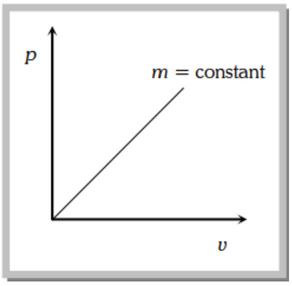
(7) If two objects of different masses have same momentum, the lighter body possesses greater velocity.

$p = m_1v_1 = m_2v_2 = \text{constant}$

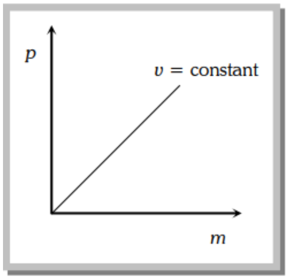
$\therefore \quad \frac{v_1}{v_2} = \frac{m_1}{m_2}$ i.e. $v \propto \frac{1}{m}$ [As p is constant]



(8) For a given body $p \propto v$



(9) For different bodies at same velocities $p \propto m$



Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

(1) If no net force acts on a body, then the velocity of the body cannot change i.e. the body cannot accelerate.

(2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types :

Inertia of rest, Inertia of motion, Inertia of direction

(3) **Inertia of rest :** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

Example :

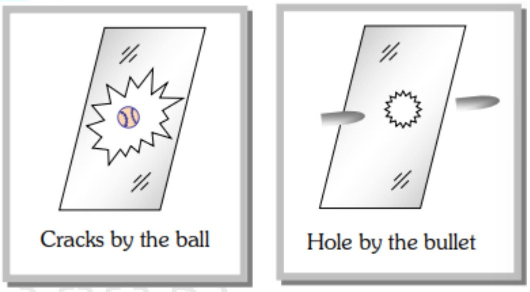
(i) A person who is standing freely in bus, thrown backward, when bus starts suddenly.

When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.

Note: If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

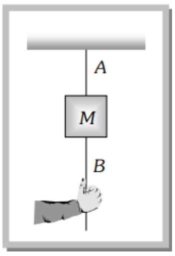
(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

(iii) A bullet fired on a window pane makes a clean hole through it while a stone breaks the whole window because the bullet has a speed much greater than the stone. So its time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.



(iv) In the arrangement shown in the figure :

(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.



(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

(v) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

(vi) The dust particles in a durree falls off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.

(4) **Inertia of motion :** It is the inability of a body to change itself its state of uniform motion i.e., a body in uniform motion can neither accelerate nor retard by its own.

Example :

(i) When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.

(ii) A person jumping out of a moving train may fall forward.

(iii) An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

(5) **Inertia of direction :** It is the inability of a body to change by itself direction of motion.

Example :

(i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

(ii) The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Newton's Second Law

(1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass m, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = \frac{d\vec{p}}{dt}$$

or $\vec{F} = \dfrac{d\vec{p}}{dt}$ (K = 1 in C.G.S. and S.I. units)

$$\text{or } \vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

(As $a = \dfrac{d\vec{v}}{dt}$ = acceleration produced in the body)

$$\therefore \vec{F} = m\vec{a}$$

Force = mass × acceleration

Force

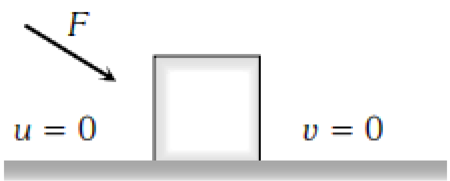
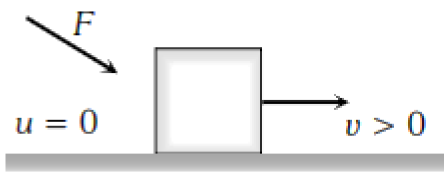
(1) Force is an external effect in the form of a push or pull which

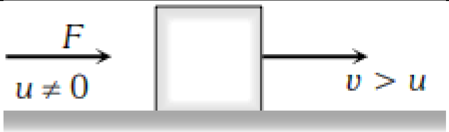
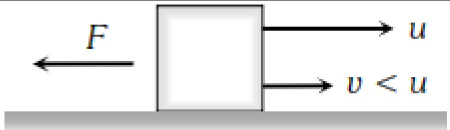
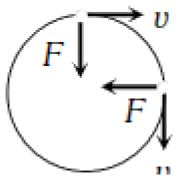
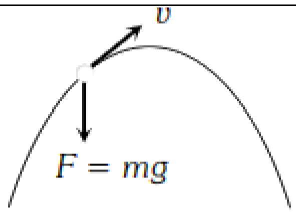
(i) Produces or tries to produce motion in a body at rest.

(ii) Stops or tries to stop a moving body.

(iii) Changes or tries to change the direction of motion of the body.

Table : Various condition of force application

	Body remains at rest. Here force is trying to change the state of rest.
	Body starts moving. Here force changes the state of rest.

	In a small interval of time, force increases the magnitude of speed and direction of motion remains same.
	In a small interval of time, force decreases the magnitude of speed and direction of motion remains same.
	In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity.
	In non-uniform circular motion, elliptical, parabolic or hyperbolic motion force acts at an angle to the direction of motion. In all these motions. Both magnitude and direction of velocity changes.

(2) Dimension : Force = mass × acceleration

$[F] = [M][LT^{-2}] = [MLT^{-2}]$

(3) Units : Absolute units : (i) Newton (S.I.) (ii) Dyne (C.G.S)

Gravitational units : (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)

Newton : One Newton is that force which produces an acceleration of $1 m/s^2$ in a body of mass 1 Kilogram.

$\therefore$ 1 Newton = $1 kg-m/s^2$

Dyne : One dyne is that force which produces an acceleration of $1 cm/s^2$ in a body of mass 1 gram.

$\therefore$ 1 Dyne = $1 gm\ cm/sec^2$

Relation between absolute units of force 1 Newton = 10^5 Dyne

Kilogram-force : It is that force which produces an acceleration of $9.8 m/s^2$ in a body of mass 1 kg.

$\therefore$ 1 kg-f = 9.80 Newton

Gram-force : It is that force which produces an acceleration of $980 cm/s^2$ in a body of mass 1gm.

$\therefore$ 1 gm-f = 980 Dyne

(4) $\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

(5) If m is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

(6) If force and acceleration have three component along x, y and z axis, then

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

From above it is clear that $F_x = ma_x$, $F_y = ma_y$, $F_z = ma_z$

(7) No force is required to move a body uniformly along a straight line with constant speed.

$\vec{F} = m\vec{a}$ $\qquad \therefore \vec{F} = 0 \ \ (\text{As } \vec{a} = 0)$

(8) When force is written without direction then positive force means repulsive while negative force means attractive.

Example : Positive force – Force between two similar charges

Negative force – Force between two opposite charges

(9) Out of so many natural forces, for distance 10^{-15} metre, nuclear force is strongest while gravitational force weakest. $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$

(10) Ratio of electric force and gravitational force between two electron's $F_e/F_g = 10^{43}$ $\therefore F_e >> F_g$

(11) **Constant force :** If the direction and magnitude of a force is constant. It is said to be a constant force.

(12) Variable or dependent force :

(i) Time dependent force : In case of impulse or motion of a charged particle in an alternating electric field force is time dependent.

(ii) Position dependent force : Gravitational force between two bodies $\frac{Gm_1m_2}{r^2}$

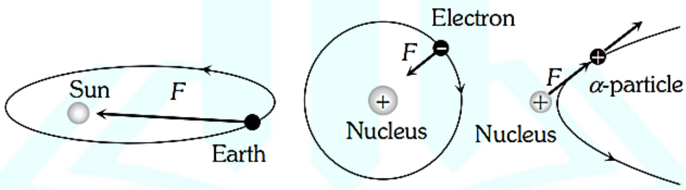
or Force between two charged particles = $\frac{q_1q_2}{4\pi\varepsilon_0 r^2}$.

(iii) Velocity dependent force : Viscous force ($6\pi\eta rv$)

Force on charged particle in a magnetic field ($qvB\sin\theta$)

(13) **Central force :** If a position dependent force is directed towards or away from a fixed point it is said to be central otherwise non-central.

Example : Motion of Earth around the Sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.



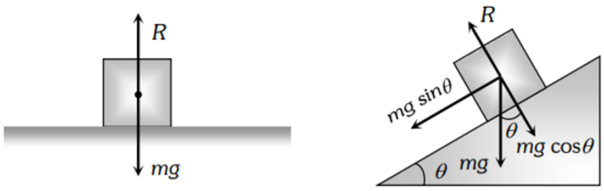
(14) **Conservative or non conservative force :** If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.

Example : Conservative force : Gravitational force, electric force, elastic force.

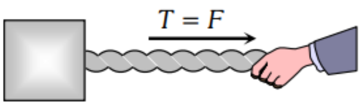
(15) Common forces in mechanics :

(i) **Weight :** Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

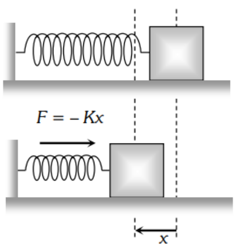
(ii) **Reaction or Normal force :** When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.



(iii) **Tension :** The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



(iv) **Spring force :** Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by F = −Kx; where x is the change in length and K is the spring constant (unit N/m).



Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

(1) When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.

(2) Forces in nature always occurs in pairs. A single isolated force is not possible.

(3) Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called 'Action' and the counter force experienced by it is called 'Reaction'.

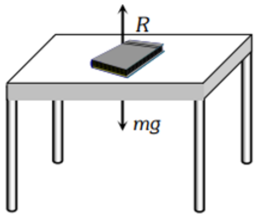
(4) Action and reaction never act on the same body. If it were so, the total force on a body would have always been zero i.e. the body will always remain in equilibrium.

(5) If $\vec{F}_{AB}$ = force exerted on body A by body B (Action) and $\vec{F}_{BA}$ = force exerted on body B by body A (Reaction)

(6) **Example :** (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.

The table supports the book, by exerting an equal force on the book. This is the force of reaction.

As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.

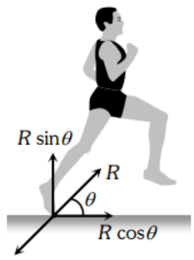


(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

(iv) Rebounding of rubber ball takes place due to third law of motion.

(v) While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.



(vi) It is difficult to walk on sand or ice.

(vii) Driving a nail into a wooden block without holding the block is difficult.

Frame of Reference

(1) A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

(2) The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities like position, velocity, acceleration etc. of an object in this coordinate system.

(3) Frame of reference are of two types : (i) Inertial frame of reference (ii) Non-inertial frame of reference.

(i) **Inertial frame of reference :**

(a) A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.

(b) In inertial frame of reference Newton's laws of motion holds good.

(c) Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.

(d) Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.

(e) To measure the acceleration of a falling apple, earth can be considered as an inertial frame.

(f) To observe the motion of planets, earth can not be considered as an inertial frame but for this purpose the sun may be assumed to be an inertial frame.

Example : The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road.

(ii) **Non-inertial frame of reference**

(a) Accelerated frame of references are called non-inertial frame of reference.

(b) Newton's laws of motion are not applicable in non-inertial frame of reference.

Example : Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

Impulse

- Impulse is defined as a force multiplied by time it acts over.
- For example: Tennis racket strikes a ball, an impulse is applied to the ball. The racket puts a force on the ball for a short time period.

$$F\frac{\triangle t}{\triangle p}$$

$$F = \frac{\triangle p}{\triangle t} = \textit{Rate of Change of momentum}$$

Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$ In the absence of external force $\vec{F} = 0$ then

$\vec{p}$ = constant

i.e. $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ = constant.

or $m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...$ = constant.

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference, though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force, by law of conservation of linear momentum.

$\vec{p}_1 + \vec{p}_2$ = constant.

$\therefore\ m_1\vec{v}_1 + m_2\vec{v}_2$ = constant.

Differentiating above with respect to time

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore\ \vec{F}_2 = -\vec{F}_1$

i.e. for every action there is an equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum.

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.

Let m_G = mass of gun, m_B = mass of bullet,

v_G = velocity of gun, v_B = velocity of bullet

Initial momentum of system = 0

Final momentum of system = $m_G \vec{v}_G + m_B \vec{v}_B$

By the law of conservation of linear momentum $m_G \vec{v}_G + m_B \vec{v}_B = 0$

So recoil velocity $\vec{v}_G = \frac{m_B}{m_G} \vec{v}_B$

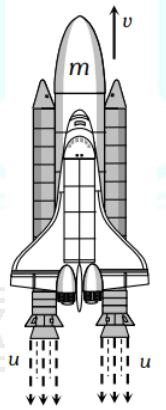
(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \frac{1}{m_G}$ i.e. higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{man}}$$

(iv) **Rocket propulsion :** The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.



Let m_0 = initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

m_r = residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\frac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption = rate of ejection of the fuel.

(a) Thrust on the rocket : $F = -u\frac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$F = -u\frac{dm}{dt}$ (if effect of gravity is neglected)

(b) Acceleration of the rocket : $a = \frac{u}{m}\frac{dm}{dt} - g$

and if effect of gravity is neglected $a = \frac{u}{m}\frac{dm}{dt}$

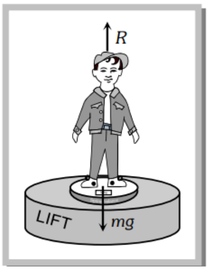
(c) Instantaneous velocity of the rocket : $v = u\log_e\left(\frac{m_0}{m}\right) - gt$

and if effect of gravity is neglected $v = u\log_e\left(\frac{m_0}{m}\right) = 2.303u\log_{10}\left(\frac{m_0}{m}\right)$

(d) Burnt out speed of the rocket : $v_b = v_{max} = u\log_e\left(\frac{m_0}{m_r}\right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

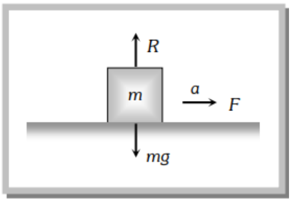
Apparent Weight of a Body in a Lift:



When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg.

This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the apparent weight of the body.

Acceleration of Block on Horizontal Smooth Surface:



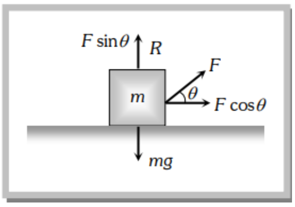
(1) **When a pull is horizontal**

$R = mg$

and $F = ma$

$\therefore$ $a = F/m$

(2) **When a pull is acting at an angle (θ) to the horizontal (upward)**



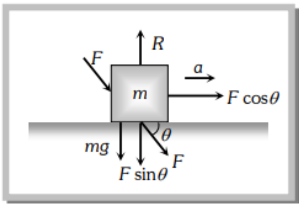
$R + F \sin\theta = mg$

$\Rightarrow$ $R = mg - F \sin\theta$

and $F \cos\theta = ma$

$\therefore$ $a = \frac{F\cos\theta}{m}$

(3) **When a push is acting at an angle (θ) to the horizontal (downward)**

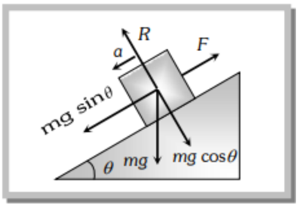


$R = mg + F \sin\theta$

and $F \cos\theta = ma$

$a = \frac{F\cos\theta}{m}$

Acceleration of Block on Smooth Inclined Plane:



(1) **When inclined plane is at rest**

Normal reaction $R = mg \cos\theta$

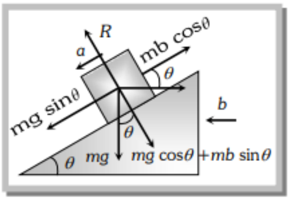
Force along a inclined plane

$F = mg \sin\theta$

$ma = mg \sin\theta$

$\therefore$ $a = g \sin\theta$

(2) **When a inclined plane given a horizontal acceleration 'b'**



Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.

Normal reaction $R = mg\cos\theta + mb\sin\theta$

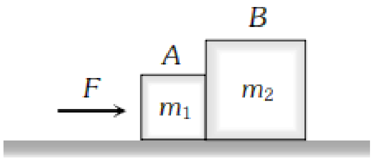
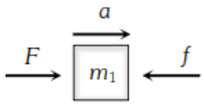
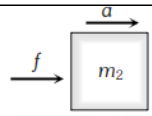
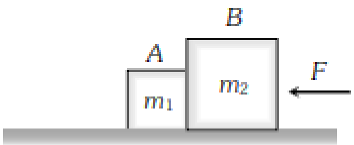
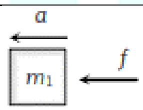
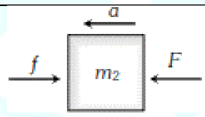
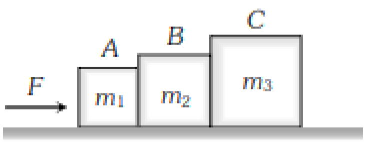
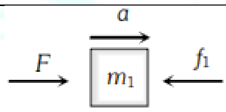
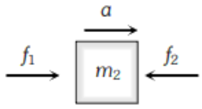
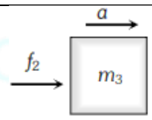
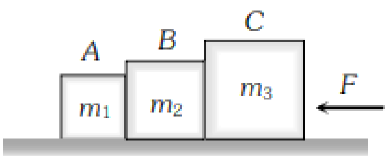
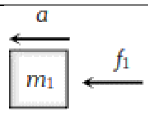
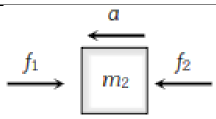
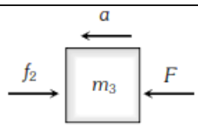
and $ma = mg\sin\theta - mb\cos\theta$

$\therefore\ a = g\sin\theta - b\cos\theta$

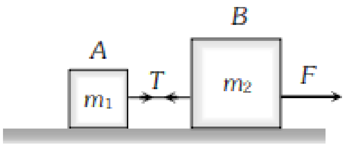
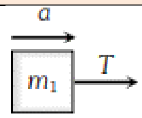
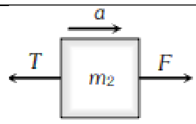
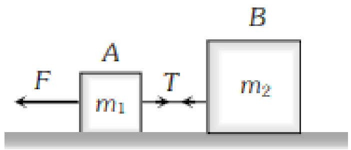
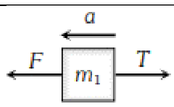
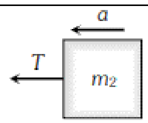
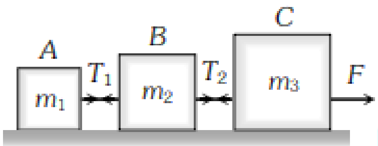
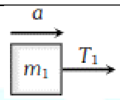
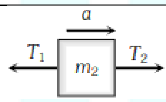
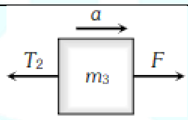
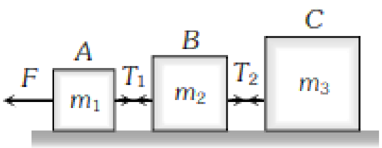
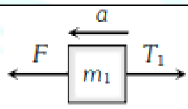
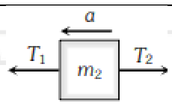
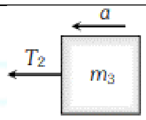
Note : The condition for the body to be at rest relative to the inclined plane : $a = g\sin\theta - b\cos\theta = 0$

$\therefore\ a = g\tan\theta$

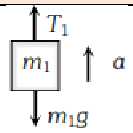
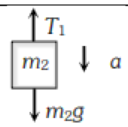
Motion of Blocks in Contact

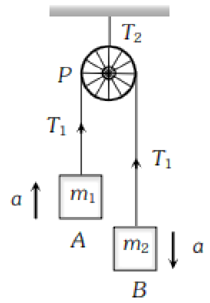
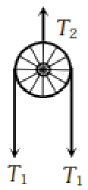
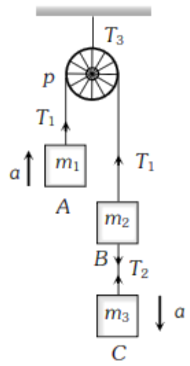
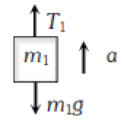
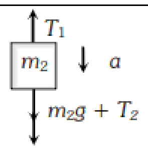
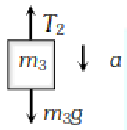
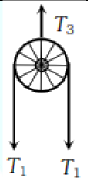
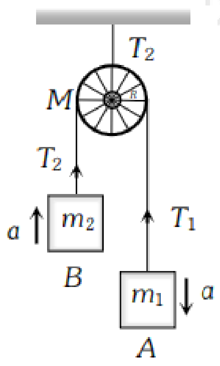
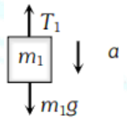
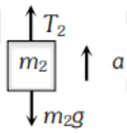
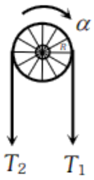
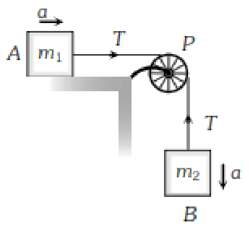
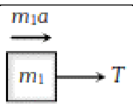
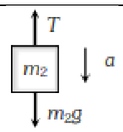
Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2a$	$f = \frac{m_2F}{m_1 + m_2}$
		$f = m_1a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2a$	$f = \frac{m_1F}{m_1 + m_2}$
		$F - f_1 = m_1a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3a$	$f_2 = \frac{m_3F}{m_1 + m_2 + m_3}$
		$f_1 = m_1a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2a$	$f_1 = \frac{m_1F}{m_1 + m_2 + m_3}$
		$F - f_2 = m_3a$	$f_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

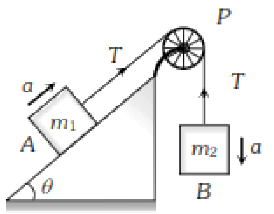
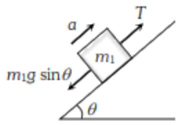
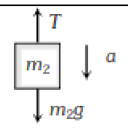
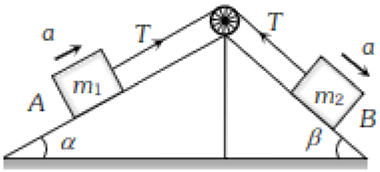
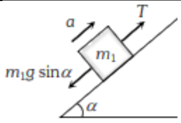
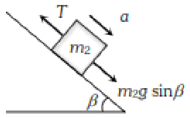
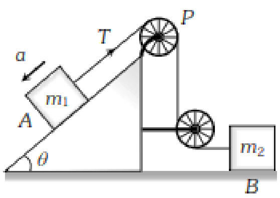
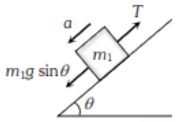
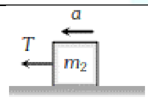
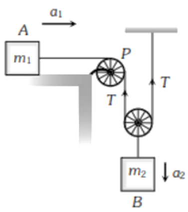
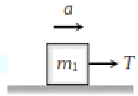
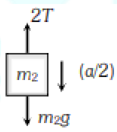
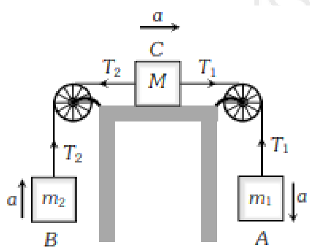
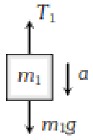
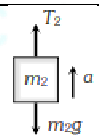
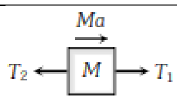
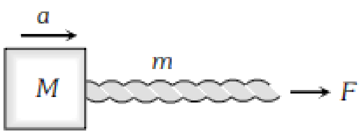
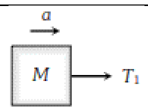
Motion of Blocks Connected by Mass Less String

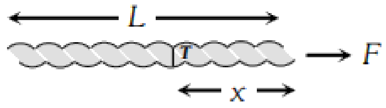
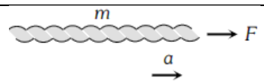
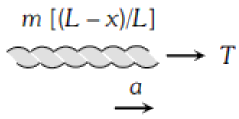
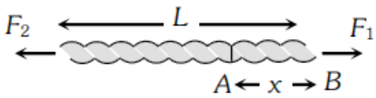
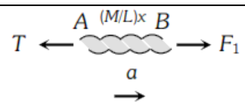
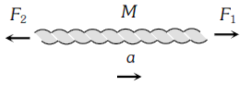
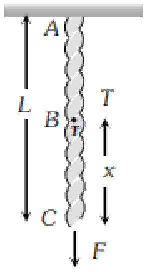
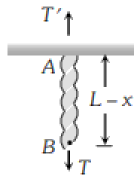
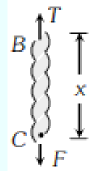
Condition	Free body diagram	Equation	Tension and acceleration
		T = m_1a	$a=\dfrac{F}{m_1+m_2}$
		F – T = m_2a	$T=\dfrac{m_1F}{m_1+m_2}$
		F – T = m_1a	$a=\dfrac{F}{m_1+m_2}$
		T = m_2a	$T=\dfrac{m_2F}{m_1+m_2}$
		$T_1 = m_1a$	$a=\dfrac{F}{m_1+m_2+m_3}$
		$T_2 - T_1 = m_2a$	$T_1=\dfrac{m_1F}{m_1+m_2+m_3}$
		$F - T_2 = m_3a$	$T_2=\dfrac{(m_1+m_2)F}{m_1+m_2+m_3}$
		$F - T_1 = m_1a$	$a=\dfrac{F}{m_1+m_2+m_3}$
		$T_1 - T_2 = m_2a$	$T_1=\dfrac{(m_2+m_3)F}{m_1+m_2+m_3}$
		$T_2 = m_3a$	$T_2=\dfrac{m_3F}{m_1+m_2+m_3}$

Motion of Connected Block Over A Pulley

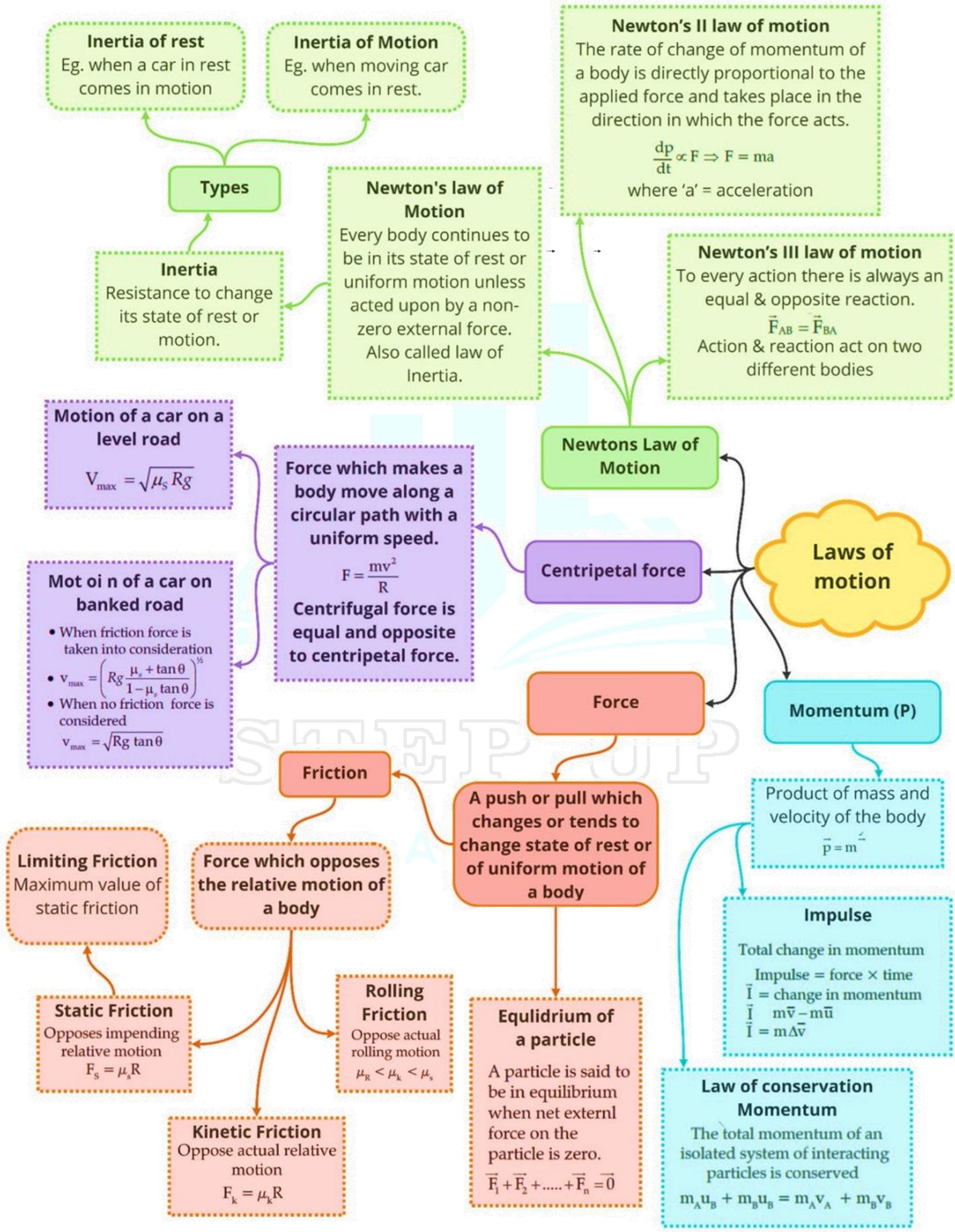
Condition	Free body diagram	Equation	Tension and acceleration
		$m_1a = T_1 - m_1g$	$T_1=\dfrac{2m_1m_2}{m_1+m_2}g$
		$m_2a = m_2g - T_1$	$T_2=\dfrac{4m_1m_2}{m_1+m_2}g$

		$T_2 = 2T_1$	$a = \left[\frac{m_2 - m_1}{m_1 + m_2}\right]g$
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1[m_2 + m_3]}{m_1 + m_2 + m_3}g$
		$m_2a = m_2g + T_2 - T_1$	$T_2 = \frac{2m_1m_3}{m_1 + m_2 + m_3}g$
		$M_3a = m_3g - T_2$	$T_3 = \frac{4m_1[m_2 + m_3]}{m_1 + m_2 + m_3}g$
		$T_3 = 2T_1$	$a = \frac{[(m_2 + m_3) - m_1]g}{m_1 + m_2 + m_3}$
When pulley have a finite mass M and radius R then tension in two segments of string are different 		$m_1a = m_1g - T_1$	$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}}$
		$M_2a = T_2 - m_2g$	$T_1 = \frac{m_1\left[2m_2 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}}g$
		Torque$=(T_1-T_2)R = I\alpha$ $(T_1-T_2)R = I\frac{a}{R}$ $(T_1-T_2)R = I\frac{1}{2}MR^2\frac{a}{R}$ $T_1-T_2 = \frac{Ma}{2}$	$T_1 = \frac{m_2\left[2m_2 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}}g$
		$T = m_1a$	$a = \frac{m_2}{m_1 + m_2}g$
		$m_2a = m_2g - T$	$T = \frac{m_1m_2}{m_1 + m_2}g$

		$m_1a - T - m_1g\sin\theta$	$a = \left[\dfrac{m_2 - m_1\sin\theta}{m_1 + m_2}\right]g$
		$m_2a = m_2g - T$	$T = \dfrac{m_1m_2(1+\sin\theta)}{m_1 + m_2}g$
		$T - m_1g\sin\alpha = m_1a$	$a = \dfrac{(m_2\sin\beta - m_1\sin\alpha)}{m_1 + m_2}g$
		$m_2a = m_2g\sin\beta - T$	$T = \dfrac{m_1m_2(\sin\alpha + \sin\beta)}{m_1 + m_2}g$
		$m_1g\sin\theta - T = m_1a$	$a = \dfrac{m_1g\sin\theta}{m_1 + m_2}$
		$T = m_2a$	$T = \dfrac{2m_1m_2}{4m_1 + m_2}g$
As $\dfrac{d^2(x_2)}{dt^2} = \dfrac{1}{2}\dfrac{d^2(x_1)}{dt^2}$  a_1 = acceleration of block A a_2 = acceleration of block B		$T = m_1a$	$a_1 = a = \dfrac{2m_2g}{4m_1 + m_2}$
		$m_2\dfrac{a}{2} = m_2g - 2T$	$a_2 = \dfrac{m_2g}{4m_1 + m_2}$ $T = \dfrac{2m_1m_2g}{4m_1 + m_2}$
		$m_1a = m_2g - T_1$	$a = \dfrac{(m_1 - m_2)}{[m_1 + m_2 + M]}g$
		$m_2a = T_2 - m_2g$	$T = \dfrac{m_1(2m_1 + M)}{[m_1 + m_2 + M]}g$
		$T_1 - T_2 = Ma$	$T_2 = \dfrac{m_2(2m_2 + M)}{[m_1 + m_2 + M]}g$
	 T_1 = force applied by the string on the block	$F = (M + m)a$ $T_1 = Ma$	$a = \dfrac{F}{M + m}$ $T_1 = M\dfrac{F}{(M + m)}$
		$T_2 = \left(M + \dfrac{m}{2}\right)a$	$T_2 = \dfrac{(2M + m)}{2(M + m)}F$

	T_2 = Tension at mid point of the rope		
		F = ma	a = F/m
m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$T = 1\left(\frac{L-x}{L}\right)a$	$T = \left(\frac{L-x}{L}\right)F$
		$F_1 - T = \frac{Mxa}{L}$	$a = \frac{F_1 - F_2}{M}$
M = Mass of uniform string L = Length of string		$F_1 - F_2 = Ma$	$T = F_1\left(1 - \frac{x}{L}\right) + F_2\left(\frac{x}{L}\right)$
		$T = \frac{M}{L}(L-x)g + T$	T = F = Mg
Mass of segment BC = $\left(\frac{M}{L}\right)x$		$T = F + \frac{M}{L}xg$	$T = F + \frac{M}{L}xg$

Class : 11th Physics
Chapter- 5 : Laws of motion



Important Questions

Multiple Choice Questions-

1. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration of 5 m/s, the reading of the spring balance will be

(a) 15 N

(b) 24 N

(c) 49 N

(d) 74 N

2. Two forces 6 N and 8 N act at a point O. If the angle between the lines of action of the force is 90°, then their resultant is

(a) 14 N

(b) 12 N

(c) 10 N

(d) 48 N

3. A body of mass 15 kg moving with a velocity of 10 m/s has its velocity reduced to 6 m/s in two seconds. The force that produced this change in velocity is

(a) 60 N

(b) 30 N

(c) 45 N

(d) 20 N

4. The frame of reference attached to a satellite of the earth is

(a) an inertial frame

(b) an absolute frame at rest with respect to the stars

(c) a non – inertial frame

(d) a gravitational frame

5. A machine gun fires a bullet of mass 40 g with a velocity of 1200 ms-1. The man holding it can exert a maximum force on 144 N on the gum. How many bullets can he fire per second at the most?

(a) one

(b) four

(c) two

(d) three

6. A block of mass M is placed on a flat surface. A force is applied to move it parallel to the surface. The frictional force f developed is proportional to the

(a) square of the mass of the body

(b) mass of the body

(c) reciprocal of the mass of the body

(d) reciprocal of the square of the body

7. A passenger in a moving bus is thrown forward when the bus is suddenly stopped. This is explained

(a) by Newtons first law

(b) by Newtons second law

(c) by Newtons third law

(d) by the principle of conservation of momentum

8. A rocket with a lift-off mass of 3.5 × 10 kg is blasted upwards with an acceleration of 10 m/s^2. The initial thrust of the blast is (take g = 10 m/s^2)

(a) 1.75×10^{5N}

(b) 3.5×10^{5N}

(c) 7.0×10^{5N}

(d) $14 .0 \times 10^{5N}$

9. A gun of mass 1000 kg fires a projectile of mass 1 kg with a horizontal velocity of 100 m/s. The velocity of recoil of the gun in the horizontal direction is

(a) 5 m/s

(b) 0.1 m/s

(c) 15 m/s

(d) 20 m/s

10. A body is sliding down a rough inclined plane which makes an angle of 30 degree with the horizontal. If the co-efficient of friction is 0.26, the acceleration in m/s^2 is

(a) 1.95

(b) 2.78

(c) 3.47

(d) 4.6

Very Short:

1. (a) Why do we beat dusty blankets with a stick to remove dust particles?
   (b) A stone when thrown on a glass window smashes the window pane to pieces. But a bullet fired from a gun passes through it making a hole. Why?
2. (a) If you jerk a piece of paper from under a book quick enough, the book will not move, why?
   (b) Passengers sitting or standing in a moving bus fall in forward direction when the bus suddenly stops. Why?
3. (a) Why are passengers thrown outward when a bus in which they are travelling suddenly takes a turn around a circular road?
   (b) Is any force required to move a body with constant velocity?
4. (a) Why a one-rupee coin placed on a revolving table flies off tangentially?
   (b) Why mud flies off tangentially to the wheel of a cycle?
5. (a) When the electric current is switched off, why the blades of a fan keep on moving for some time?
   (b) Why the passengers fall backward when a bus starts moving suddenly?
6. (a) A body of mass m is moving on a horizontal table with constant velocity. What is the force on the table?
   (b) Name a factor on which the inertia of a body depends.
7. (a) Rocket works on which principle of conservation?
   (b) Is the relation F→=ma→ applicable to the motion of a rocket?
8. (a) Will a person while firing a bullet from a gun experience a backward jerk? Why?
   (b) A bomb explodes in mid-air into two equal fragments. What is the relation between the directions of their motion? **Answer:**
9. (a) What happens to the acceleration of an object if the net force on it is doubled?
   (b) An electron moving with a certain velocity collides against a stationary proton and sticks to it. Is the law of conservation of linear momentum true in this case?
10. (a) According to Newton's third law of motion, every force is accompanied by an equal (in magnitude) and opposite (in direction) force called reaction, then how can a movement take place?
    (b) You can move a brick easily by pushing it with your foot on a smooth floor, but, if you kick it, then your foot is hurt. Why?

Short Questions:

1. (a) A learner shooter fired a shot from his rifle and his shoulder got injured ¡n the process. What mistake did he commit?
   (b) When the horse suddenly stops, the rider falls in the forward direction. Why? Explain it.
2. (a) Newton's first law of motion is the law of Inertia. Explain.
   (b) What happens to a stone tied to the end of a string and whirled in a circle if the string suddenly breaks? Explain why?
3. (a) An astronaut accidentally gets separated out of his small spaceship accelerating in inter-steller space at a constant rate of 100 ms-2. What is the acceleration of the astronaut at the instant after he is outside the spaceship?
   (b) How is it that a stone dropped from a certain height falls much more rapidly as compared to a parachute under similar conditions?
4. (a) When a man jumps out of a boat, then it is pushed away. Why?
   (b) Explain how lubricants reduce friction?
5. Two hoys on ice-skates hold a rope between them. One boy is much heavier than the other. The lightweight boy pulls on the rope. How will they move?
6. Explain why ball bearings are used in machinery?
7. Why a horse has to apply more force to start a cart than to keep it moving? Explain.
8. Sand is thrown on tracks or roads covered with snow. Explain why?

Long Questions:

1. (a) State and prove impulse-momentum Theorem.
   (b) Prove that Newton's Second law is the real law of motion.

2. Derive the general expression for the velocity of a rocket in flight and obtain the expression for the thrust acting on it.
3. (a) Define inertia. What are its different types? Give examples.
   (b) Explain Newton's First law of motion. Why do we call it the law of inertia?
   (c) State Newton's Second law of motion. How does it help to measure force? Also, state the units of force.
4. (a) State Newton's Third law of motion. Discuss its consequences.
   (b) State the law of conservation of linear momentum and illustrate it with examples.
   (c) Define the terms – momentum and impulse. What are their units in the S.I. system?

Assertion Reason Questions:

1. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: On a rainy day, it is difficult to drive a car or bus at high speed.

   Reason: The value of coefficient of friction is lowered due to wetting of the surface.
2. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: For the motion of electron around nucleus, Newton's second law is used. **Reason:** Newton's second law can be used for motion of any object.

Case Study Questions:

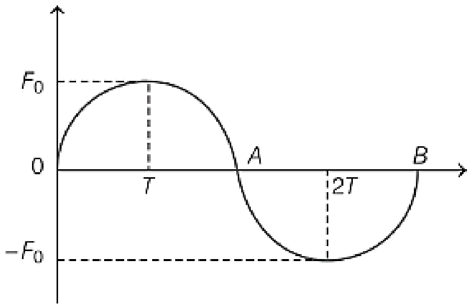
1. This principle is a consequence of Newton's second and third laws of motion. In an isolated system (i.e., a system having no external force), mutual forces (called internal forces) between pairs of particles in the system causes momentum change in individual particles. Let a bomb be at rest, then its momentum will be zero. If the bomb explodes into two equals parts, then the parts fly off in exactly opposite directions with same speed, so that the total momentum is still zero. Here, no external force is applied on the system of particles (bomb).

   i. A bullet of mass 10 g is fired from a gun of mass 1kg with recoil velocity of gun 5 m/s. The muzzle velocity will be
   (a) 30 km/min
   (b) 60 km/min
   (c) 30 m/s
   (d) 500 m/s

   ii. A shell of mass 10kg is moving with a velocity of $10ms^{-1}$ when it blasts and forms two parts of mass 9kg and 1kg respectively. If the first mass is stationary, the velocity of the second is
   (a) $1m\ s^{-1}$
   (b) $10m\ s^{-1}$
   (c) $100m\ s^{-1}$
   (d) $1000m\ s^{-1}$

   iii. A bullet of mass 0.1kg is fired with a speed of 100 ms^{-1} The mass of gun being 50kg, then the velocity of recoil becomes
   (a) $0.05\ m\ s^{-1}$
   (b) $0.5\ m\ s^{-1}$
   (c) $0.1\ m\ s^{-1}$
   (d) $0.2\ m\ s^{-1}$

   iv. A unidirectional force F varying with time T as shown in the figure acts on a body initially at rest for a short duration 2T. Then, the velocity acquired by the body is



(a) $\frac{\pi F0\, T}{4\, m}$

(b) $\frac{\pi F0\, T}{2\, m}$

(c) $\frac{F0\, T}{4\, m}$

(d) Zero

v. Two masses of M and 4M are moving with equal kinetic energy. The ratio of their linear momenta is

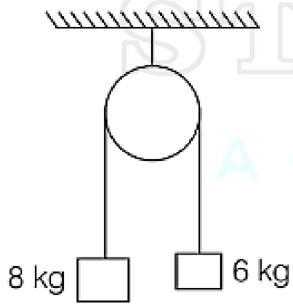
(a)1 : 8

(b)1 : 4

(c)1 : 2

(d)4 : 1

2. When bodies are in contact, there are mutual contact forces satisfying the third law of motion. The component of contact force normal to the surfaces in contact is called normal reaction. The component parallel to the surfaces in contact is called friction.



In the above figure, 8kg and 6kg are hanging stationary from a rough pulley and are about to move. They are stationary due to roughness of the pulley.

i. Which force is acting between pulley? and rope?

(a) Gravitational force

(b) Tension force

(c) Frictional force

(d) Buoyant force

ii. The normal reaction acting on the system is

(a) 8 g

(b) 6 g

(c) 2 g

(d) 14 g

iii. The tension is more on side having mass of

(a) 8kg

(b) 6kg

(c) Same on both

(d) Nothing can be said

iv. The force of friction acting on the rope is

(a) 20 N

(b) 30 N

(c) 40 N

(d) 50 N

v. Coefficient of friction of the pulley is

(a) $\frac{1}{6}$

(b) $\frac{1}{7}$

(c) $\frac{1}{5}$

(d) $\frac{1}{4}$

Answer Key

Multiple Choice Answers-

1. **Answer:** (b) 24 N
2. **Answer:** (c) 10 N
3. **Answer:** (b) 30 N
4. **Answer:** (c) a non – inertial frame
5. **Answer:** (d) three
6. **Answer:** (b) mass of the body
7. **Answer:** (a) by Newtons first law
8. **Answer:** (c) 7.0×10^{5N}
9. **Answer:** (b) 0.1 m/s
10. **Answer:** (b) 2.78

Very Short Answers:

1. **Answer:**
   (a) It is done due to inertia of rest.
   (b) This is due to the inertia of rest.
2. **Answer:**
   (a) It is due to the inertia of rest.
   (b) This is due to the inertia of motion.
3. **Answer:**
   (a) This is due to the inertia of direction.
   (b) No.
4. **Answer:**
   (a) This is due to the inertia of direction.
   (b) This is due to the inertia of direction.
5. **Answer:**
   (a) This is due to the inertia of motion.
   (b) This is due to the inertia of rest.
6. **Answer:**
   (a) mg i.e., equal to the weight of the body.
   (b) Mass.
7. **Answer:**
   (a) Law of conservation of linear momentum.
   (b) No.
8. **Answer:**
   (a) Yes, it is due to the law of conservation of linear momentum.
   (b) The two fragments will fly off in two opposite directions.
9. **Answer:**
   (a) As $a = \frac{F}{m}$ i.e., $a \propto F$, so acceleration will be doubled when m the force is doubled.
   (b) Yes, it is true.
10. **Answer:**
   (a) As the action and reaction never act on the same body, so the motion is possible.
   (b) As Ft remains constant, so if t is reduced, then F will be increased and hence hurt our foot.

Short Questions Answers:

1. **Answer:**
   (a) We know that a gun recoils i.e. moves back after firing. To avoid injury to the shoulder, the gun must he held tightly against the shoulder. The learner shooter might have not held it tightly against his shoulder and hence the gun must have injured his shoulder after firing.
   (b) When the horse suddenly stops, the rider falls in forwarding direction due to the inertia of motion.
   Explanation: The lower portion of the rider comes to rest along with the horse while the upper portion of the rider continues to move forward. Hence, he falls forward.
2. **Answer:**
   (a) According to Newton's first law of motion, a body can't change its state of rest or of uniform motion along a straight line unless an external force acts on it. It means that the natural tendency of the material body is to continue in the state of rest or that of uniform motion which is termed as inertia. Thus, Newton's first law is the law of inertia.
   (b) The stoneflies off tangentially to the circle along a straight line at the point where the string breaks. It is due to the inertia of direction. When the string breaks, the force acting on the stone ceases. In the absence of force, the stoneflies away in the direction of instantaneous velocity which is along the tangent to the circular path.
3. **Answer:**
   (a) According to Newton's first law of motion, the moment he is out of the spaceship, there is no external force on the astronaut, thus his acceleration is zero. Here we are assuming that he is out of the gravitational field of heavenly bodies i.e. there are no nearby stars to exert a gravitational force on him and the small spaceship exerts a negligible gravitational attraction on him.
   (b) As the surface area of a parachute is much larger as compared to the surface area of a stone, so the air resistance, i. e. fluid friction in the case of the parachute is much larger than in the case of stone. Hence the parachute falls slowly.
4. **Answer:**
   (a) This is due to Newton's third law of motion. When the man jumps out of the boat, he applies a force on it in the backward direction, and in turn, the reaction of the boat on the man pushes him out of the boat.
   (b) The lubricants spread as a thin layer between the two surfaces. The motion now

is between the surface and the lubricant layer which changes the dry friction into wet friction. As wet friction is less than dry friction, hence lubricants reduce friction.

5. **Answer:** The light-weight boy is doing the action on the heavy boy by pulling the rope. According to Newton's third law, equal and opposite force (reaction) also acts on the light boy. As the mass of the boy pulling the rope is lesser, so the acceleration produced in him will be more. Thus, both the boys move tow; rds each other and the lighter boy will move faster.
6. **Answer:** We know that rolling friction is much lesser than sliding friction, so we convert the sliding friction into rolling friction which is done using ball bearings that are placed in between the axle and the hub of the wheel. The ball bearings tend to roll around the axle as the wheel turns and as such the frictional force is reduced.
7. **Answer:** Static friction comes into play when the horse applies force to start the motion in the cart. On the other hand, kinetic friction comes into play when the cart is moving.

   Also, we know that the static friction is greater than the kinetic friction, so the horse has to apply more force to start a cart than to keep it moving.
8. **Answer:** When the roads (or tracks) are covered with snow, then there is a considerable reduction of frictional force between the tires of the vehicles and the road (or between the track and the wheels of the vehicle or train) which leads to the skidding of the ehicles (or trains). Thus, driving is not safe. When sand is thrown on the snow-covered roads (or tracks), then the force of friction increases, so safe driving is possible.

Long Questions Answers:

1. **Answer:**

   1. It states that the impulse of force on a body is equal to the change in momentum of the body.

      i.e. $J = F_t = P_2 - P_1$

      Proof: From Newton's Second law of motion, we know that

      $$F = \frac{dp}{dt} \text{ or } Fdt = dp \qquad ....(i)$$

      Let P_1 and P_2 be the linear momenta of the body at time t = 0 and t respectively.

      ∴ integrating equation (i) within these limits, we get

      $$\int_0^t Fdt = \int_{P_1}^{P_2} dp$$

      or $$F\int_0^t dt = \int_{P_1}^{P_2} p^\circ dp$$

      or $$F[t]_0^t = [p]_{p_1}^{p_2}$$

      $$Ft = p_2 - p_1$$

      or $$J = p_2 - p_1$$

      Hence proved.

2. **Proof:** If we can show that Newton's first and third laws are contained in the second law, then we can say that it is the real law of motion.

   1. **First law is contained in second law:** According to Newton's second law of motion,

      F = ma ...(i)

      where m = mass of the body on which an external force F is applied and a = acceleration produced in it.

      If F = 0, then from equation (1), we get

      ma = 0, but as m ≠ 0

      ∴ a = 0

      which means that there will be no acceleration in the body if no external force is applied. This shows that a body at rest will remain at rest and a body in uniform motion will continue to move along the same straight line in the absence of an external force. This is the statement of Newton's first law of motion. Hence, the First law of motion is contained in the Second law of motion.

   2. Third law is contained in second law: Consider an isolated system of two bodies A and B. Let them act and react internally.

      Let FAB = force applied on body A by body B

      and FBA = force applied on body B by body A

It $\frac{dpA}{dt}$ = rate of change of momentum of body A

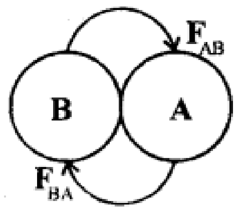
and

$\frac{dpB}{dt}$ = rate of change of momentum of body B

Then according to Newton's second law of motion,

$$F_{AB} = \frac{dp_A}{dt} \quad ....(2)$$

$$F_{BA} = \frac{dp_B}{dt} \quad ....(3)$$



(2) and (3) gives

$$F_{AB} + F_{BA} = \frac{d}{dt}(p_A) + \frac{d}{dt}(p_B)$$

$$= \frac{d}{dt}(p_A + p_B)$$

As no external force acts on the system (∵ it is isolated), therefore according to Newton's second law of motion,

$$\frac{d}{dt}(p_1 + p_2) = 0$$

or $\quad F_{AB} + F_{BA} = 0$

or $\quad F_{AB} = -F_{BA}$

or $\quad$ Action = – Reaction,

which means that action and reaction are equal and opposite. It is the statement of Newton's 3rd law of motion. Thus 3rd law is contained in the second law of motion.

As both First and Third Law is contained in Second law, so Second law is the real law of motion.

2. **Answer:**

The working of a rocket is based upon the principle of conservation of momentum. Consider the flight of the rocket in outer space where no external forces act on it.

Let m_o = initial mass of rocket with fuel.

V_u = initial velocity of the rocket,

m = mass of the rocket at any instant t.

v = velocity of the rocket at that instant.

d_m = mass of the gases ejected by the rocket, in a small-time it.

u =H velocity of exhaust gases,

DV = increase in the velocity of the rocket in a time dt.

∴ Change in the momentum of exhaust gases = dm. u

Change in momentum of rocket = – (m – dm) dv.

A negative sign shows that the rocket is moving in a direction opposite to the motion of exhaust gases.

Applying the law of conservation of linear momentum,

$$dm.u = -(m - dm)\,dv \quad ...(1)$$

As dm being very small as compared to m, so it can be neglected, Thus, eqn. (1) reduces to

$$dm.u = -m\,dv$$

or $$dv = -u\frac{dm}{m} \quad ...(2)$$

Instantaneous velocity of the rocket:

At t = 0, mass of rocket = m0, velocity of rocket = v_o.

At t = t, mass of rocket = m, velocity of rocket = v.

∴ Integrating Eqn. (1) within these limits, we get

$$\int_{v_0}^{v} dv = -\int_{m_0}^{m} u\frac{dm}{m}$$

In actual practice, the velocity of exhaust gases nearly remains constant.

∴ $$\int_{v_0}^{v} dv = -u\int_{m_0}^{m} \frac{dm}{m}$$

or $$[u]_{v_0}^{v} = -[\log_e m]_{m_0}^{m}$$

or $$v - v_0 = -u(\log_e m - \log_e m_0)$$

$$= u(\log_e m_0 - \log m)$$

$$= u\log_e\left(\frac{m_0}{m}\right)$$

or $$v = v_0 + u\log_e\left(\frac{m_0}{m}\right) \quad ....(3)$$

equation (3) gives the instantaneous velocity of the rocket. In general v_o = 0 at t = 0,

∴ Eqn. (3) reduces to

$$v = u\log_e\frac{m_0}{m} \quad ....(4)$$

From Eqn. (4), we conclude that the velocity of the rocket at any instant depends upon:

- speed (u) of the exhaust gases.
- Log of the ratio of initial mass (m0) of the rocket to its mass (m) at that instant of time.

Upthrust on the rocket (F): It is the upward force exerted on the rocket by the expulsion of exhaust gases. It is obtained as follows:

Dividing Eqn. (2) by dt, we get

$$\frac{dv}{dt}=-\frac{u}{m}\frac{dm}{dt}$$

or $$m\frac{dv}{dt}=-u\frac{dm}{dt}$$

But $$\frac{dv}{dt}=a,$$

is the instantaneous acceleration

∴ $$ma=-u\frac{dm}{dt}$$

or $$F=-u\frac{dm}{dt} \quad ....(5)$$

where F = ma is the instantaneous force (thrust).

From Eqn. (5), we conclude that the thrust (F) on the rocket at any instant is the product of the velocity of exhaust gases and the rate of combustion of fuel at that instant. Here negative sign shows that the thrust and velocity of exhaust gases are in opposite direction.

3. **Answer:**

(a) The tendency of bodies to remain in their state of rest or uniform motion along a straight line in the absence of an external force is called inertia.

Inertia is of the following three types:

1. The inertia of rest: When a body continues to lie at the same position with respect to its surrounding, it is said to possess inertia of rest. This situation may be changed only by the application of external force. For example, if a cot or sofa is lying in a particular place in the house, it will remain there even after days or years unless someone removes (by applying force) the same from its position. This is an example of the inertia of rest.
2. The inertia of motion: When a body is moved on a frictionless surface or a body is thrown in a vacuum, it will continue to move along its original path unless acted upon by an external force. In actual situations, air or floor etc. exert friction on the moving bodies so we are unable to visualize a force-free motion. This type of inertia when a body continues to move is called the inertia of direction.
3. In the above examples it is found that the direction of motion of the body or particle also does not change unless an external force acts on it. This tendency to preserve the direction of motion is called the inertia of direction.

(b) According to the First law of motion, "Everybody continues to be in the state of rest or of uniform motion along a straight line until it is acted upon by an external force."

It means that if a book lying on a table, it will remain there for days or years together unless force is applied on it from outside to pick it.

Similarly, if a body is moving along a straight line with some speed, it will continue to do so until some external force is applied to it to change its direction of motion.

Thus, First law tells us the following:

It tells us about the tendency of bodies to remain in the state of rest or of motion and the bodies by themselves can neither change the state of rest nor of uniform motion. This tendency is called inertia. To break the inertia of rest or motion or direction, we need an external force. Thus the definition of the first law matches with the definition of inertia and hence first law is called the law of inertia.

The first law of motion also provides the definition of another important physical quantity called force. Thus force is that agency which changes or tends to change the state of rest or of uniform motion of a body along a straight line.

(c) It states that the time rate of change of momentum of a body is directly proportional to the force applied to it.

i.e. mathematically, $F \propto \frac{d}{dt}(p)$

$$\propto \frac{d}{dt}(ma)$$

$$\propto m\frac{dv}{dt}$$

or $F = kma$ ....(1)

where a = $\frac{dv}{dt}$ = acceleration produced in the body of mass m.

k = proportionality constant which depends on the system of units chosen to measure F, m, and a.

In the S.I. system, k = l,

∴ F = ma

The magnitude of the force is given by

$$F = ma \quad .... (2)$$

Note: We have assumed that the magnitude of velocity is smaller and much less than the speed of light. Only under this condition Eqns. (1) and (2) hold good.

The definition of the Second law and its mathematical form is given in Eqn. (2) provide us a mean of measuring force.

One can easily find the change in velocity of a body in a certain interval of time. Both velocity and time can be easily measured. Thus, by knowing the mass of the body one can determine both change in momentum as well as the acceleration of the body produced by an external force. If the force is increased, the rate of change of momentum is also found to increase. So also, is the acceleration. Now with known values of m and we can find F.

Units of force: Force in S.I. units is measured in newton or N. From Eqn. (1) or (2) we can see that a newton of force is that fore? which produces 1 ms^{-2} acceleration in the body of mass 1 kg.

1 newton = 1 kilogram × 1 metre/(second)^2

or

1 N = 1 kg × 1 ms^{-2} = 1 kg ms^{-2}

In CGS system force is measured in dyne

1 dyne = 1 gram × 1 cm/s^2 = 1 g cm s^{-2}

Since 1 N = 1 kgm s^{-2} = 1000 g × 100 cm s^{-2}

= 10^5 g cm s^{-2} = 10^5 dyne

1 N = 10^5 dyne

or

1 dyne = 10^5 N

Gravitational Unit: If a falling mass of 1 kg is accelerated towards the Earth with 9.8 ms^{-2}, then the force generated is called 1 kg wt (1-kilogram weight) force. It is the S.I. gravitational unit of force.

We know that the earth accelerates the mass with g = 9.8 ms^{-2}

1 Kg wt = 9.8 N [1 kg × 9.8 ms 2 = 9.8 N]

C.G.S. gravitational unit is gf or g wt.

1 gf = 1g × 980 cms^{-2}

= 980 dyne

4. **Answer:**

(a) Newton's Third law of motion states that "to every action, there is always an equal and opposite reaction."'

So, if a body 1 applies a force F12 on body 2 (action), then body 2 also applies a force F2] on body 1 but in opposite direction, then

$$F_{21} = -F_{12}$$

In terms of magnitude

$$|F_{21}| = |-F_{12}|$$

It is very important to note that F_{12} and F_{21} though are equal in magnitude and opposite in direction yet act on different points or else no motion will be possible.

For example, hands pull up a chest expander (spring), and spring in turn exerts a force on the arms. A football when pressed reacts on the foot with the same force and so on. The most important consequence of the third law of motion is the law of conservation of linear momentum and its application in collision problems.

Since $F_{12} = -F_{21}$

and $F = m\frac{\Delta v}{\Delta t}$

∴ $m_1\frac{\Delta v_1}{\Delta t} = -m_2\frac{\Delta v_2}{\Delta t}$

Here Δt is the time for which the bodies come in contact during impact. This is the same for the two bodies of masses m_1 and m_2 and having velocity changes Δv_1 and Δv_2 respectively. Therefore,

$$m_1 \Delta v_1 = - m_2 \Delta v_2$$

or $$m_1 \Delta v_1 + m_2 \Delta v_2 = 0$$

Let u_1, u_2 and v_1 and v_2 be initial and final velocities of the two masses before and after collision, then

$$m_1(v_1 - u_1) = - m_2(v_2 - u_2)$$

or $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Momentum before impact = momentum after impact

This is the law of conservation of momentum.

(b) 'The linear momentum of an isolated system always remains the same provided no external force is applied on it.' This is the law of conservation of linear momentum.

The linear momentum of a body = mass × velocity

$$p = mv$$

If a system has several bodies initially at rest then initial momentum = 0.

The final momentum = $p_1 + p_2 + p_3 + \ldots\ldots$

According to law of conservation of linear momentum

$$p_1 + p_2 + p_3 + \ldots = 0$$

Linear momentum is a vector quantity and is measured. in kg ms^{-1}or Ns.

For example, a gun and a bullet make a system in which both are initially at rest. When the bullet of mass m is fired with muzzle velocity v, the gun of mass M gets a recoil velocity V. Since the initial linear momentum of the system is zero.

$$MV + mv = 0$$

or

$$MV = - mv$$

Thus, gun moves in the opposite direction to that of the bullet.

(c) The total quantity of motion possessed by the body is called is momentum. Mathematically, it is equal to the product of the mass of the body and the velocity of the body.

In linear motion, this term is called linear momentum P.

It is a vector quantity.

$$p = mv$$

The units of linear momentum are kg ms^{-1} or NS in S.I. units.

Impulse: The action or impact of force is called the impulse of force. Mathematically, impulse J is equal to the product of the force F acting on the body and the time for which the force acts on it. Thus

$$J = F \times t = Ft$$

J is a vector quantity and is measured in Ns or kg ms^{-1}

The action of force or impulse is increased if the force acts for a smaller interval.

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

On a rainy day, the roads are wet. Wetting of roads lowers the coefficient of friction between the types and the road. Therefore, grip on a road of car reduces and thus chances of skidding increases.

2. (c) Assertion is correct, reason is incorrect+

Explanation:

Newton's second law cannot be used for any object.

Case Study Answer:

1. i. (d) 500 m/s

Explanation:

Conservation of linear momentum gives

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$\Rightarrow \quad v_1 = \frac{-m_2v_2}{m_1}$$

Given, $m_1 = 10g = \left(\frac{10}{1000}\right)kg$

$m_2 = 1\ kg$ and $v_2 = -5\ m/s$

∴ Velocity of muzzle,

$$v_1 = \frac{+1 \times 5}{10/1000} = 500\, m/s$$

ii. (c) 100m s⁻¹

Explanation:

Given that, $v_1 = 10\ ms^{-1}$,

$m_1 = 10\ kg, v_2 = 0,$

$m_2 = 9\ kg, v_3 = v,$

$m_3 = 1\ kg$

According to conservation of momentum,

$$m_1v_1 = m_2v_2 + m_3v_3$$

$$10 \times 10 = 9 \times 0 + 1 \times v$$

$$\Rightarrow \quad v = 100\ ms^{-1}$$

iii. (d)0.2 m s⁻¹

Explanation:

From the law of conservation of momentum,
Initial momentum = Final momentum

$$\Rightarrow \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\therefore \quad 0.1\times0+50\times0=0.1\times100+50(-v_2)$$

$$\Rightarrow \quad 0 = 10 - 50\ v_2$$

$$\therefore \quad v_2 = \frac{10}{50} = 0.2\ ms^{-1}$$

iv. (d) Zero

Explanation:

From 0 to T, area is positive and from T to 2T, area is negative, so net area is zero. Hence, there is no change in momentum.

v. (c) 1 : 2

Explanation:

Two masses are moving with equal kinetic energy.

$$\frac{1}{2}Mv_1^2 = \frac{1}{2}4Mv_2^2$$

or $$\frac{v_1}{v_2} = 2$$

The ratio of linear momentum is

$$\frac{p_1}{p_2} = \frac{Mv_1}{4Mv_2}$$

or $$\frac{p_1}{p_2} = \frac{1}{4}\left(\frac{v_1}{v_2}\right)$$

or $$\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \quad p_1 : p_2 = 1 : 2$$

2. i. (c) Frictional force

Explanation:

Frictional force acts between pulley and rope.

ii. (d) 14 g

Explanation:

The reaction force is

R = T_1 T_2 = (8 + 6) g = 14 g

iii. (d) Nothing can be said

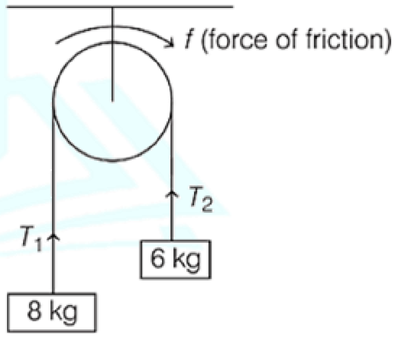
Explanation:

As, tension, $T = mg \Rightarrow T \propto m$

So, the side having 8 kg mass will have more tension.

iv. (a) 20 N

Explanation:



Due to friction, tension at all points of the thread is not alike.

$$T_1 - T_2 = f$$

$$\Rightarrow \quad f = 8g - 6g = 2g$$

$$= 20\ N \qquad (\because g = 10\ ms^{-2})$$

iv. (b) $\frac{1}{7}$

Explanation:

As, $\mu R = f$ = 20 N

$$\mu = \frac{20}{R} = \frac{20}{14\times10} = \frac{1}{7} \qquad (\because R = mg)$$

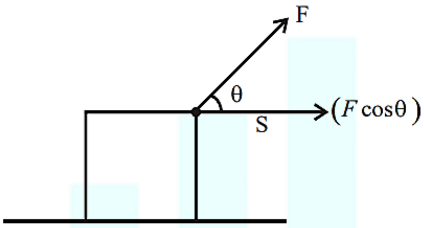
Work Energy and Power 6

Work done by a Constant Force:

Work is said to be done if a body displaces by the application of force. Work done depends on

(i) applied force

(ii) displacement in any direction except the perpendicular direction of force.



Work done by the force is measured as the product of displacement and the component of force in the direction of displacement

$$W=(F\cos\theta)S=FS\cos\theta=\vec{F}.\vec{S}$$

Work is a scalar quantity

Its S.I Unit is N-m or joule (J).

Joule:

Work is said to be one joule if a force of 1 newton displaces a body through a distance of 1m along the direction of force

1J = IN × 1m

C.G.S unit of work is erg.

Erg:

Work is said to be one erg if a force of 1 dyne displaces a body through a distance of 1cm along the direction of force.

I erg = 1 dyne × 1cm

Relation between joule and erg:

1 joule = 10^7 erg

Other units of work

Electron Volt (ev) = 1.6×10^{-19} J

Calorie = 4.2 J

Kilowatt hour = 3.6×10^6 J

Dimensional formula of work is ML^2T^{-2}

Work done by Multiple Forces:

When several forces act on a body, the net work done on the body is

$$W_{net}=\vec{F}_1.\vec{S}+\vec{F}_2.\vec{S}+\vec{F}_3.\vec{S}+..............$$

$W_{net} = W_1 + W_2 + W_3 + \ldots\ldots\ldots\ldots$

Net work done is the sum of the works done by all the forces acting on the particle.

Nature of Work:

Work done by force may be positive, negative or zero.

Positive Work:

Work done is said to positive if applied force or one of its component is in the direction of displacement when $0° \le \theta < 90°$.

Therefore $W = (FS \cos\theta)$ is positive.

Ex 1: When a horse pulls a cart the applied force and displacement are in the same direction so work done by horse is positive.

Ex 2: Work done by the gravitational force on a freely falling body is positive.

Ex 3: When a spring is stretched, both the stretching force and displacement act in the same direction so work done is positive.

Ex 4: When a block is lifted from the ground the work done by the lifting force is positive.

Negative Work:

Work done by a force is said to be negative if the applied force has a component in a direction opposite to that of the displacement when $90 < \theta \le 180°$ then $\cos\theta$ is negative therefore

$W = (FS \cos\theta)$ is negative.

Ex 1: When a body is dragged on rough surface, work done by frictional force is negative.

Ex 2: Work done by the gravitational force on a vertically projected up body is negative.

Zero Work:

- If a body displaces perpendicular to the direction of force then work done is zero ($\theta = 90°$).
- If there is no displacement [S=0] then work done is zero.
- If the applied force is zero(F=0), then work done is zero.

Examples:

- A person carrying a load and moving horizontally on a platform does no work against gravity.
- When a body moves in a circle the work done by the centripetal force is zero.
- The tension in the string of a simple pendulum is always perpendicular to its displacement, So work done by tension is zero.
- When a person tries to displace a wall by applying a force and if it does not move the work done by him is zero.
- A person carrying a load on his head and standing at a given place does no work.

Work done by Variable Force:

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement ds is given by $dW = \overline{F}.\overline{ds}$

The total work done in going from A to B is

$$W_{AB} = \int_A^B \vec{F}.\overrightarrow{ds} = \int_A^B (F\cos\theta)ds$$

In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z k$$

$\vec{ds} = dx\hat{i} + dy\hat{j} + dzk$

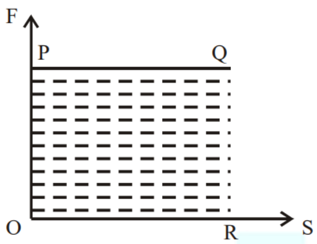
$$W = \int_A^B F_x dx + \int_A^B F_y dy + \int_A^B F_z dz$$

Graphical Interpretation of Work done:

Area of F-S graph gives work

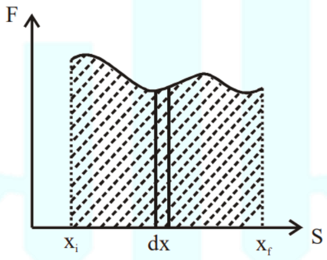
- Work done by constant force.

  The area enclosed by the graph on displacement axis gives the amount of work done by the force



Work = FS = Area of OPQR

- Work done by variable force

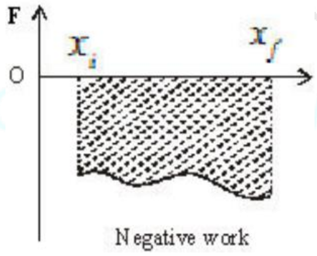


For a small displacement dx the work done will be the area of the strip of width dx

$$W = \int_{X_i}^{X_f} dw = \int_{X_i}^{X_f} F dx$$

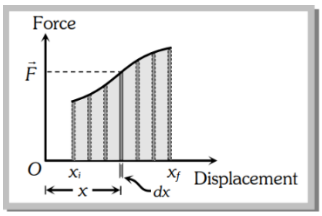
In this case work done is positive.

If area lies above X-axis work done is +ve if the area lies below X-axis work done is –ve.



Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.



Let $\vec{F}$ be the average value of variable force within the interval dx from position x to (x + dx) i.e. for small displacement dx. The work done will be the area of the shaded strip of width dx. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips

$dW = \vec{F} dx$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F} dx$$

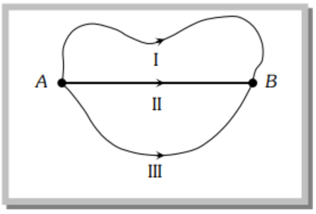
$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$\therefore$ W = Area undercurve Between x_i and x_f

i.e. Area under force displacement curve with proper algebraic sign represents work done by the force.

Work Done in Conservative and Non-Conservative Field

(1) In conservative field work done by the force (line integral of the force i.e. $\int \vec{F}.d\vec{l}$) is independent of the path followed between any two points.



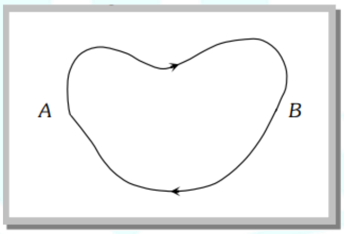
$$\underset{\text{Path I}}{W_{A \to B}} = \underset{\text{Path II}}{W_{A \to B}} = \underset{\text{Path III}}{W_{A \to B}}$$

or $$\int_{\text{Path I}} \vec{F}.d\vec{l} = \int_{\text{Path II}} \vec{F}.d\vec{l} = \int_{\text{Path III}} \vec{F}.d\vec{l}$$

(2) In conservative field work done by the force (line integral of the force i.e. $\int \vec{F}.d\vec{l}$) over a closed path/loop is zero.

$$W_{A \to B} + W_{B \to A} = 0$$

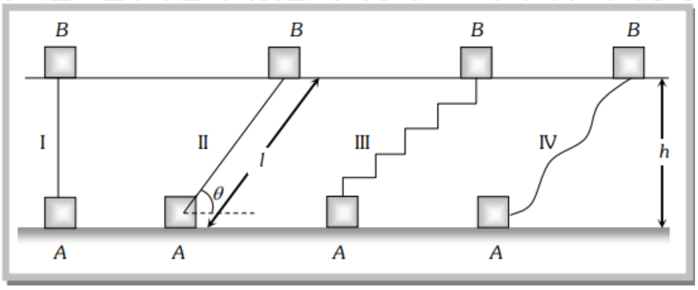
or $$\oint \vec{F}.d\vec{l} = 0$$



Conservative force : The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces etc. and all the central forces are conservative in nature.

If a body of man m lifted to height h from the ground level by different path as shown in the figure



Work done through different paths

$W_I = F.s = mg \times h = mgh$

$W_{II} = F.s = mg \sin\theta \times l = mg \sin\theta \times \dfrac{h}{\sin\theta} = mgh$

$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mg(h_1 + h_2 + h_3 + h_4) = mgh$

$W_{IV} = \int \vec{F}.d\vec{s} = mgh$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position A, similar amount of work (energy) is released from the system it means $W_{AB} = mgh$

and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round strip is zero.

$W_{Net} = W_{AB} + W_{BA}$

$= mgh + (-mgh) = 0$

i.e. the gravitational force is conservative in nature.

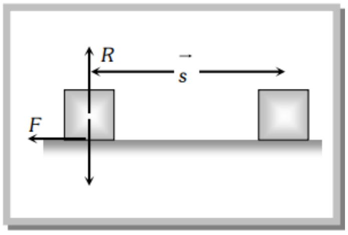
Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Airdrag etc.

If a body is moved from position A to another position B on a rough table, work done against frictional force shall depends on the length of the path between A and B and not only on the position A and B.

$W_{BA} = \mu mgs$

Further if the body is brought back to its initial position A, work has to be done against the frictional force, which always opposes the motion. Hence the net work done against the friction over a round trip is not zero.



$W_{BA} = \mu mgs.$

$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$

i.e. the friction is a non-conservative force.

Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done, therefore it is a scalar quantity.

(2) **Dimension:** $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) **Units :** Joule [S.I.], erg [C.G.S.]

Practical units : electron volt (eV), Kilowatt hour (KWh), Calories (cal)

Relation between different units:

1 Joule = 10^7 erg

1 eV = 1.6×10^{-19} Joule

1 kWh = 3.6×10^6 Joule

1 calorie = 4.18 Joule

(4) **Mass energy equivalence :** Einstein's special theory of relativity shows that material particle itself is a form of energy.

The relation between the mass of a particle m and its equivalent energy is given as

$E = mc^2$ where c = velocity of light in vacuum.

If m = 1 amu = 1.67×10^{-27} kg

then E = 931 MeV = 1.5×10^{-10} Joule

If m = 1 kg then E = 9×10^{16} Joule

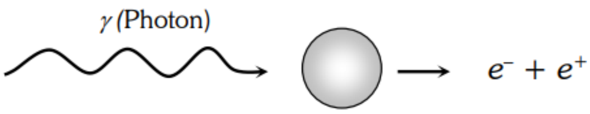
Examples : (i) Annihilation of matter when an electron (e^-) and a positron (e^+) combine with each other, they annihilate or destroy each other. The masses of electron and positron are converted into energy. This energy is released in the form of γ-rays.

$e^- + e^+ \rightarrow \gamma + \gamma$

Each γ photon has energy = 0.51 MeV.

Here two γ photons are emitted instead of one γ photon to conserve the linear momentum.

(ii) **Pair production :** This process is the reverse of annihilation of matter. In this case, a photon (γ) having energy equal to 1.02 MeV interacts with a nucleus and give rise to electron (e^-) and a positron (e^+). Thus energy is converted into matter.



(iii) **Nuclear bomb :** When the nucleus is split up due to mass defect (The difference in the mass of nucleons and the nucleus), energy is released in the form of γ-radiations and heat.

(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential)

(ii) Chemical energy

(iii) Electrical energy

(iv) Magnetic energy

(v) Nuclear energy

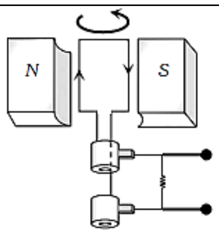
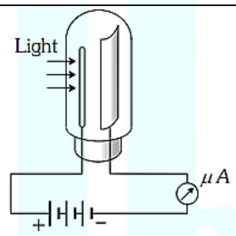
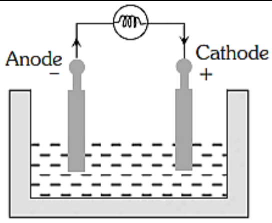
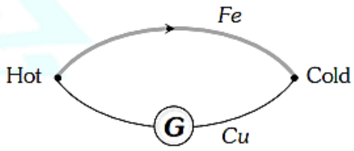
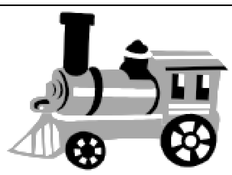
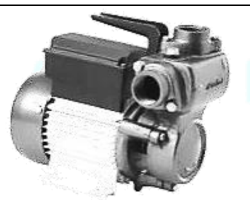
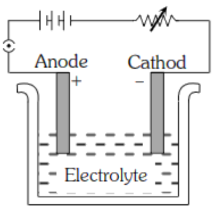
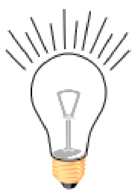
(vi) Sound energy

(vii) Light energy

(viii) Heat energy

(6) **Transformation of energy :** Conversion of energy from one form to another is possible through various devices and processes.

Various devices for energy conversion from one form to another

Mechanical → electrical	**Light → Electrical**	**Chemical → electrical**
Dynamo 	Photoelectric cell 	Primary cell 
Chemical → heat	**Sound → Electrical**	**Heat → electrical**
Coal Burning 	Microphone 	Thermo-couple 
Heat → Mechanical	**Electrical → Mechanical**	**Electrical → Heat**
Engine 	Motor 	Heater 
Electrical → Sound	**Electrical → Chemical**	**Electrical → Light**
Speaker 	Voltameter 	Bulb 

Kinetic Energy

The energy possessed by a body by virtue of its motion, is called kinetic energy.

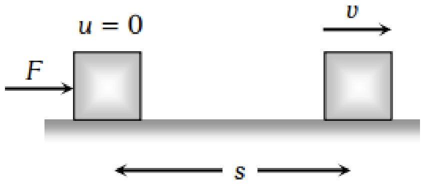
Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.

(iii) Moving air (i.e. wind) possesses kinetic energy which is used to run wind mills.

(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.

(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.



(1) **Expression for kinetic energy :**

Let m = mass of the body,

u = Initial velocity of the body (= 0)

F = Force acting on the body,

a = Acceleration of the body,

s = Distance travelled by the body,

v = Final velocity of the body

From $v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0 + 2as \qquad \therefore s = \frac{v^2}{2a}$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$W = F \times s = ma \times \frac{v^2}{2a}$

$\Rightarrow W = \frac{1}{2} v^2$

This work done appears as the kinetic energy of the body KE = W = $\frac{1}{2}v^2$

(2) **Calculus method :** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through small displacement $d\vec{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = Fds$$

$\Rightarrow$ dW = m a ds [As F = ma]

$\Rightarrow dW = m\frac{dv}{dt}ds$ [As $a = \frac{dv}{dt}$]

$\Rightarrow dW = mdv.\frac{ds}{dt}$

$\Rightarrow$ dW = mv dv ...(i)

[As $\frac{ds}{dt} = v$]

Therefore, work done on the body in order to increase it velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body KE = $\frac{1}{2}mv^2$.

In vector form KE = $\frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar i.e. kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Kinetic energy according to relativity :** As we know E = $\frac{1}{2}mv^2$.

But this formula is valid only for (v << c) If v is comparable to c (speed of light in free space = 3×10^8 m/s) then according to Einstein theory of relativity

$$E = \frac{mc^2}{\sqrt{1-(v^2/c^2)}} - mc^2$$

(5) **Work-energy theorem:** From equation (i) dW = mvdv .

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow W = \frac{1}{2}m[v^2 - u^2]$$

Work done = change in kinetic energy

W = ΔE

This is work energy theorem, it states that work done by a force acting on a body is equal to the change in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive i.e. body moves in the direction of the force (or field) and if kinetic energy decreases, work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

(7) **Various graphs of kinetic energy**

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P = mv]$$

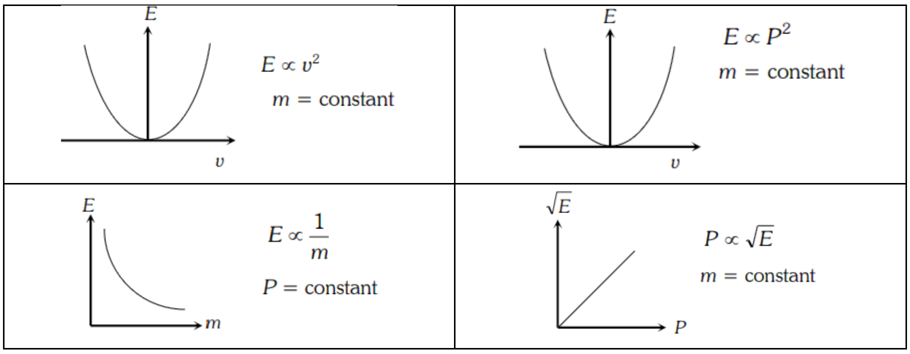
$$\therefore E = \frac{1}{2}Pv$$

or $E = \frac{P^2}{2m}$ [As $v = \frac{P}{m}$]

So we can say that kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2}Pv = \frac{p^2}{2m}$ and Momentum P = $\frac{2E}{v} = \sqrt{2mE}$

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) **Various graphs of kinetic energy**



Stopping of Vehicle by Retarding Force

If a vehicle moves with some initial velocity and due to some retarding force it stops after covering some distance after some time.

(1) **Stopping distance :** Let m = Mass of vehicle,

v = Velocity, P = Momentum, E = Kinetic energy

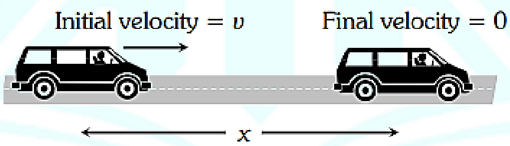
F = Stopping force, x = Stopping distance,

t = Stopping time

Then, in this process stopping force does work on the vehicle and destroy the motion.

By the work- energy theorem

$$W = \Delta K = \frac{1}{2}mv^2$$



⇒ Stopping force (F) × Distance (x) = Kinetic energy (E)

$$\Rightarrow \text{Stopping distance (x)} = \frac{\text{Kinetic energy (E)}}{\text{Stopping force (F)}}$$

$$\Rightarrow x = \frac{mv^2}{2F} \quad ...(i)$$

(2) **Stopping time :** By the impulse-momentum theorem

$F \times t = \Delta P \Rightarrow F \times t = P$

$$\text{or } t = \frac{P}{F} \quad ...(ii)$$

(3) **Comparison of stopping distance and time for two vehicles :** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\frac{x_1}{x_2} = \frac{E_1}{E_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$

and the ratio of their stopping time $\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{m_1 v_1}{m_2 v_2}$

(i) If vehicles possess same velocities

$v_1 = v_2$

(ii) If vehicle possess same kinetic momentum

$P_1 = P_2$

$$\frac{x_1}{x_2}=\frac{E_1}{E_2}=\left(\frac{P_1^2}{2m_1}\right)=\left(\frac{2m_2}{P_2^2}\right)=\frac{m_1}{m_2}$$

$$\frac{t_1}{t_2}=\frac{P_1}{P_2}=1$$

(iii) If vehicle possess same kinetic energy

$$\frac{x_1}{x_2}=\frac{E_1}{E_2}=1$$

$$\frac{t_1}{t_2}=\frac{P_1}{P_2}=\frac{\sqrt{2m_1E_1}}{\sqrt{2m_2E_2}}=\sqrt{\frac{m_1}{m_2}}$$

Note: If vehicle is stopped by friction then

Stopping distance $x=\dfrac{\frac{1}{2}mv^2}{F}=\dfrac{\frac{1}{2}mv^2}{ma}=\dfrac{v^2}{2\mu g}$ [As $a=\mu g$]

Stopping time $t=\dfrac{mv}{F}=\dfrac{mv}{m\mu g}=\dfrac{v}{\mu g}$

Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy.

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2-U_1=-\int_{r_1}^{r_2}\vec{F}.d\vec{r}=-W \qquad ...(i)$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinity and assume potential energy to be zero there, i.e. if we take $r_1=\infty$ and $r_2=r$ then from equation (i)

$$U=-\int_{\infty}^{r}\vec{F}.d\vec{r}=-W$$

In case of conservative force (field) potential energy is equal to negative of work done by conservative force in shifting the body from reference position to given position.

This is why, in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive i.e. potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative i.e. potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $\left(-\vec{\nabla}\right)$ of the potential energy.

So $\vec{F}=\vec{\nabla}U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla}=\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}k$)

$$\Rightarrow\vec{F}=-\left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}k\right]$$

where,

$\frac{\partial U}{\partial x}$ = Partial derivative of U w.r.t. x (keeping y and z constant)

$\frac{\partial U}{\partial y}$ = Partial derivative of U w.r.t. y (keeping x and z constant)

$\frac{\partial U}{\partial z}$ = Partial derivative of U w.r.t. z (keeping x and y constant)

(3) **Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

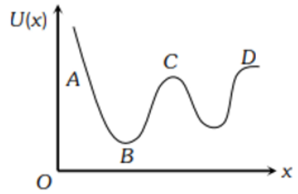


Figure shows a graph of potential energy function U(x) for one dimensional motion.

As we know that negative gradient of the potential energy gives force.

$$\therefore -\frac{\partial U}{dx} = F$$

(4) **Nature of force**

(i) Attractive force : On increasing x, if U increases,

$\frac{\partial U}{dx}$ = positive, then F is in negative direction

i.e. force is attractive in nature.

In graph this is represented in region BC.

(ii) Repulsive force : On increasing x, if U decreases,

$\frac{\partial U}{dx}$ = negative, then F is in positive direction

i.e. force is repulsive in nature.

In graph this is represented in region AB.

(iii) Zero force : On increasing x, if U does not change,

$\frac{\partial U}{dx}$ = 0 then F is zero

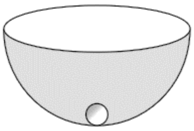
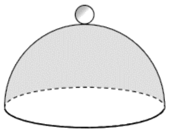
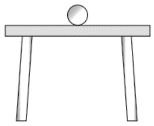
i.e. no force works on the particle.

Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

(5) **Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

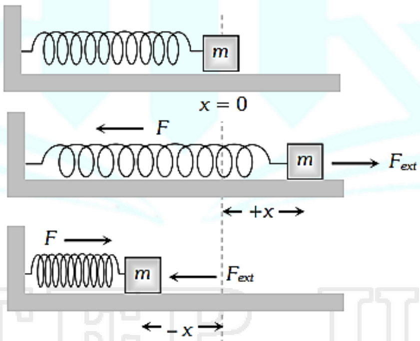
For equilibrium $\frac{\partial U}{dx}$ = 0, but the equilibrium of particle can be of three types :

Stable	Unstable	Neutra
When a particle is displaced slightly from its present position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from its present position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from its position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F=-\frac{dU}{dx}=0$	$F=-\frac{dU}{dx}=0$	$F=-\frac{dU}{dx}=0$
i.e. rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2}$ = negative i.e. rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2}=0$ i.e. rate of change of $\frac{dU}{dx}$ is zero.
Example :  A marble placed at the bottom of a hemispherical bowl.	Example :  A marble balanced on top of a hemispherical bowl.	Example :  A marble placed on horizontal table.

Elastic Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position (x=0) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position. According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e. $\vec{F}\propto-\vec{x}$

or $\vec{F}=-k\vec{x}$ ...(i)

where k is called spring constant.

If x = 1, F = k (Numerically)

Or k = F

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k=\frac{F}{x}$

$$\therefore[k]=\frac{[F]}{[x]}=\frac{[MLT^{-2}]}{L}=[MT^{-2}]$$

Units : S.I. unit Newton/metre, C.G.S unit Dyne/cm.

Note: Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position (x=0), work has to be done by external force against restoring force.

$\vec{F}_{ext} = -\vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

dW = $\vec{F}_{ext}.d\vec{x} = F_{ext}.dx\cos 0°$ = kx dx [As cos 0° = 1]

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W\int_0^x dW = \int_0^x kx\,dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy in the stretched spring.

∴ Elastic potential energy $U = \frac{1}{2}kx^2$

$U = \frac{1}{2}Fx$ $\left[As\ k = \frac{F}{x}\right]$

$U = \frac{F^2}{2k}$ $\left[As\ x = \frac{F}{k}\right]$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$$

Note: If spring is stretched from initial position x_1 to final position x_2 then work done = Increment in elastic potential energy

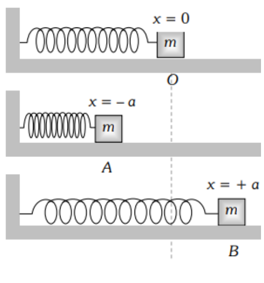
$$= \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

Work done by the spring-force on the block in various situation are shown in the following table.

Work done for spring

Initial state of the spring	Final state of the spring	Initial position (x_1)	Final position (x_2)	Work done (W)
Natural	Compressed	0	−x	$-1/2\ kx^2$
Natural	Elongated	0	x	$-1/2\ kx^2$
Elongated	Natural	x	0	$1/2\ kx^2$
Compressed	Natural	− x	0	$1/2\ kx^2$
Elongated	Compressed	x	− x	0
Compressed	Elongated	− x	x	0

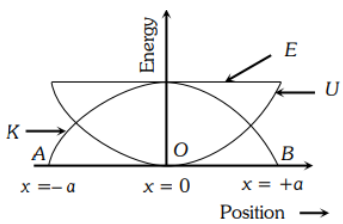
(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$U = \frac{1}{2}kx^2$ ...(i)

So for the extreme position

$U = \frac{1}{2}kx^2$ [As x = ± a for extreme]



This is maximum potential energy or the total energy of mass.

$\therefore$ Total energy $E = \frac{1}{2}kx^2$ ...(ii)

[Because velocity of mass is zero at extreme position]

$K = \frac{1}{2}mv^2 = 0$]

Now kinetic energy at any position

$K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx^2$

$K = \frac{1}{2}k\left(a^2 - x^2\right)$ ...(iii)

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme $x = \pm a$]

and $U_{min} = 0$ [At mean $x = 0$]

$K_{max} = \frac{1}{2}ka^2$ [At mean $x = 0$]

and $K_{min} = 0$ [At extreme $x = \pm a$]

$E = \frac{1}{2}ka^2$ = constant (at all positions)

It means kinetic energy and potential energy changes parabolically w.r.t. position but total energy remain always constant irrespective to position of the mass.

Electrical Potential Energy

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance r.

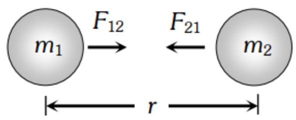
$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

While for a point charge q at a point in an electric field where the potential is V

U = qV

As charge can be positive or negative, electric potential energy can be positive or negative.

Gravitational Potential Energy



It is the usual form of potential energy and this is the energy associated with the state of separation between two bodies that interact via gravitational force.

For two particles of masses m_1 and m_2 separated by a distance r

Gravitational potential energy $U = \frac{Gm_1m_2}{r}$

(1) If a body of mass m at height h relative to surface of earth then

Gravitational potential energy $U = \dfrac{mgh}{1 + \frac{h}{R}}$

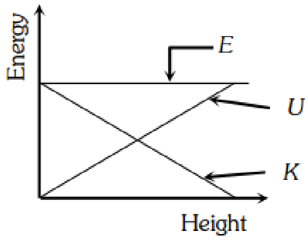
Where R = radius of earth, g = acceleration due to gravity at the surface of the earth.

(2) If h << R then above formula reduces to U = mgh.

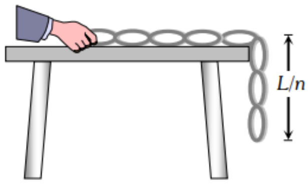
(3) If V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be U = mV

(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its initial potential energy is zero.

As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion, total energy remains constant as shown in the figure.



Work Done in Pulling the Chain Against Gravity



A chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.

Let $m = \frac{M}{L}$ mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$dW = F(-dy)$ [As y is decreasing]

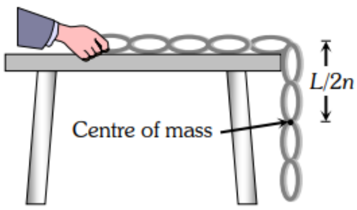
i.e. $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

$$W = -\int_{L/n}^{x} mgy\,dy = -mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mgL^2}{2n^2}$$

$\therefore W = \frac{MgL}{2n^2}$ [As m = M/L]

Alternative method :



If point mass m is pulled through a height h then work done W = mgh

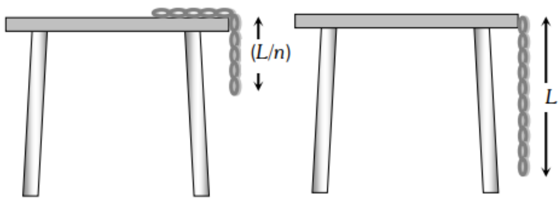
Similarly for a chain we can consider its centre of mass at the middle point of the hanging part i.e. at a height of L/(2n) from the lower end and mass of the hanging part of chain $= \frac{M}{n}$

So work done to raise the centre of mass of the chain on the table is given by

$W = \frac{M}{n} \times g \times \frac{L}{2n}$ [As W = mgh]

or $W = \frac{MgL}{2n^2}$

Velocity of Chain While Leaving the Table



Taking surface of table as a reference level (zero potential energy)

Potential energy of chain when $1/n^{th}$ length hanging from the edge $= \frac{-MgL}{2n^2}$

Potential energy of chain when it leaves the table $= -\frac{MgL}{2}$

Kinetic energy of chain = loss in potential energy

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{MgL}{2} - \frac{MgL}{2n^2}$$

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{MgL}{2}\left[1 - \frac{1}{n^2}\right]$$

$\therefore$ Velocity of chain = $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}$

Law of Conservation of Energy

(1) **Law of conservation of energy :** For a body or an isolated system by work-energy theorem we have

$K_2 - K_1 = \int \vec{F}.d\vec{r}$ ...(i)

But according to definition of potential energy in a conservative field

$U_2 - U_1 = -\int \vec{F}.d\vec{r}$ ...(ii)

So from equation (i) and (ii) we have

$K_2 - K_1 = -(U_2 - U_1)$

or $K_2 + U_2 = K_1 + U_1$

i.e. K + U = constant.

For an isolated system or body in presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as the law of conservation of mechanical energy.

$\Delta(K + U) = \Delta E = 0$ [As E is constant in a conservative field]

$\therefore \Delta K + \Delta U = 0$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount equal to work done by the frictional force.

$\Delta(K + U) = \Delta E = W_f$ [where W_f is the work done against friction]

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$ [where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy which is conserved, but it is the total energy, may be heat, light, sound or mechanical etc., which is conserved. In other words : *"Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system remain constant". This is the law of conservation of energy.*

Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.})\frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.})\frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Types of equilibrium :

(a) **Stable equilibrium :** When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions: $-\frac{dU}{dx}=0, \frac{d^2U}{dx^2}=+ve$

(b) **Unstable Equilibrium :** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition : $\frac{dU}{dx}$ = 0 potential energy is max i.e. $\frac{d^2U}{dx^2}=-ve$

(c) **Neutral equilibrium :** In the neutral equilibrium potential energy is constant .When a particle is displaced from its position it does not experience any force to acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

Law of Conservation of linear momentum

(a) According to this principle, when the value of external force acting on a particle or system is zero, its linear momentum remains conserved. On the other hand in the absence of external force, the linear momentum of a particle or system remains unchanged. This is known as law of conservation of linear momentum.

(b) $\therefore \ \vec{F}_{ext}=\frac{d\vec{p}}{dt}$

If $\vec{F}_{ext}=0 \Rightarrow \frac{d\vec{p}}{dt}=0 \Rightarrow d\vec{p}=0$

$\Rightarrow$ Change in momentum = 0

$\Rightarrow$ Momentum = Constant

If $\vec{p}_1, \vec{p}_2, \vec{p}_3$,................, be the linear momentum of elements of system, then

$\vec{p}_1+\vec{p}_2+\vec{p}_3+$ ....Constant

Hence if the external force acting on a system is zero, the resultant momentum remains conserved.

(c) The above equation is equivalent to three scalar equations.

$p_{1x}+p_{2x}+p_{3x}+----+p_{nx}$ = constant

$p_{1y}+p_{2y}+p_{3y}+----+p_{ny}$ = constant

$p_{1z}+p_{2z}+p_{3z}+----+p_{nz}$ = constant

On the other hand in the absence of external force, the components, of momentum in different direction remains conserved or the component of momentum along X-axis, Y-axis and Z-axis remains conserved.

Collision of bodies

The event or the process, in which wo bodies either coming in contact with each other or due to mutual interaction at distance apart, affect each others motion (velocity, momentum, energy or direct of motion) is defined as a collision between the bodies. **In short, the mutual interaction between two bodies or particles is defined as a collision.** In collision –

(a) The particles come closer, before collision and after collision, they either stick together or move away from each other.

(b) The particles need not come in contact with each other for a collision.

(c) The law of conservation of linear momentum is necessarily conserved in a collision, where as the law of conservation of mechanical energy is not.

NOTE – If $\vec{F}$ is the average of the time verifying force during collision and ?t is the duration of collision then impulse $J=\vec{F}\Delta t$.

TYPES OF COLLISIONS

On the basis of direction :

- **One - dimensional collision or Head on collision or Direction collision**
  The collision , in which the particles move along the same straight line before and after the collision, is defined as one dimensional collision.
- **Two dimensional collision or Oblique collision**
  The collision , in which the particles move in the same plane at different angles before and after collision , is defined as oblique - collsion.

On the basis of conservation of kinetic energy:

- **Elastic- collision**
  A collision is said to be elastic , if the total kinetic energy before and after collision remains the smae
- **In-elastic- collision**
  A collision is said to be in-elastic , if the total kinetic energy does not remains constant
- **Perfectly inelastic collision**
  The collision , in which particles gets sticked together after the collision , is called perfectly inelastic collision. In this type of inelastic collsion , lose of energy is maximum.

NOTE : Linear momentum is essentially conserved in any collision.

Newton's law of collision

$$\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}} = -e\,,$$

where e = coefficient of restitution

Let u_1 and u_2 be the velocities of two bodies before collision and v_1 and v_2 that after the collision, then

$$\frac{v_2 - v_1}{u_2 - u_1} = -e$$

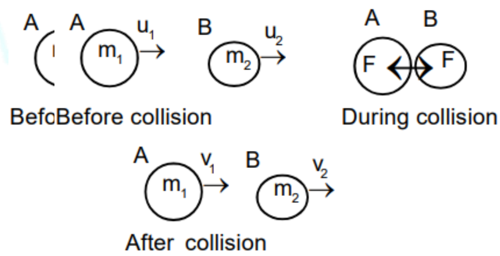
e = 1, for elastic collision

e < 1, for inelastic collision

e = 0, for perfectly inelastic collision

Now we discuss the collision as follows

Direct elastic collision



(a) In this collision, both momentum and kinetic energy remains conserved. Thus for such collision

$(p)_{b.c} = (p)_{a.c}$

$$\Rightarrow m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

[where, p stands for linear momentum

b.c for - before collision

a.c for - after collision]

[KE. for - kinetic energy.]

and $(K.E.)_{b.c} = (K.E.)_{a.c}$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(b) Newton's law for elastic direct collision :

$v_2 - v_1 = -(u_2 - u_1)$

(c) Important formula and features for direct elastic collision :

(i) The velocity of first body after collision

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

(ii) The velocity of second body after collision

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

(iii) If the body with mass m_2 is initially at rest i.e. $u_2 = 0$,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 \text{ and } v_2 = \frac{2m_2}{m_1 + m_2}u_1$$

(iv) When a particles of mass m1 moving with velocity u1 collides with another particle with m_2 at rest and if-

↓	↓	↓
$m_1 = m_2$	$m_1 >> m_2$	$m_1 <<< m_2$
In this case,	In this case,	In this case,
$v_1 = 0$ and	$v_1 = u_1$ and	$v_1 = -u_1$ and
$v_2 = u_1$	$v_1 = 2u_1$	$v_2 = \frac{2m_1}{m_2}u_1 \approx 0$

(v) When $m_1 = m_2 = m$ but $u_2 \neq 0$, then $v_1 = u_2$ and $v_2 = u_1$ i.e. the particles mutually exchange their velocities.

(vi) Exchange of energy is maximum, when $m_1 = m_2$. This fact is utilised in atomic reactor in slowing down the neutrons. To slow down the neutrons , these are made to collide with nuclei of almost similar mass. For this hydrogen nuclei are most appropriate.

NOTE : To solve problems based on direct elastic collisions, the momentum conservation law and Newton's law of collision are to be applied. In special circumstances law of conservation of kinetic energy should be applied.

Direct inelastic collision

(a) In this case, $(p)_{b.c} = (p)_{a.c}$

$$\Rightarrow m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

but

$(K.E.)_{b.c.} \neq (K.E)_{a.c}$

$\Rightarrow (K.E)_{b.c} + (K.E.)_{a.c} + Q$,

(where Q = heat energy, sound energy etc.)

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(b) According to Newton's law, for inelastic collision we have, $v_1 - v_2 = -e(u_1 - u_2)$

(c) In inelastic collision , velocity of first body after collision :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{m_2(1+e)}{m_1 + m_2}\right)u_2 \text{ and}$$

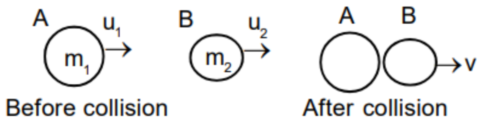
velocity of second body,

$$v_2 = \frac{m_2(1+e)}{m_1 + m_2}u_1 + \frac{m_2 - em_1}{m_1 + m_2}u_2$$

(d) Loss of energy in inelastic collision :

$$\Delta E_k = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2(1-e)^2$$

Direct, perfectly inelastic collision



In this type of collision,

(a) $(p)_{b.c} = (p)_{a.c}$

$$\Rightarrow m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)V$$

but

$(K.E.)_{b.c.} \neq (K.E)_{a.c}$

$\Rightarrow (K.E)_{b.c} + (K.E.)_{a.c} + Q,$

$$\Rightarrow \frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m_2u_2{}^2 = \frac{1}{2}(m_1 + m_2)v^2 + Q$$

(b) According to Newton's law for this collision :

$v_1 = v_2$ $\qquad (\because e = 0)$

(c) Velocity after collision of the combined body :

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

(d) Loss of energy :

$$\Delta E_k = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2$$

(e) If $u_2 = 0$, ratio of final energy to initial energy :

$$\frac{E_f}{E_i} = \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2}m_1u_1^2} = \frac{m_1}{m_1 + m_2} < 1$$

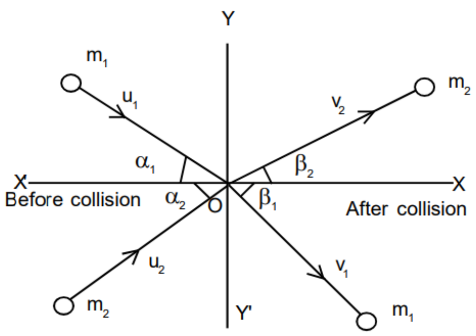
$\Rightarrow E_f < E_i$

i.e. there is loss of kinetic energy (f)

If $u_2 = 0$, $\dfrac{\Delta E_k}{E_i} = \dfrac{m_2}{(m_1 + m_2)}$

NOTE : In some cases the kinetic energy of combined body gets increased, but this is compensated by the lose in potential energy of the body

Oblique elastic collision



In this case , both momentum and K.E. remains conserved. Momentum is a vector quantity and it can be resolved into any two perpendicular directions (say, x and y)

(a) Applying law of conservation of linear momentum :

along x-axis : $(p_x)_{b.c} = (p_x)_{a.c}$

$\Rightarrow m_1u_1 \cos \alpha_1 + m_2u_2 \cos \alpha_2$

$= m_1v_1 \cos \beta_1 + m_2v_2 \cos \beta_2$ and

along y-axis : $(p_y)_{b.c} = (p_y)_{a.c}$

$\Rightarrow m_1u_1 \sin \alpha_1 + m_2u_2 \sin \alpha_2$

$= m_1v_1 \sin \beta_1 + m_2v_2 \sin \beta_2$

(b) Applying law of conservation of KE. : $(K.E.)_{b.c} = (K.E.)_{a.c}$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(c) If $m_1 = m_2$ and $(\alpha_1 + \alpha_2) = 90°$, $(\beta_1 + \beta_2) = 90°$. Which means, if, two particles of same mass moving at right angles to each other collide elastically, after collision also they move at right angles to each other.

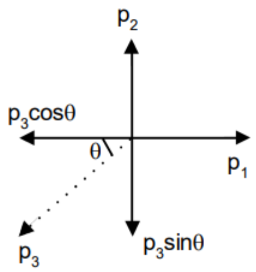
(d) If a body A collides elastically with another body B of same mass at rest at a glancing angle, then after collision the two bodies move at right angles to each other i.e. $\alpha + \beta = 90°$.

(e) If a stationary body breaks due to some interaction in three parts, out of which the first two parts, move at right angles to each other with momenta p_1 and p_2 respectively, then the momenta of third part is determined as follows :

According to law of conservation of momentum :

Along horizontal direction : $p_3 \cos \theta = p_1$

Along vertical direction : $p_3 \sin \theta = p_2$



Magnitude of $p_3 = \sqrt{p_1^2 + p_2^2}$ and its, direction from horizontal,

$$\tan\theta = \frac{p_2}{p_1} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{p_2}{p_1}\right)$$

Direction of p_3, from the direction of motion of first part = $[\pi + \tan^{-1}\frac{p_2}{p_1}]$ and that from the direction motion second part

$= [\frac{\pi}{2} + \tan^{-1}\frac{p_2}{p_1}]$

Work Kinetic Energy Theorem

It is possible to relate the work done by all the forces on a body (or a system) to the change in kinetic energy of the body (or, the system).

Consider a rigid body of acted upon by forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ moving with a velocity $\vec{v}$ which is in general a function of time. Newton's second law gives us :

$$m\frac{d\vec{v}}{dt} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots\ldots = \vec{F}_{net} \qquad \ldots(1)$$

where RHS represents the resultant (net) of all the forces $\vec{F}_1, \vec{F}_2,$, ... Etc.

Taking the dot product of both sides of equation (1) by $\vec{v}$ dt $(= d\vec{r}$, since $\vec{v} = \frac{d\vec{r}}{dt}$), we get,

$$m\vec{v}.\frac{d\vec{v}}{dt}dt = \vec{F}_1.d\vec{r} + \vec{F}_2d\vec{r} + \ldots\ldots\ldots = \vec{F}_{net}.d\vec{r}$$

or $$m\bar{v}.d\vec{v} = \vec{F}_1.dr + \vec{F}_2d\vec{r} + \qquad \ldots(2)$$

Now, $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z k$, and $d\vec{v} = dv_x\hat{i} + dv_y\hat{j} + dv_z k$; so,

$$m\left(v_x dv_x + v_y dv_y + v_z dv_z\right) = \vec{F}_{net}.d\vec{r}$$

or $m\int_A^B v_x dv_x + \int_A^B v_y dv_y + \int_A^B v_z dv_z = \int_A^B \vec{F}_{net}.d\vec{r}$, if the motion takes place from A to B, we get by integration

or, $$m\left(\frac{v_x^2 + v_y^2 + v_z^2}{2}\right)\Bigg|_A^B = \int_A^B \vec{F}_{net}.d\vec{r}$$

or, $$\frac{1}{2}m\left(v_B^2 - v_A^2\right) = \int_A^B \vec{F}_{net}.d\vec{r}$$

or, $$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \int_A^B \vec{F}_{net}.d\vec{r} \qquad \text{...(3)}$$

Wnet = ΔKE

Class : 11th Physics
Chapter- 6 : Work, Energy, and Power

Work, Energy, and Power

Collision

An instance of one moving body striking against another

Types

Perfectly Elastic Collision
- Total mechanical/kinetic energy is conserved.
- Momentum is conserved.

Perfectly Inelastic Collision
- Total energy and linear momentum conserved.
- Kinetic energy not conserved.

power (P)

Rate of doing work

$$P = \frac{\text{Work}(w)}{\text{Time}(t)}$$

$$P_{inst.} = \vec{F}\cdot\frac{\vec{ds}}{dt} = \vec{F}\cdot\vec{v}$$

1hp=746watt

1kWH=3.6×10^6J

Work (w)

Work is said to be done when force produces displacement.

Work done by constant force

$$W = Fs\cos\theta = \vec{F}\cdot\vec{s}$$

Work done by a variable force

$$W = \int_{s_1}^{s_2} F ds\cos\theta = \int_{s_1}^{s_2}\vec{F}\cdot\vec{ds}$$

Types

Positive Work
When force (F) and displacement (S) are in same direction.
W will be +ve when $0<\theta<\frac{\pi}{2}$

Negative Work
When force (F) and displacement (S) are in opposite direction.
W will be (-ve) when $\frac{\pi}{2}<\theta<\pi$

Zero work
When force and displacement are perpendicular to each other. W = zero when $\theta = \frac{\pi}{2}$

Energy

Capacity of doing work

Various forms
- Heat Energy
- Chemical Energy
- Electrical Energy
- Nuclear Energy

Formula
Equivalence of mass and Energy
$E=\Delta mc^2$

law of conservation of Energy
Energy can neither be created nor destroyed; it can only be transformed from one form to another.

Work Energy theorem
Change in Kinetic Energy = Work done by net force on the body i.e;

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W$$

Types of mechanical energy

Potential Energy
By virtue of position
$E_p = mgh$

Kinetic Energy
By virtue of velocity
$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential Energy of Spring
$U = \frac{1}{2}kx^2$
k= spring constant

Important Questions

Multiple Choice Questions-

1. When a force of 50 N acts on a body, the body is displaced through a distance of 3 m in a direction normal to the direction of the force. The work done by the force
   (a) 150 J
   (b) 1470 J
   (c) Zero
   (d) -150 J
2. A body of mass 20 kg is initially at a height of 3 m above the ground. It is lifted to a height of 2 m from that position. Its increase in potential energy is
   (a) 100 J
   (b) 392 J
   (c) 60 J
   (d) -100 J
3. A wooden cube having mass 10 kg is dropped from the top of a building. After 1 s, a bullet of mass 20 g fired at it from the ground hits the block with a velocity of 1000 m/s at an angle of 30° to the horizontal moving upwards and gets imbedded in the block. The velocity of the block/bullet system immediately after the collision is
   (a) 17 m/s
   (b) 27 m/s
   (c) 52 m/s
   (d) 10 m/s
4. A body of mass 10 kg is moved parallel to the ground, through a distance of 2 m. The work done against gravitational force is
   (a) 196 J
   (b) -196 J
   (c) 20 J
   (d) zero
5. A quantity of work of 1000 J is done in 2 seconds. The power utilized is
   (a) 998 W
   (b) 1002 W
   (c) 2000 W
   (d) 500 W
6. A body of mass 1 kg travels with a velocity of 10 m/s, this a wall and rebounds. If 50% of its initial energy is wasted as heat, its kinetic energy at the instant of rebounding is
   (a) 20 J
   (b) 60 J
   (c) 50 J
   (d) 25 J
7. A marble moving with some velocity collides perfectly elastically head-on with another marble at rest having mass 1.5 times the mass of the colliding marble. The percentage of kinetic energy by the colliding marble after the collision is
   (a) 4
   (b) 25
   (c) 44
   (d) 67
8. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force given by ($-kr^2$) where k is a constant , then
   (a) the total energy of the particle is (- k/2r)
   (b) the kinetic energy of the particle is (k/r)
   (c) the potential energy of the particle is (k/2r)
   (d) the kinetic energy of the particle is (- k/r)
9. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass. If e is the coefficient of restitution, then the ratio of velocity of the two spheres after the collision will be
   (a) 1– e / 1 + e
   (b) 1 + e / 1 – e
   (c) e + 1 / e – 1
   (d) e – 1 / e + 1
10. Two masses 1 g and 4 g are moving with equal kinetic energies.s The ratio of the magnitudes of their linear momenta is
    (a) 4 : 1
    (b) 0 : 1
    (c) 1 : 2
    (d) 1 : 6

Very Short:

1. What is the source of the kinetic energy of the falling raindrops?
2. A spring is stretched. Is the work done by the stretching force positive or negative?
3. What is the type of collision when?
   (a) Does a negatively charged body collide with a positively charged body?
   (b) Do macroscopic bodies collide?
   (c) Do two quartz balls collide?
4. (a) Give two examples of potential energy other than gravitational potential energy.
   (b) Give an example of a device that converts chemical energy into electrical energy.
   (c) Heat energy is converted into which type of energy in a steam engine?
   (d) Where is the speed of the swinging pendulum maximum?
   (e) A heavy stone is lowered to the ground. Is the work done by the applied force positive or negative?
5. What is the work done by the centripetal force? Why?
6. (a) What is the work done by the tension in the string of simple pendulum?
   (b) What is the work done by a porter against the force of gravity when he is carrying a load on his hand and walking on a horizontal platform?
   (c) Name the force against which the porter in part (A) is doing some work.
7. When an arrow is shot, wherefrom the arrow will acquire its K.E.?
8. When is the exchange of energy maximum during an elastic collision?
9. Does the work done in raising a load onto a platform depend upon how fast it is raised?
10. Name the parameter which is a measure of the degree of elasticity of a body.

Short Questions:

1. An airplane's velocity is doubled,
   (a) What happens to its momentum? Is the law of conservation of momentum obeyed?
   (b) What happens to its kinetic energy? Is the law of conservation of energy obeyed?
2. In a thermal station, coal is used for the generation of electricity. Mention how energy changes from one form to the other. before it is transformed into electrical energy?
3. Chemical, gravitational and nuclear energies are nothing but potential energies for different types of forces in nature. Explain this statement clearly with examples.
4. What went wrong at the Soviet atomic power station at Chernobyl?
5. A man can jump higher on the moon than on Earth. With the same effort can a runner improve his timing for a 100 m race on the moon as compared to that on Earth?
6. How many MeV are there in a 1-watt hour?
7. What is Newton's experimental law of impact?
8. Two masses one n times as heavy as the other have the same K.E. What is the ratio of their momenta?

Long Questions:

1. (a) State work-energy theorem or principle.
   (b) State and prove the law of conservation of energy.

Assertion Reason Questions:

1. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: A work done by friction is always negative.
   Reason: If frictional force acts on a body its K.E. may decrease.
2. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

Assertion: The work done in moving a body over a closed loop is zero for every force in nature. **Reason:** Work done depends on nature of force.

Case Study Questions:

1. The scalar product or dot product of any two vectors A and B, denoted as A.B (read A dot B) is defined as

$$A.B = A\ B \cos\theta$$

Where q is the angle between the two vectors. Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction. Following are properties of dot product

- the scalar product follows the commutative law: A.B = B.A
- Scalar product obeys the distributive law: (B + C) = A.B + A.C Further, A. (λ B) = λ (A.B) where λ is a real number.
- For unit vectors i, j, k we have

i × i = j × j = k × k = 1 and i × j = j × k = k × i = 0

- A x A = |A ||A| cos 0 = A^2.
- B = 0, if A and B are perpendicular.

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

W = (F cos θ) d = F.d (We see that if there is no displacement, there is no work done even if the force is large. Work has only magnitude and no direction. Its SI unit is (N m) or joule (J). Thus, when you push hard against a rigid brick wall, the force you exert on the wall does not work.

No work is done if:

- The displacement is zero.
- The force is zero. A block moving on a smooth horizontal table is not acted upon by Horizontal force (since there is no friction) but may undergo a large displacement.
- The force and displacement are mutually perpendicular. This is so since, for θ = π/2 rad
- Cos (π/2) = 0. For the block moving on a smooth horizontal table, the gravitational force mg does no work since it acts at right angles to the displacement. If we assume that the moon's orbits around the earth are perfectly circular, then the earth's gravitational force does not work. The moon's instantaneous displacement is tangential while the earth's force is radially inwards and θ = π/2.

i. Scalar product A.B = B.A is

a. Commutative law

b. Distributive law

c. Both a and b

d. None of these

ii. When force acts in the direction of displacement then work done will be

a. Positive

b. Negative

c. Both a and b can possible

d. None of these

iii. Define scalar product. give its properties

iv. Define work done. Give its SI unit

v. Write down the conditions for which work done is zero

2. The kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is

$$K = \frac{1}{2} * mv^2 = \frac{1}{2} v.v$$

Kinetic energy is a scalar quantity. The kinetic energy of an object is a measure of the work and the energy possessed by an object is thus measured in terms of its capacity of doing work. The unit of energy is, therefore, the same as that of work, that is, joule (J).

Work energy theorem: The change in kinetic energy of a particle is equal to the work done on it by the net force. Mathematically

$$\mathbf{K_f - K_i = W}$$

Where K_i and K_f are respectively the initial and final kinetic energies of the object. Work refers to the force and the displacement over which it acts. Work is done by a force on the body over a certain displacement.

i. Kinetic energy is
   a. Scalar quantity
   b. Vector quantity
   c. None of these
ii. Which of the following has same unit?
   a. Potential energy and work
   b. Kinetic energy and work
   c. Force and weight
   d. All of the above
iii. What is work energy theorem?
iv. Kinetic energy is scalar quantity. Justify the statement.
v. Give formula for kinetic energy of body.

Answer Key

Multiple Choice Answers-

1. **Answer:** (c) Zero
2. **Answer:** (b) 392 J
3. **Answer:** (a) 17 m/s
4. **Answer:** (d) zero
5. **Answer:** (d) 500 W
6. **Answer:** (d) 25 J
7. **Answer:** (a) 4
8. **Answer:** (a) the total energy of the particle is (- k/2r)
9. **Answer:** (a) 1 – e / 1 + e
10. **Answer:** (c) 1 : 2

Very Short Answers:

1. **Answer:** It is the gravitational potential energy that is converted into kinetic energy.
2. **Answer:** Positive because the force and the displacement are in the same direction.
3. **Answer:**
   (a) Perfectlyjnelastic collision.
   (b) Inelastic collision.
   (c) Perfectly elastic collision.
4. **Answer:**
   (a) Electrostatic P.E. and elastic P.E.
   (b) Daniell cell.
   (c) Mechanical energy.
   (d) At the bottom of the swing.
   (e) Negative work.
5. **Answer:** Zero. This is because the centripetal is always perpendicular to the displacement.
6. **Answer:**
   (a) zero
   (b) zero
   (c) Frictional force.
7. **Answer:** It is the potential energy of the bent bow which is converted into K.E.
8. **Answer:** When two colliding bodies are of the same mass, there will be a maximum exchange of energy.
9. **Answer:** The work done is independent of time.
10. **Answer:** Coefficient of restitution.

Short Questions Answers:

1. **Answer:**
   (a) The momentum of the airplane will be doubled. Yes, the law of conservation of momentum will also be obeyed because the increase in momentum of the airplane is simultaneously accompanied by an increase in momentum of exhaust gases.
   (b) K.E. becomes four times. Yes, the law of conservation of energy is obeyed with the increase in K.E. coming from the chemical energy of fuel i. e. from the burning of its fuel.
2. **Answer:** When coal is burnt, heat energy is produced which converts water into steam. This steam rotates the turbine and thus heat energy is converted into mechanical energy of rotation. The generator converts this mechanical energy into electrical energy.
3. **Answer:** A system of particles has potential energy when these particles are held a certain distance apart against some force. For example, chemical energy is due to the chemical bonding

between the atoms. Gravitational energy arises when the objects are held at some distance against the gravitational attraction.

Nuclear energy arises due to the nuclear force acting between the nuclear particles.

4. **Answer:** In this reactor, graphite was used as a moderator. The fuel elements were cooled by water and steam was produced from within the reactor. Both water and the steam came in contact with hot graphite. Due to this hydrogen and carbon-monoxide (CO) were released. When they came in contact with air, there was a big explosion.

5. **Answer:** Man can jump higher on the moon because the acceleration due to gravity on the moon is less than that on the Earth. But acceleration due to gravity does not affect the horizontal motion. Hence the runner can't improve his timing on the moon for the 100 m race.

6. **Answer:**

We know that 1 watt hour = 1 JS^{-1} × 3600 s = 3600 J

Also we know that 1 eV = 1.6×10^{-19} J

or $$1\,J = \frac{1}{1.6\times10^{-19}}\,eV$$

$\therefore$ $$1 \text{ watt hour} = 3600\times\frac{1}{1.6\times10^{-19}}\,eV$$

$= 2.25 \times 10^{22}$ eV

Now 1 MeV = 10^6 eV

or 1 eV = 10^{-6} MeV

1 watt hour = $2.25 \times 10^{22} \times 10^{-6}$ MeV

$= 2.25 \times 10^{16}$ MeV

7. **Answer:** The ratio of the relative speed of separation after a collision to the relative speed of approach before the collision is always constant. This constant is known as the coefficient of restitution. It is denoted by e.

$\therefore$ $$e = \frac{V_{2f} - v_{1f}}{u_{1i} - u_{2i}}$$

where u_{1i} and u_{2i}, are the velocities of the bodies before collision and v_{2f}, v_{1f} are the velocities of the bodies after the collision.

8. **Answer:**

We know that $p = \sqrt{2mE_k}$ or $E_k = \dfrac{p^2}{2m}$

Since E_k is constant

$\therefore$ $$p \,\alpha\, \sqrt{m}$$

$\therefore$ $$p_1 \,\alpha\, \sqrt{nm} \text{ and } p_2 \,\alpha\, \sqrt{m}$$

$\therefore$ $$\frac{p_1}{p_2} = \frac{\sqrt{nm}}{\sqrt{m}} = \frac{\sqrt{n}}{1}.$$

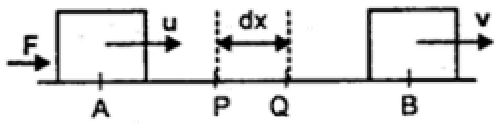
Long Questions Answers:

1. **Answer:**

(a) It states that the work done on a body is equal to the change in its kinetic energy.

i.e., W = change in kinetic energy

Proof: Let m = mass of a body moving in a straight line with a constant initial velocity u.



Let F = force applied on it at point A to B so that its velocity is V at B.

If dx = small displacement from P to Q and a = acceleration produced in the body, then

F = ma

If dw be the work done from P to Q, then

$$dw = F.dx = F\,dx\cos 0$$

$$= ma\,dx = m\frac{dv}{dt}dx$$

$$= mdv\cdot\left(\frac{dx}{dt}\right) = mv\cdot dv$$

If W = total work done from A to B, then

$$W = \int_u^v dW = m\int_u^v u\,dv$$

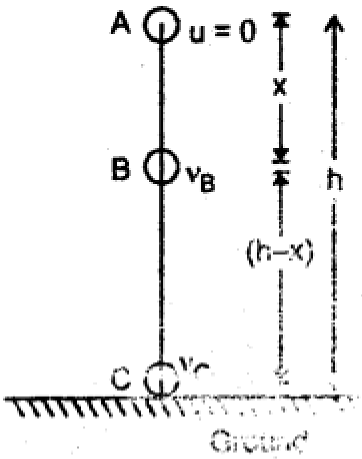
$$m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

= change in K.E. of body.

(b) It states that energy can neither be created nor can be destroyed but it can be changed from one form of energy into another i.e. total energy = constant.

Proof: Let a body of mass m be lying at rest at point A at a height h above the ground. Let it be allowed to fall freely and reaches a point B after falling through a distance x and it finally hits the ground at point C. Let v and V be its velocities at points B and C respectively.

∴ AB = x and BC = h – x



At point A: u = 0

∴ K.E. = 0

P.E. = mgh

If E be the total energy of the body, then

E = K.E. + P.E. = 0 + mgh

or

E = mgh ....(i)

At point B: using the relation,

$v^2 - u^2 = 2$ as, we get

$v_B^2 - 0 = 2gx$

$(\because \text{here } v = v_B, u = 0, a = g, s = x)$

or $v_B^2 = 2gx$

∴ $K.E. = \frac{1}{2}mv_B^2 = \frac{1}{2}m \times (2gx) = mgx$

and $P.E. = mg\ (h - x)$

∴ $E = K.E. + P.E. = mgx + mgh - mgx$

or $E = mgh$ ....(ii)

At point C: Here, v = v_c, a = g, s = h

∴ $v_C^2 - 0^2 = 2gh$

$v_C^2 = 2gh$

Thus $K.E. = \frac{1}{2}mv_C^2 = \frac{1}{2}m \times (2gh) = mgh$

and $P.E. = mg(0)$

(∵ height from ground at point C = 0)

= 0

∴ $E = K.E. + P.E. = mgh + 0 = mgh$

or $E = mgh$ ....(iii)

Thus, from (i), (ii), and (iii), it is clear that total energy at points A, B, and C is the same. It is purely P.E. at A and purely K.E. at point C.

Assertion Reason Answer:

1. (d) Assertion is incorrect, reason is correct.

   Explanation

   When frictional force is opposite to velocity, kinetic energy will decrease.

2. (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   Explanation:

   In close loop, s = 0, and so W = Fs = 0.

Case Study Answer:

1. i. (a) commutative law

   ii. (a) positive

   iii. the scalar product or dot product of any two vectors A and B, denoted as A.B (read A dot B) is defined as

   A.B = A B cos q. where q is the angle between the two vectors. Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction. Following are properties of dot product

   - the scalar product follows the commutative law:

     A.B = B.A

   - Scalar product obeys the distributive law:

     (B + C) = A.B + A.C

     Further, A. (λ B) = λ (A.B) where λ is a real number.

   - For unit vectors i, j, k we have

   i × i = j × j = k × k = 1 and i × j = j × k = k × i = 0

   - A×A = |A ||A| cos 0 = A².
   - B = 0, if A and B are perpendicular.

   iv. The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

   W = (F cos θ) d = F.d

   Work has only magnitude and no direction. Its SI unit is (N m) or joule (J).

v. **No work is done if:**
- The displacement is zero.
- The force is zero. A block moving on a smooth horizontal table is not acted upon by a Horizontal force (since there is no friction) but may undergo a large displacement.
- The force and displacement are mutually perpendicular. This is so since, for $\theta = \pi/2$ rad $\cos(\pi/2) = 0$

2. i. (a) Scalar quantity

ii. (c)All of the above

iii. **Work energy theorem:** The change in kinetic energy of a particle is equal to the work done on it by the net force. Mathematically

$$\mathbf{K_f - K_i = W}$$

Where K_i and K_f are respectively the initial and final kinetic energies of the object. Work refers to the force and the displacement over which it acts. Work is done by a force on the body over a certain displacement. Energy possessed by object due to its motion is called as kinetic energy. Its SI unit is N-m or Joule (J).

iv. Kinetic energy is scalar quantity as it is a work done and work done is scalar quantity hence kinetic energy is also scalar quantity and doesn't have any direction.

v. The kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is

$$K = \frac{1}{2} * mv^2 = \frac{1}{2} v.v$$

Kinetic energy is a scalar quantity. Having unit, the same as that of work, that is, joule (J).

System of Particles and Rotational Motion 7

Introduction

Rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.

Bridge can be taken as rigid body as the distance between any two particles does not change or it is so small that it can be neglected

The lemon is not a rigid body since we can see significant change in its shape after application of force

In pure translational motion at any instant of time all particles of the body have the same velocity.

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

The motion of a rigid body which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation. The motion of a rigid body which is pivoted or fixed in some way is rotation.

Rotational Motion

Let's us understand this by an example. Now let us imagine a circular block going down the edge of the right-angled triangle. Examining the location and orientation of different points on the cylindrical block will tell us something new. The points on the cylindrical body experience something much different than the rectangular block. As shown by the arrows in the diagram representing the velocity, each point experiences a different magnitude of velocity in a different direction. Here the points are arranged with respect to an axis of rotation.

Rotation is what you achieve when you constrain a body and fix it along with a straight line. This means that the body can only turn around the line, which is defined as rotational motion. A ceiling fan, a potter's wheel, a vehicle's wheel are all examples of rotational motion.

Translational Motion

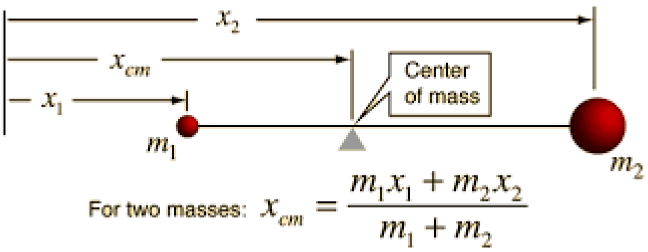
Let us understand translational motion with the help of examples. Let's imagine a rectangular block placed on the slanting edge of a right-angled triangle. If the block is assumed to slide down this edge without any side movement, every point in the rectangular block experiences the same displacement and more importantly, the distance between the points is also maintained.

In a pure translational motion, every point in the body experiences the same velocity be it at any instant of time. Both the points, P1 and P2 undergo the exact same motions. A car moving in a straight line, the path of a bullet out of a gun etc. are examples of translational motion.

Centre of mass

Centre of mass of a body or system of a particle is defined as, a point at which the whole of the mass of the body or all the masses of a system of particle appeared to be concentrated. In physics, we can say that the centre of mass is a point at the centre of the distribution of mass in space (also known as balance point) wherein the weighted relative position of the distributed mass has a sum of zero. In simple words, the centre of mass is a position that is relative to an object. We can say that it is the average position of all the parts of the system or it is the mean location of a distribution of mass in space. It is a point where force is usually applied that results in linear acceleration without any angular acceleration.

When we are studying the dynamics of the motion of the system of a particle as a whole, then we need not bother about the dynamics of individual particles of the system. But only focus on the dynamic of a unique point corresponding to that system.

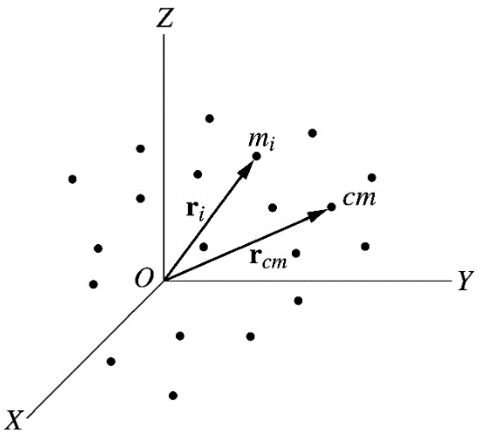


Motion of this unique point is identical to the motion of a single particle whose mass is equal to the sum of all individual particles of the system and the resultant of all the forces exerted on all the particles of the system by surrounding bodies (or) action of a field of force is exerted directly to that particle. This point is called the centre of mass of the system of particles. The concept of centre of mass (COM) is useful in analyzing the complicated motion of the system of objects, particularly when two and more objects collide, or an object explodes into fragments.

Centre of Gravity

The Centre of gravity can be taken as the point through which the force of gravity acts on an object or system. It is basically the point around which the resultant torque due to gravity forces disappears. In cases where the gravitational field is assumed to be uniform, the centre of gravity and centre of mass will be the same. Sometimes these two terms – the centre of gravity and centre of mass are used interchangeably as they are often said to be at the same position or location.

System of Particles



The term system of particles means a well-defined collection of a large number of particles that may or may not interact with each other or are connected to each other. They may be actual particles of rigid bodies in translational motion. The particle which interacts with each other apply force on each other.

The force of interaction $\overrightarrow{F_{\hat{i}\hat{j}}}$ and $\overrightarrow{F_{\hat{j}\hat{i}}}$ between a pair of i^{th} and i^{th} particle.

These forces of mutual interaction between the particle of the system are called the internal force of the system.

These internal forces always exist in pairs of equal magnitude and opposite directions. Other than internal forces, external forces may also act on all or some of the particles. Here the term external force means a force that is acting on any one particle, which is included in the system by some other body outside the system.

Rigid body

In practice, we deal with extended bodies, which may be deformable or non-deformable (or) rigid. An extended body is also a system of an infinitely large number of particles having an infinitely small separation between them. When a body deforms, the separation between the distance between its particles and their relative locations changes. A rigid body is an extended object in which the separations and relative location of all of its constituent particles remain the same under all circumstances.

It is the average position of all the parts of the system, weighted according to their masses. For a simple rigid object which has a uniform density, the centre of mass is located at the centroid.

Linear Momentum of a System of Particles

Linear momentum is a product of the mass (m) of an object and the velocity (v) of the object. If an object has higher momentum, then it harder to stop it. The formula for linear momentum is p = mv. The total amount of momentum never changes, and this property is called conservation of momentum. Let us study more about Linear momentum and conservation of momentum.

We know that the linear momentum of the particle is

$$\mathbf{p = mv}$$

Newton's second law for a single particle is given by,

$$\mathrm{F} = \frac{dP}{dt}$$

where F is the force of the particle. For ‘ n ‘ no. of particles total linear momentum is,

$$P = p_1 + p_2 + \ldots\ldots + p_n$$

each of momentum is written as m_1v_1 + m_2v_2 + ... + m_nv_n. We know that velocity of the centre of mass is

$$V = \sum \frac{m_i v_i}{M}\ ,\ mv = \sum m_i v_i$$

So comparing these equations we get,

P = M V

Therefore, we can say that the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass. Differentiating the above equation we get,

$$\frac{dP}{dt} = \mathrm{M}\ \frac{dV}{dt} = \mathrm{MA}$$

$\frac{dv}{dt}$ is acceleration of centre of mass, MA is the force external. So,

$$\frac{dP}{dt} = \mathrm{F_{ext}}$$

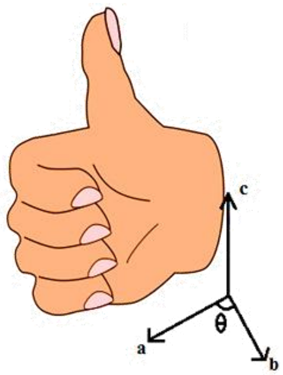
This above equation is nothing but Newton’s second law to a system of particles. If the total external force acting on the system is zero,

$$\mathrm{F_{ext}} = 0 \text{ then, } \frac{dP}{dt} = 0$$

This means that P = constant. So, whenever the total force acting on the system of a particle is equal to zero then the total linear momentum of the system is constant or conserved. This is nothing but the law of conservation of total linear momentum of a system of particles.

Vector Product

Vector product (cross product) of two vectors a and b is a × b = ab $\boldsymbol{sin\theta} = \boldsymbol{c}$, where $\boldsymbol{\theta}$ is angle between a & b



Vector product c is perpendicular to the plane containing a and b.

If you keep your palm in direction of vector a and curl your fingers to the direction a to b, your thumb will give you the direction of vector product c

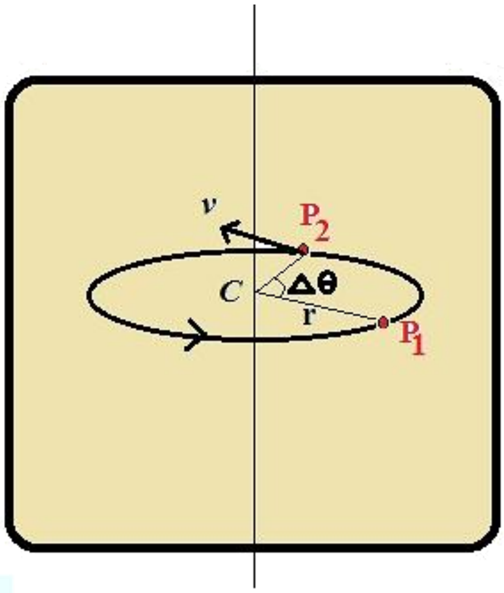
Properties of vector product

- $a \times b \neq b \times a$
- $a \times b = -b \times a$
- $a \times (b + c) = a \times b + a \times c$
- $a \times a = 0$, 0 is called null vector *i.e.,* having zero magnitude
- $\hat{i} \times \hat{j} = \hat{k}$
- $\hat{j} \times \hat{k} = \hat{i}$
- $\hat{k} \times \hat{i} = \hat{j}$

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ then their vector product is given by

$$\begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{a} \times \vec{b} = & a_x & a_y & a_z \\ & b_x & b_y & b_z \end{array}$$

Angular velocity & its relation with linear velocity



Every particle of a rotating body moves in a circle. Angular displacement of a given particle about its centre in unit time is defined as angular velocity.

Average angular velocity = $\frac{\Delta\theta}{\Delta t}$

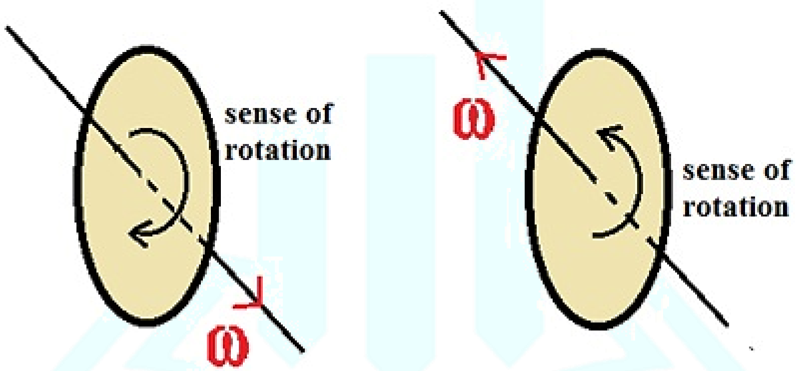
Instantaneous angular velocity, $\boldsymbol{\omega} = \frac{d\theta}{dt}$

v = w r, where v – linear velocity of particle moving in a circle of radius r

All parts of a moving body have the same angular velocity in pure rotation motion.

Angular velocity, ω, is a vector quantity

If you curl your fingers of right hand in the sense of rotation, thumb will give direction of angular velocity.



v = ω x r

Angular acceleration is given by rate of change of angular velocity with respect to time.

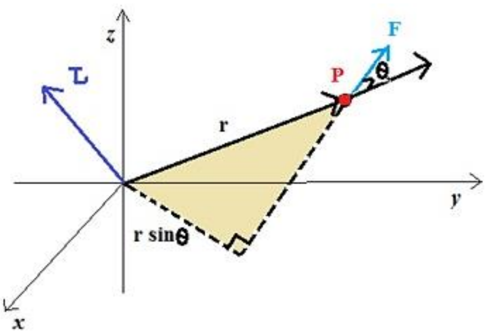
$$\alpha = \frac{d\omega}{dt}$$

Torque & Angular Momentum

The rotational analogue of force is moment of force (Torque).

If a force acts on a single particle at a point P whose position with respect to the origin O is given by the position vector r the moment of the force acting on the particle with respect to the origin O is defined as the vector product $t = r \times F = rF\,sin\theta$

Torque is vector quantity.



The moment of a force vanishes if either

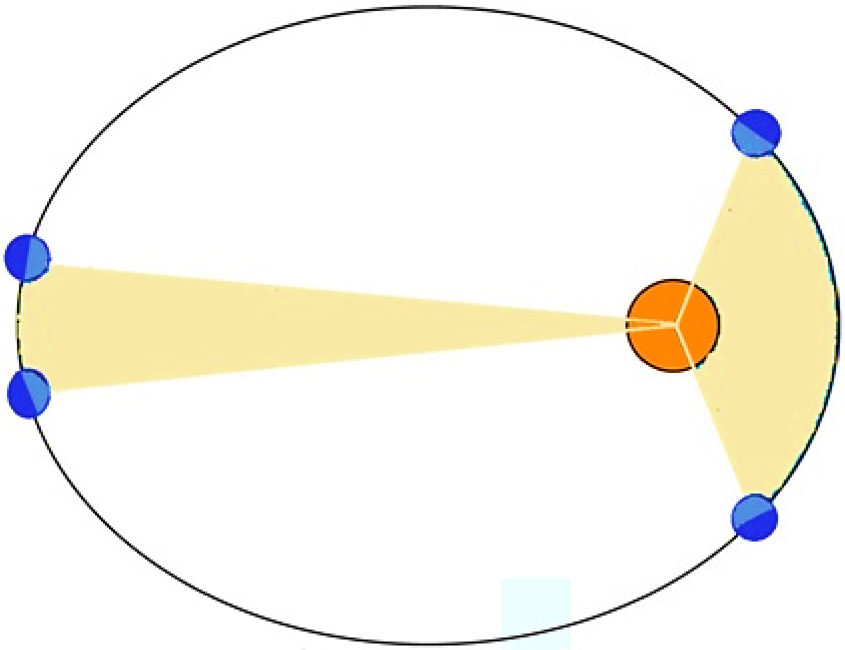
The magnitude of the force is zero, or

The line of action of the force ($r\,sin\theta$) passes through the axis.

Conservation of Angular Momentum

if the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved

If text = 0, then $\frac{dL}{dt} = \mathbf{0}$ => L = constant



Angualar Momentum of planet is conserved. L = mvr = constant.
When close to Sun their speed will slow down.

Equilibrium of Rigid Body

A force changes the translational state of the motion of the rigid body, i.e., it changes its total linear momentum.

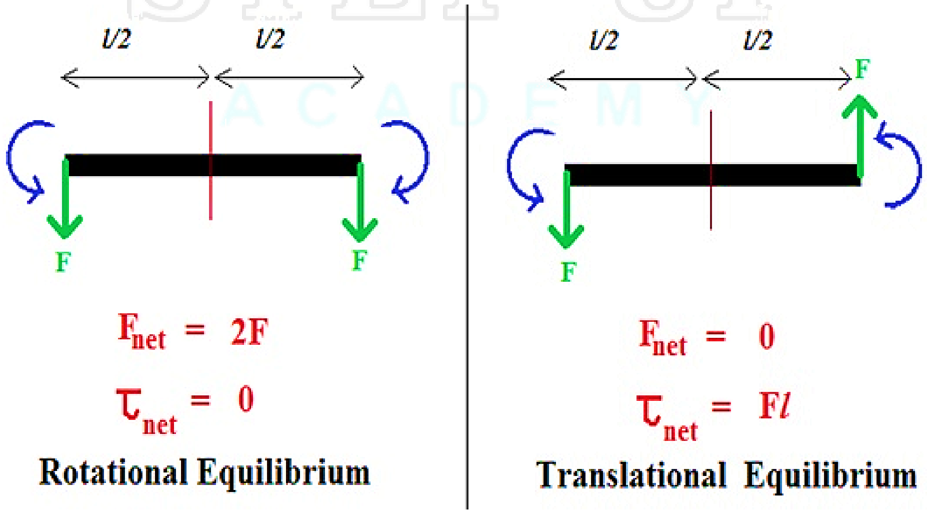
A torque changes the rotational state of motion of the rigid body, i.e., it changes the total angular momentum of the body

Note: Unless stated otherwise, we shall deal with only external forces and torques.

A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time. This means

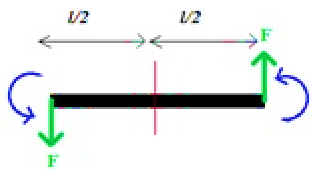
Total force should be zero => Translational Equilibrium

Total torque should be zero => Rotational Equilibrium

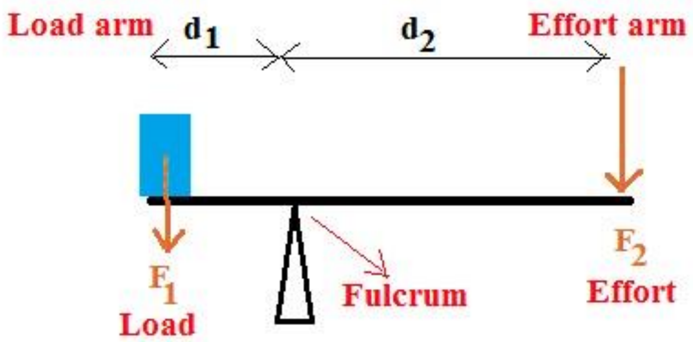


A pair of equal and opposite forces with different lines of action is known as a couple or torque. A couple produces rotation without translation.

When you open the lid of a jar , you apply couple on it



An ideal lever is essentially a light rod pivoted at a point along its length. This point is called the fulcrum The lever is a system in mechanical equilibrium.



$$\text{Mechanical Advantage} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Mechanical advantage greater than one means that a small effort can be used to lift a large load.

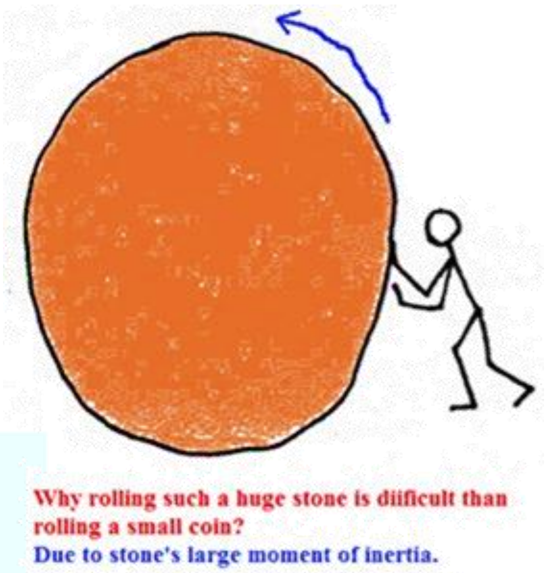
Centre of Gravity

The centre of gravity of a body is that point where the total gravitational torque on the body is zero.

The centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity-free space.

If g varies from part to part of the body, then the centre of gravity and centre of mass will not coincide.

Moment of Inertia

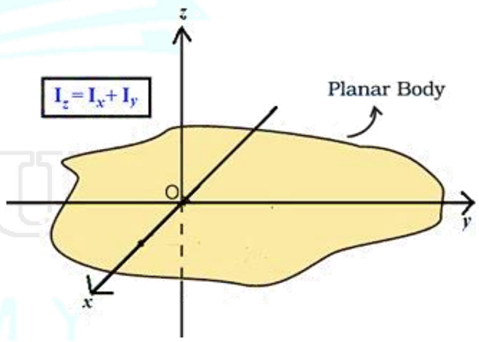


- Moment of inertia (I) is analogue of mass in rotational motion.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

- Moment of inertia about a given axis of rotation resists a change in its rotational motion; it can be regarded as a measure of rotational inertia of the body.
- It is a measure of the way in which different parts of the body are distributed at different distances from the axis.
- the moment of inertia of a rigid body depends on
- The mass of the body,
- Its shape and size
- Distribution of mass about the axis of rotation
- The position and orientation of the axis of rotation.
- The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.
- I = M k^2, where k is radius of gyration.

Theorem of perpendicular axis

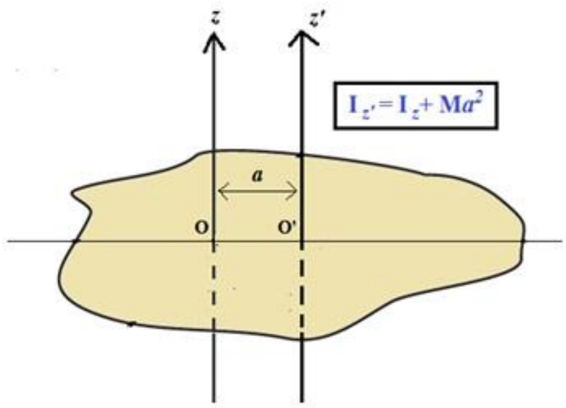


Perpendicular Axis Theorem: The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

Applicable only to planar bodies.

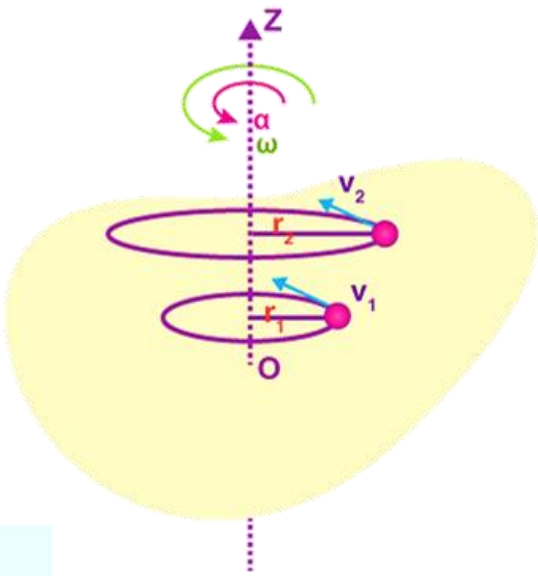
Theorem of parallel axis

Parallel Axis Theorem: The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.



This theorem is applicable to a body of any shape.

Kinematics of Rotational Motion around a Fixed Axis



As we know, the rotational motion and translational motion are analogous to each other in any respect. Also, the terms we use in rotational motion such as the angular velocity and angular acceleration as analogous to the terms velocity and acceleration in translational motion. In that respect, we see that the rotation of a body about a fixed axis is analogous to the linear motion of a body in translational motion. In this section, we will discuss the kinematics of a body undergoing rotational motion about a fixed axis.

Dynamics of Rotational Motion about a Fixed Axis

Only those components of torques, which are along the direction of the fixed axis, need to be considered because the component of the torque perpendicular to the axis of rotation will tend to turn the axis from its position.

This means

We need to consider only those forces that lie in planes perpendicular to the axis. Forces which are parallel to the axis will give torques perpendicular to the axis.

We need to consider only those components of the position vectors which are perpendicular to the axis. Components of position vectors along the axis will result in torques perpendicular to the axis

- Work done by torque is given by: $dW = \tau d\theta$
- Power (instantaneous) is given by: $P = \frac{dW}{dt} = \tau\omega$
- Kinetic Energy is given by: $K = \frac{1}{2} I\omega^2$
- The rate of increase of kinetic energy is $\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\omega\frac{d\omega}{dt}$

  (This is considering the moment of inertia does not change with time.)
- Since $\alpha = \frac{d\omega}{dt}, \frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\omega\alpha$
- We know that Work Done is equal to change in Kinetic Energy, $\tau = I\alpha$
- The angular acceleration is directly proportional to the applied torque and is inversely proportional to the moment of inertia of the body.
- $\tau = I\alpha$ can be called as Newton's second law for rotation about a fixed axis.

Linear Motion	Rotational Motion about a Fixed Axis
1. Displacement x	Angular displacement θ
2. Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3. Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4. Mass M	Moment of inertia I
5. Force $F = Ma$	Torque $\tau = I\alpha$
6. Work $dW = F\,ds$	Work $W = \tau d\theta$

7.	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8.	Power $P = Fv$	Power $P = \tau\omega$
9.	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

Angular Momentum in Case of Rotation about a Fixed Axis

- We know that $L = r \times p$

  $L = r \times (mv)$

  Now, v = $v = \omega r \Rightarrow L = mr^2\omega$

- $L = I\omega$

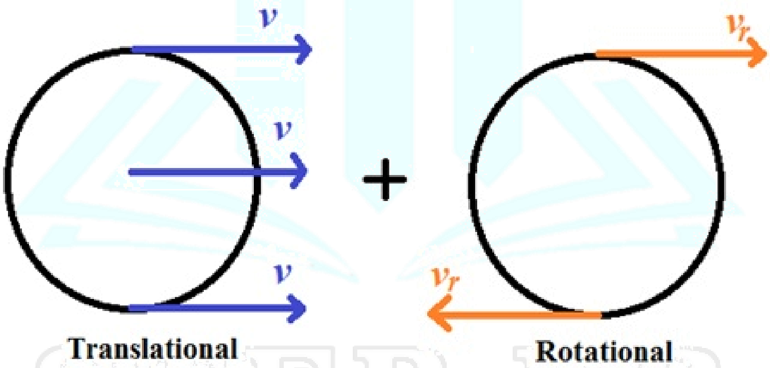
  Conservation of angular momentum

- We know that, $\frac{dL}{dt} = \frac{d}{dt}(I\omega) = \tau$

- If $\tau_{ext} = 0$ then $I\omega$ = constant

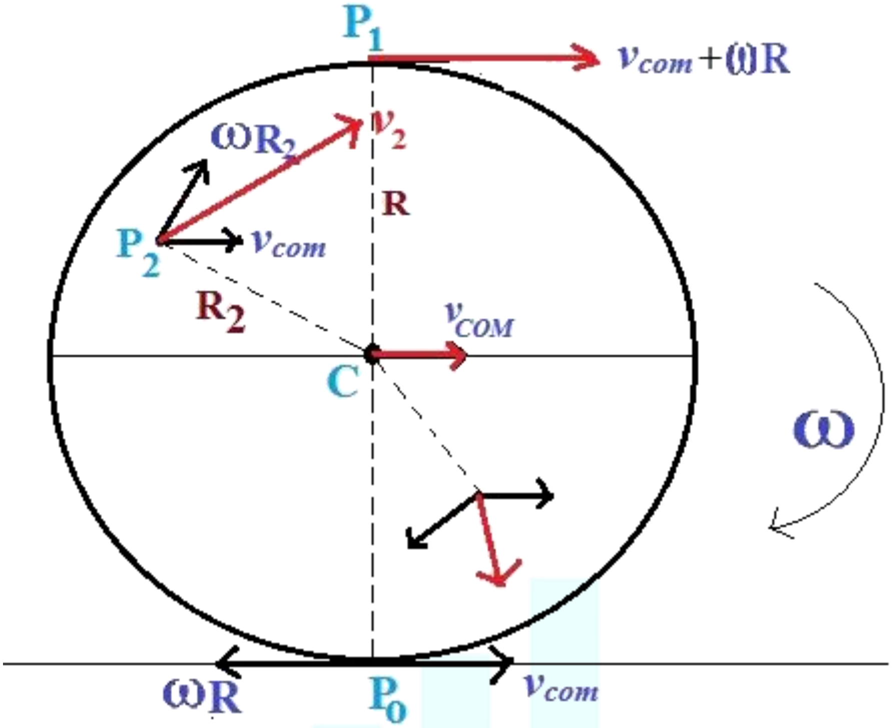
Rolling motion

Rolling motion is a combination of rotation and translation.



All the particles on a rolling body have two kinds of velocity

- Translational, which is velocity of COM.
- Linear velocity on account of rotational motion.



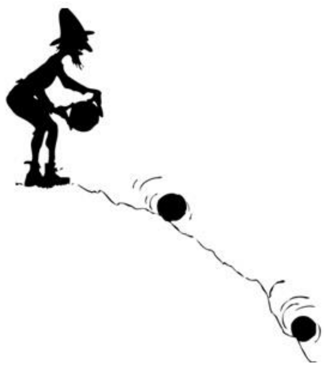
- Here in the figure we can see that every point have two velocities, one in the direction of velocity of COM and other perpendicular to the line joining centre and the point.
- Point P_o have opposite velocities, and if condition of no-slipping is there then it must have zero velocity, so $V_{com} = \omega R$
- At point P_1 both the velocities add up.
- At any other point, add both the velocities vectorially to get the resultant, which are shown for some of the cases in red color in figure.
- The line passing through P_0 and parallel to w is called the instantaneous axis of rotation.
- The point P_0 is instantaneously at rest.
- Kinetic Energy of Rolling Motion
- $KE_{rolling} = KE_{translation} + KE_{rotation}$

  $$KE = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{com}^2$$

  Substituting $I = mk^2$ (where k is radius of gyration) and $v_{com} = R\omega$

  We get

  $$KE = \frac{1}{2}mv_{com}^2\left(1 + \frac{k^2}{R^2}\right)$$



Top Formulae

Position vector of COM of a system	$\vec{R} = \dfrac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + ...}{m_1 + m_2 + m_3 + ...}$
Coordinates of COM	$x = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$ $y = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...}$ $z = \dfrac{m_1z_1 + m_2z_2 + m_3z_3 + ...}{m_1 + m_2 + m_3 + ...}$
Velocity of COM of a system of two particles	$\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_1}{m_1 + m_2}$
Equations of rotational motion	i) $\omega = \omega_1 \quad \alpha t$ ii) $\theta = \omega_1 t + \dfrac{1}{2}\alpha t^2$ iii) $\omega_2 - \omega^2 = 2\alpha\theta$
Centripetal acceleration	$= \dfrac{v^2}{r} = r\omega^2$
Linear acceleration	$a = r\alpha$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p}$
Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Kinetic energy of rotation	$= \dfrac{1}{2}\mathrm{I}\omega^2$
Kinetic energy of translation	$= \dfrac{1}{2}mv^2$
Total kinetic energy	$= \dfrac{1}{2}\mathrm{I}\omega^2 + \dfrac{1}{2}mv^2$
Angular momentum	$L = \mathrm{I}\omega$
Torque	$\tau = I\alpha$
Relation between torque and angular momentum	$\vec{\tau} = \dfrac{d\vec{L}}{dt}$
Moment of inertia in terms of radius of gyration	$I = \sum_{i=1}^{i=n} m_i r_i^2 = MK^2$

Moment of inertia of a uniform circular ring about an axis passing through the centre and perpendicular to the plane of the ring	$I = MR^2$
For a uniform circular disc	$I = \frac{1}{2}MR^2$
For a thin uniform rod	$I = \frac{1}{12}M\ell^2$
For a hollow cylinder about its axis	$I = MR^2$
For a solid cylinder about its axis	$I = \frac{1}{2}MR^2$
For a hollow sphere about its diameter	$I = \frac{2}{3}MR^2$
For a solid sphere about its diameter	$I = \frac{2}{5}MR^2$
Coefficient of friction for rolling of solid cylinder without slipping down the rough inclined plane	$\mu = \frac{1}{3}\tan\theta$

Class : 11th Physics
Chapter- 7 : System of Particles and Rotational Motion

System of Particles and Rotational Motion

Theorem of perpendicular axes
$I_z = I_x + I_y$
I_x, I_y & I_z moments of inertia about perpendicular axes x, y and z respectively

Theorem of parallel axes
$I = I_{cm} + mr^2$
I_{cm} = M.I.about the parallel axis through the centre of mass

Moment of Inertia of some regular shaped body
(1) Rod $I_{cm} = \frac{ml^2}{12}$
I_{one} edge $= \frac{ml^2}{3}$
(2) Ring $I_{cm} = mR^2$
$I_{diameter} = \frac{mR^2}{2}$
(3) Disc $I_{cm} = \frac{mR^2}{2}$
$I_{diameter} = \frac{mR^2}{4}$
(4) Solid sphere
$I_{diameter} = \frac{2}{5}mR^2$

Theorems of moment of Inertia
Inertia of rotational motion, M.I.,
$I = \sum_{i=1}^{n} m_i r_i^2$

Moment Inertia

Equations of rotational motion
(i) $\omega_2 = \omega_1 + \alpha t$
(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
(iii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

Radius of gyration
$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$
$K = \sqrt{\frac{I}{m}}$

Angular momentum
L= Iω = mvr = r × mv
L= r × p = rpsinθ

Torque or couple or moment of force
$\vec{\tau} = \vec{r} \times \vec{F} = \hat{r}\, F\sin\theta\, \hat{n}$
$\tau_{rotational} = I\alpha$

Rigid body
A body with perfectly definite and unchanging shape.

Centre of mass

The point where the whole mass of the system is supposed to be concentrated

- Position of center of mass of an object changes in translatory motion but remains unchanged in rotatory motion
- Position of center of mass depends upon shape, size, distribution of mass of the body

Position of Centre mass of the system
$V_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i}$

Acceleration of centre of mass of the system
$a_{cm} = \sum_{i=1}^{n} \frac{m_i a_i}{\sum m_i}$

Velocity of Centre mass of the system
$V_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$

Centre of mass for rigid bodies is independent of the state i.e., rest or motion of the body

Important Questions

Multiple Choice Questions-

1. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

   (a) L/2

   (b) L/4

   (c) 2 L

   (d) 4 L

2. A car is moving with a speed of 108 km/hr on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s. What is the acceleration of the car?

   (a) 9.8 m/s^2

   (b) 2.7 m/s^2

   (c) 3.6 m/s^2

   (d) 1.8 m/s^2

3. The moment of inertia of uniform circular disc about an axis passing its center is $6kgm^2$. its M.I. about an axis perpendicular to its plane and just touching the rim will be

   (a) 18 kg m^2

   (b) 30 kg m^2

   (c) 15 kg m^2

   (d) 3 kg m^2

4. A particle undergoes uniform circular motion. About which point on the plane of the circle will the angular momentum of the particle remain conserved?

   (a) center of the circle

   (b) on the circumference of the circle

   (c) inside the circle

   (d) outside the circle

5. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is u and the speed of B is 2 u, the speed of center of mass is,

   (a) Zero

   (b) u

   (c) 1.5 u

   (d) 3 u

6. The moment of inertia of a body about a given axis is 1.2 kg $metre^2$. Initially, the body is at rest. In order to produce a rotating kinetic energy of 1500 joules, an angular acceleration of 25 $radian/sec^2$ must be applied about that axis for a duration of

   (a) 4 sec

   (b) 2 sec

   (c) 8 sec

   (d) 10 sec

7. Two discs has same mass rotates about the same axes. r1 and r2 are densities of two bodies (r1 > r2) then what is the relation between l1 and

   (a) l2.

   (b) l1 > l2

   (c) l1 < l2

   (d) l1 = l2

8. The kinetic energy of a body is 4 joule, and its moment of inertia is 2 kg m^2 then angular momentum is

   (a) 4 kg m^2/sec

   (b) 5 kg m^2/sec

   (c) 6 kg m^2/sec

   (d) 7 kg m^2/sec

9. A mass is revolving in a circle which is in the plane of the paper. The direction of angular acceleration is

   (a) Upward to the radius

   (b) Towards the radius

   (c) Tangential

   (d) At right angle to angular velocity

10. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body

    (a) Remains constant

    (b) Becomes half

    (c) Doubles

    (d) Quadruples

Very Short Question:

1. Can the geometrical centre and C.M. of a body coincide? Give examples.

2. How does the M.I. change with the speed of rotation?
3. Under what conditions, the torque due to an applied force is zero?
4. Is it correct to say that the C.M. of a system of n-particles is always given by average position vectors of the constituent particles? If not, when the statement is true?
5. A cat is able to land on her feet after a fall. Which principle of Physics is being used by her?
6. What is conserved when a planet revolves around a star?
7. If no external torque acts on a body, will its angular velocity remain conserved?
8. A body is rotating at a steady rate. Is a torque acting on the body?
9. What is the other name for angular momentum?
10. Out of two spheres of equal masses, one rolls down a smooth inclined plane of height h and the other is falling freely through height h. In which case, the work done is more?

Short Questions:

1. What is the difference between the centre of gravity and C.M.?
2. There are two spheres of the same mass and radius, one is solid, and the other is hollow. Which of them has a larger moment of inertia about its diameter?
3. What shall be the effect on the length of the day if the polar ice caps of Earth melt?
4. If only an external force can change the state of motion of the C.M. of a body, how does it happen that the internal force of brakes can bring a vehicle to rest?
5. What do you understand by a rigid body?
6. What do you understand by a rigid body?
7. Two equal and opposite forces act on a rigid body. Under what conditions will the body (a) rotate, (Z>) not rotate?
8. (a) Why is it easier to balance a bicycle in motion?

   (b) Why spokes are fitted in the cycle wheel?

Long Questions:

1. Discuss the rolling of a cylinder (without slipping) down a rough inclined plane and obtain an expression for the necessary coefficient of friction between the cylinder and the surface.
2. Prove that

   (a) $\Delta\omega = \tau\, \Delta\theta$

   (b) $P = \tau\, \omega$.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: The Centre of mass of a body may lie where there is no mass.

   Reason: Centre of mass of body is a point, where the whole mass of the body is supposed to be concentrated.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

   (c) Assertion is correct, reason is incorrect

   (d) Assertion is incorrect, reason is correct.

   Assertion: The earth is slowing down and as a result the moon is coming nearer to it.

   Reason: The angular momentum of the earth moon system is conserved.

Case Study Questions:

1. The cross product of two vectors is given by Vector C = A × B. The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors. Direction of the

vectors is given by right hand corkscrew rule and is perpendicular to the plane containing the vectors.

∴ |vector C| = AB sin θ and Vector C = AB sin θ n

Where, cap n is the unit vector perpendicular to the plane containing the vectors A and B. Following are properties of vector product

a) Cross product does not obey commutative law. But its magnitude obeys commutative low.

$$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}\Rightarrow\left(\vec{A}\times\vec{B}\right)=-\left(\vec{B}\times\vec{A}\right),$$

$$\left|\vec{A}\times\vec{B}\right|=\left|\vec{B}\times\vec{A}\right|$$

c) It obeys distributive law

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

d) The magnitude cross product of two vectors which are parallel is zero. Since θ = 0.

vector |A x B| = AB sin 0° = 0

e) For perpendicular vectors, θ = 90°, vector |A x B| = AB sin 90° |cap n| = AB

$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

$\hat{i}\times\hat{j}=\hat{k}$; $\hat{j}\times\hat{k}=\hat{i}$; $\hat{k}\times\hat{i}=\hat{j}$

$\hat{j}\times\hat{i}=-(\hat{i}\times\hat{j})=-\hat{k}$; $\hat{k}\times\hat{j}=-(\hat{j}\times\hat{k})=-\hat{i}$; $\hat{i}\times\hat{k}=-(\hat{k}\times\hat{i})=-\hat{j}$

f) The expression for a × b can be put in a determinant form which is easy to remember

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

i. If θ is angle between two vectors, then resultant vector is maximum when θ is

a) 0

b) 90

c) 180

d) None of these

ii. Cross product is operation performed between

a) Two scalar numbers

b) One scalar other vector

c) 2 vectors

d) None of these

iii. Define cross product of two vectors

iv. State right hand screw rule for finding out direction of resultant after cross product of two vectors.

v. Give properties of cross product of parallel vector.

2. Radius of gyration: The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis. the moment of inertia of a rigid body analogous to mass in linear motion and depends on the mass of the body, its shape and size, distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

Theorem of perpendicular axes

It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body. If we consider a planar body, An axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that

I z = I x + I y.

Theorem of parallel axes

The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. z and z' are two parallel axes, separated by a distance a. The z-axis passes through the centre of mass O of the rigid body. Then according to the theorem of parallel axes

$I_{z'} = I_z + Ma^2$

Where I_z and I_z' are the moments of inertia of the body about the z and z¢ axes respectively, M is the total mass of the body and a is the perpendicular distance between the two parallel axes.

i. SI unit of radius of gyration
a) Metre (m)
b) M^2
c) M^3
d) None of these

ii. Moment of inertia is analogous to
a) Mass
b) Area
c) Force
d) None of these

iii. Define radius of gyration

iv. State Theorem of perpendicular axes

v. State Theorem of parallel axes

Answer Key

Multiple Choice Answers-

1. **Answer:** (b) L/4
2. **Answer:** (b) 2.7 m/s^2
3. **Answer:** (a) 18 kg m^2
4. **Answer:** (a) center of the circle
5. **Answer:** (a) Zero
6. **Answer:** (b) 2 sec
7. **Answer:** (b) l1 > l2
8. **Answer:** (a) 4 kg m^2/sec
9. **Answer:** (c) Tangential
10. **Answer:** (b) Becomes half

Very Short Answers:

1. **Answer:** Yes, C.M. and geometrical centre may coincide when the body has a uniform mass density, *e.g.* C.M. and geometrical centre are the same in case of a sphere, cube and cylinder etc.
2. **Answer:** M.I. is not affected by the speed of rotation of the body.
3. **Answer:**

   We know that $\tau = rF \sin\theta$. If $\theta = 0$ or 180,

   or

   r = 0, then $\tau = 0$, r = 0 means the applied force passes through the axis of rotation.
4. **Answer:** No, this statement is true when all the particles of the system are of the same mass.
5. **Answer:** Principle of conservation of angular momentum.
6. **Answer:** Angular momentum.
7. **Answer:** No, it is the angular momentum that will be conserved.
8. **Answer:** No, torque is required only for producing angular acceleration.
9. **Answer:** Moment of momentum.
10. **Answer:** Moment of momentum.

Short Questions Answers:

1. **Answer:** C.G.: It is the point where the whole of the weight of the body is supposed to be concentrated i.e. on this point, the resultant of the gravitational force on all the particles of the body acts.

   C.M.: It is the point where the whole of the mass of the body may be supposed to be concentrated to describe its motion as a particle.
2. **Answer:** The hollow sphere shall have greater M.I., as its entire mass is concentrated at the boundary of the sphere which is at maximum distance from the axis.
3. **Answer:** Melting of polar ice caps will produce water spread around the Earth going farther away from the axis of rotation that will increase the radius of gyration and hence M.I. In order to conserve angular momentum, the angular velocity ω shall decrease. So the length of the day $\left(T = \frac{2\pi}{\omega}\right)$ shall increase.
4. **Answer:** The internal force of brakes converts the rolling friction into sliding friction. When brakes are applied, wheels stop rotating. When they slide, the force of friction comes into play and stops the vehicle. It is an external force.
5. **Answer:** A rigid body is that in which the distance between all the constituting particles remains fixed under the influence of external force. A rigid body thus conserves its shape during its motion.

6. **Answer:**
   - The mutual forces between the particles of a system are called internal forces.
   - The forces exerted by some external source on the particles of the system are called external forces.
7. **Answer:** Two equal and opposite forces acting on a rigid body such that their lines of action don't coincide constitute a couple. This couple produces a turning effect on the body. Hence the rigid body will rotate. If the two equal and opposite forces act in such a way that their lines of action coincide, then the body will not rotate.
8. **Answer:**
   (a) The rotating wheels of a bicycle possess angular momentum. In the absence of an external torque, neither the magnitude nor the direction of angular momentum can change. The direction of angular momentum is along the axis of the wheel. So the bicycle does not get tilted.
   (b) The cycle wheel ¡s constructed in such a way so as to increase the M.I. of the wheel with minimum possible mass, which can be achieved by using spokes and the M.I. is increased to ensure the uniform speed.

Long Questions Answers:

1. **Answer:**

Consider a solid cylinder of mass m, radius R and MJ. I rolling down an inclined plane without slipping as shown in the figure. The condition of rolling down without slipping means that at each instant of time, the point of contact P of the cylinder with the inclined plane is momentarily at rest and the cylinder is rotating about that as the axis.

Let θ = angle of inclination of the plane. The forces acting on the cylinder are:

- The weight mg of the cylinder acting vertically downward.
- The force of friction F between the cylinder and the surface of the inclined plane and acts opposite to the direction of motion.
- The normal reaction N due to the inclined plane acting normally to the plane at the point of contact. The weight W of the cylinder can be resolved into two rectangular components:

(a) mg cos θ along ⊥ to the inclined plane.

(b) mg sin θ along the inclined plane and in the downward direction. It makes the body move downward.

Let a = linear acceleration produced in the cylinder,

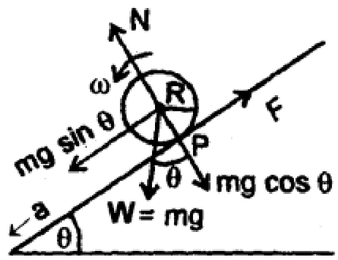
Then according to Newton's 2nd law of motion,

ma = mg sin θ – F .... (1)

and N = mg cos θ .... (2)

If α = angular acceleration of the cylinder about the axis of rotation, then

τ = I α .... (3)



Here, τ is provided by F i.e.

τ = F.R .... (4)

∴ from (3) and (4), we get

$I\,\alpha = FR$

or $$F = \frac{I\alpha}{R} = \frac{I}{R}\cdot\frac{a}{R} \qquad (\because a = R\alpha)$$

$$= \frac{Ia}{R^2} \qquad \text{....(5)}$$

∴ from (1) and (5), we get

$$ma = mg\sin\theta - \frac{Ia}{R^2}$$

or $$ma + \frac{Ia}{R^2} = mg\sin\theta$$

or $$a\left(m + \frac{I}{R^2}\right) = mg\sin\theta$$

or $$a = \frac{mg\sin\theta}{m\left(1 + \frac{I}{mR^2}\right)}$$

or $$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}} \qquad \text{....(6)}$$

For solid cylinder,

$$I = \frac{1}{2}mR^2$$

$\therefore \qquad a = \dfrac{g\sin\theta}{\left(1+\dfrac{1}{2}\right)} = \dfrac{2}{3}g\sin\theta$

$\therefore$ From (5) and (6), we get

$$F = \frac{I}{R^2}\cdot\frac{g\sin\theta}{\left(1+\dfrac{I}{mR^2}\right)}$$

$$F = \frac{mg\sin\theta}{\left(1+\dfrac{mR^2}{I}\right)} \qquad ....(7)$$

$$= \frac{mg\sin\theta}{(1+2)} \qquad \left(\because I = \frac{1}{2}mR^2\right)$$

$$F = \frac{1}{3}mg\sin\theta \qquad ....(8)$$

If μ_s be the coefficient of static friction between the cylinder and the surface,

Then

$$\mu_s = \frac{F}{N} = \frac{\frac{1}{3}mg\sin\theta}{mg\cos\theta}$$

$$= \frac{1}{3}\tan\theta$$

For rolling without slipping

$$\frac{F}{N} \le \mu$$

or $\qquad \dfrac{1}{3}\tan\theta < \mu \qquad ....(9)$

equation (9) is the required condition for rolling without slipping i.e., $\frac{1}{3}$ tan θ should be less than equal to μs i.e., the maximum allowed inclination of the plane with the horizontal is given by

$\theta_{max} = \tan^{-1}(3\mu_s)$

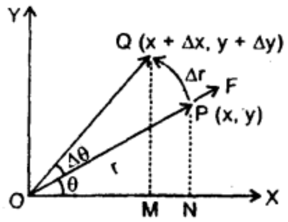
2. **Answer:**

(a) $\Delta\omega = \tau\,\Delta\theta$

Let F = force applied on a body moving in XY plane.

Δr = linear displacement produced in the body by the force F in moving from P to Q.

If Δω is the small work done by the force, then by definition of work.



$$\Delta W = F\,.\,\Delta r \qquad ....(1)$$

In component form,

$$F = F_x\hat{i} + F_y\hat{j}$$

and $\qquad \Delta r = \Delta x\hat{i} + \Delta y\hat{j} \qquad ....(2)$

$\therefore$ from (1) and (2), we get

$$\Delta W = \left(F_x\hat{i} + F_y\hat{j}\right)\cdot\left(\Delta x\hat{i} + \Delta y\hat{j}\right)$$

$$= F_x\Delta x + F_y\Delta y \qquad ....(3)$$

Let PN ⊥ on X-axis & PON = θ

$\therefore$ in rt ∠d ΔPNO,

$$\sin\theta = \frac{y}{r} \qquad ....(4)$$

and $\qquad \cos\theta = \dfrac{x}{r} \qquad ....(5)$

Also in ΔQMO,

$$x + \Delta x = r\cos(\theta + \Delta\theta)$$

and $\qquad y + \Delta y = r\sin(\theta + \Delta\theta)$

As Δθ is very small, i.e. Δθ → 0, cos Δθ → 1 and sin Δθ → Δθ

$\therefore \qquad x + \Delta x = r(\cos\theta\cos\Delta\theta - \sin\theta\sin\Delta\theta)$

$$= r(\cos\theta\,.\,1 - \sin\theta\,.\,\Delta\theta)$$

$$= x - y\,\Delta\theta$$

or $\qquad \Delta x = -y\,\Delta\theta \qquad ....(6)$

and $\qquad y + \Delta y = r(\sin\theta\cos\Delta\theta - \cos\theta\sin\Delta\theta)$

$$= r(\sin\theta\,.\,1 + \cos\theta\,.\,\Delta\theta)$$

$$= y + x\,\Delta\theta$$

or $\qquad \Delta y = x\,\Delta\theta \qquad ....(7)$

$\therefore$ from (3), (6) and (7), we get

$$\Delta\omega = F_x(-y\,\Delta\theta) + F_y(x\,\Delta\theta)$$

$$= \left(x F_y - y F_x\right)\Delta\theta$$

$$= \tau\Delta\theta$$

(b) $P = \tau\,\omega$.

We know that $\quad P = \dfrac{\Delta\omega}{\Delta t} = \dfrac{d\theta}{dt} = \omega \text{ if } \Delta t \to 0$

$\therefore \qquad P = \tau\omega\,.$

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   Explanation

   As the concept of Centre of mass is only theoretical, therefore in practice no mass may lie at the Centre of mass. For example, Centre of mass of a uniform circular ring is at the Centre of the ring where there is no mass.

2. (d) The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

   Explanation:

   The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

Case Study Answer:

1. Answer

(a) (a) 0

(b) (c) 2 vectors

(c) The cross product of two vectors is given by Vector C = A × B. The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors.

4 |vector C| = ABsinθ and Vector C = ABsinθ n. Where, cap n is the unit vector perpendicular to the plane containing the vectors A and B.

(d) We can find the direction of the unit vector with the help of the right-hand rule. In this rule, we can stretch our right hand so that the index finger of the right hand in the direction of the first vector and the middle finger is in the direction of the second vector. Then, the thumb of the right hand indicates the direction or unit vector n.

(e) The cross product of two vectors is zero vectors if both the vectors are parallel or opposite to each other. Conversely, if two vectors are parallel or opposite to each other, then their product is a zero vector. Two vectors have the same sense of direction. θ = 90^0 As we know, sin 0° = 0 and sin 90° = 1

$$\vec{X}\times\vec{Y}=|\vec{X}|\cdot|\vec{Y}|\sin\theta$$

$$\vec{X}\times\vec{Y}=|\vec{X}|\cdot|\vec{Y}|\sin 0^\circ$$

$$\vec{X}\times\vec{Y}=|\vec{X}|\cdot|\vec{Y}|\times 0$$

Hence, the cross product of the parallel vectors becomes $\vec{X}\times\vec{Y}=0$, which is a unit vector.

2. Answer

i. (a) Metre (m)

ii. (c) Mass

iii. **Radius of gyration**: The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

iv. **Theorem of perpendicular axes** It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body. If we consider a planar body, an axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that

$$\mathbf{I\,z = I\,x + I\,y}$$

v. The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. z and z' are two parallel axes, separated by a distance a. The z-axis passes through the centre of mass O of the rigid body. Then according to the theorem of parallel axes

$$\mathbf{I_{z'} = I_z + Ma^2}$$

Where I_z and I_z' are the moments of inertia of the body about the z and z¢ axes respectively, M is the total mass of the body and a is the perpendicular distance between the two parallel axes.

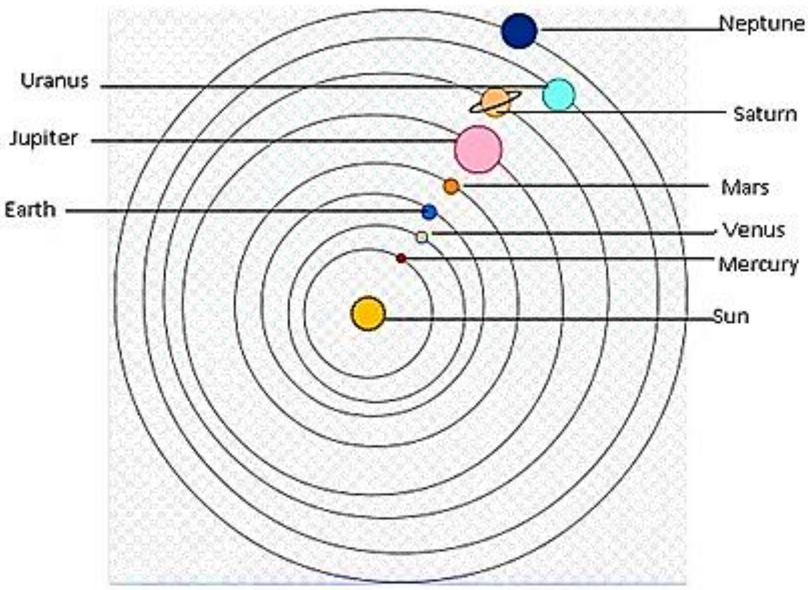
Gravitation 8

Gravitation

When we talk about gravitation or gravity it is a naturally occurring phenomenon or a force which exists among all material objects in the universe.

Whenever we throw an object towards the sky it will fall back onto the ground.

For Example: - A ball comes down when thrown up. Rain drops fall towards the ground; Planets revolve in an elliptical orbit around sun etc.



Planets revolving in the elliptical orbit.

Rain drops falling on the earth.

There is a force due to which all things are attracted towards the earth. This force is known as Gravitation.

Gravitation is the force of attraction between all masses in the universe, especially the force of attraction exerted by the earth on all the bodies near its surface.

In this chapter we will take a look at gravitation force, its laws, and we will also study about the planetary motion.

Leaves fall off the tree.

Gravitational Constant and Universal Law of Gravitation

We know that the universal law of gravitation was put forth by Sir Isaac Newton in 1687. It is one of the most important laws of physics. Let us know more about Newton's universal law of gravitation and gravitational constant.

Universal Law of Gravitation

According to Newton's Universal Law of Gravitation, the force exerted between two objects by each other is given by the following relation.

$$F_g \propto \frac{m_1 m_2}{r^2}$$

where g is the gravitational force between two bodies, m_1 is the mass of one object, m_2 is the mass of the second object and r is the distance between the centres of two objects.

Gravitational Constant

The actual force exerted between two bodies can be given by the following equation

$$F_g = G\frac{m_1 \cdot m_2}{r^2}$$

where G is the universal gravitational constant with a value (G = 6.674 × 10-11 N · (m/kg)^{2}). G here is an empirical constant of proportionality.

What is interesting here is that, even though it is Newton's Universal Law of Gravitation, the value of G wasn't given by him. This was calculated by Henry Cavendish in 1798 through a series of experiments and observations. The influence of the earth's core on the experiments is hypothesised to alter its rotational inertia, because of which the value of G given is not always constant throughout the globe.

Another theory regarding the universal gravitational constant (fun fact: it is also referred to as Big G) is that, if it is true that the universe is expanding since the Big Bang, then the value of G will keep decreasing!

The universal gravitational constant is used in Newton's Universal Law of Gravitation, Einstein's General Theory of Relativity and also Kepler's Third Law of Planetary Motion to calculate the time period of a planet to complete one full revolution in its orbit.

Acceleration due to gravity of the earth

Acceleration attained due to gravity of earth.

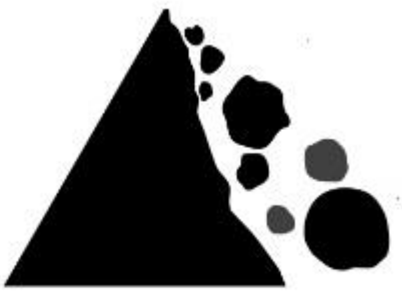
All the objects fall towards the earth because of gravitational pull of the earth.

And when a body is falling freely, it will have some velocity and therefore it will attain some acceleration. This acceleration is known as acceleration due to gravity.

It is a vector quantity.

Denoted by 'g'.

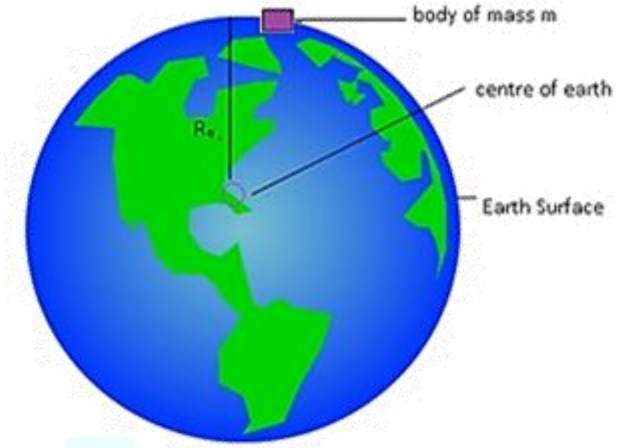
Its value is $9.8 m/s^2$.



Stones falling from rock

Example: Stones falling from a rock will have some velocity because of which some acceleration. This acceleration is due to the force exerted by the earth on the rocks. This is known as acceleration due to gravity.

Expression for Acceleration due to gravity



Consider any object of mass 'm' at a point A on the surface of the earth.

The force of gravity between the body and earth can be calculated as:

$F = G\,m\,M_e / R_e^2$ (1) where

m = mass of the body

M_e = mass of the earth

R_e = distance between the body the earth is same as the radius of the earth

Newton's Second law states that

F=ma (2)

Comparing the equations (1) and (2)

$$F = m(G\,m\,M_e / R_e^2)$$

$(G\,m\,Me / R_e^2)$ is same as g (acceleration due to gravity)

Therefore, the expression for Acceleration due to gravity.

$$g = G\,M_e / R_e^2$$

Acceleration due to gravity below the surface of earth

To calculate acceleration due to gravity below the surface of the earth (between the surface and centre of the earth).

Density of the earth is constant throughout. Therefore,

$$\rho = M_e / (4/3\,\Pi\,R_e^3) \text{ equation (1)}$$

where

M_e = mass of the earth

Volume of sphere = $4/3\,\Pi\,R_e^3$

R_e = radius of the earth

As entire mass is concentrated at the centre of the earth.

Therefore, density can be written as

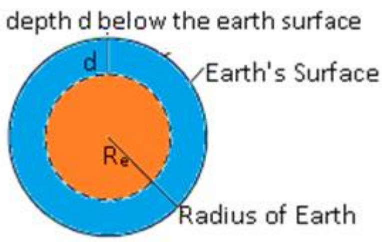
$$\rho = M_s / (4/3\,\Pi\,R_s^3) \text{ equation (2)}$$

Comparing equation (1) and (2)

$$M_e / M_s = R_e^3 / R_z^3 \text{ where } R_s = (R_e - d)^3$$

d = distance of the body form the centre to the surface of the earth

Therefore,



$$M_e / M_s = \mathrm{R}_e^3 / (R_e - d)^3$$

$$M_s = M_e (R_e - d)^3 / R_e^3 \text{ from equation (3)}$$

To calculate Gravitational force (F) between earth and point mass m at a depth d below the surface of the earth.

Above figure shows the value of g at a depth d. In this case only the smaller sphere of radius (R_e-d) contributes to g.

$$F = G\,m\,M_s / (R_e - d)^2$$

$$g = F/m$$

where g = acceleration due to gravity at point d below the surface of the earth.

$$g = GM_s / (R_e - d)^2$$

Putting the value of M_s from equation (3)

$$= GM_e (R_e - d)^3 / R_e^3 (R_e - d)^2$$

$$= GM_e (R_e - d) / R_e^3$$

We know $g = GM_e / R_e^2$ equation (4)

$$g(d) = GM_e / R_e^2 (1 - d / R_e)$$

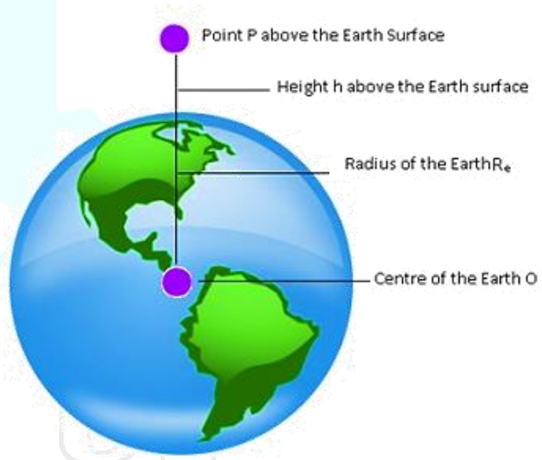
From equation (4)

$$g(d) = g(1 - d / R_e)$$

Acceleration due to gravity above the surface of earth

To calculate the value of acceleration due to gravity of a point mass m at a height h above the surface of the earth.

Above figure shows the value of acceleration due to gravity g at a height h above the surface of the earth.



Force of gravitation between the object and the earth will be

$$F = G\,mM_e / (R_e + h)^2$$

where

m = mass of the object

R_e = radius of the earth

$$g(h) = F / m = GM_e / (R_e + h)^2 = GM_e / [R_e^2 (1 + h / R_e)^2]$$

h << R_e (as radius of the earth is very large)

By calculating we will get,

$$g(h) = g(1 - 2h / R_e)$$

The value of acceleration due to gravity varies on the surface, above the surface and below the surface of the earth.

Inertial and Gravitational Mass

Inertial mass is a mass parameter giving the inertial resistance to acceleration of the body when responding to all types of force. Gravitational mass is determined by the strength of the gravitational force experienced by the body when in the gravitational field g.

Inertial Mass: - Inertial mass is defined as the mass of body by virtue of inertia of mass.

By Newton's Law F=ma

$\boldsymbol{m = \frac{F}{a}}$ where m= inertial mass (as it is because of inertia of a body)

Gravitational Mass: -Gravitational mass is defined as the mass of the body by virtue of the gravitational force exerted by the earth.

By Gravitation Force of attraction

$$F = \frac{GmM}{r^2}$$

$$M = \frac{Fr^2}{GM}$$

where

M = mass of the object

F = force of attraction exerted by the earth

R = distance between object and earth

M = mass of the earth

Experimentally, Inertial mass = Gravitational mass

Gravitational Potential Energy

Stationary roller-coaster

- Potential energy is due to the virtue of position of the object.
- Gravitational Potential Energy is due to the potential energy of a body arising out of the force of gravity.
- Consider a particle which is at a point P above the surface of earth and when it falls on the surface of earth at position Q, the particle is changing its position because of force of gravity.
- The change in potential energy from position P to Q is same as the work done by the gravity.
- It depends on the height above the ground and mass of the body.

Expression for Gravitational Potential Energy

Case1:- 'g' is constant.

Consider an object of mass 'm' at point A on the surface of earth.

Work done will be given as:

W_{BA} = F X displacement where F = gravitational force exerted towards the earth)

= mg (h_2 - h_1) (body is brought from position A to B)

= $mgh_2 - mgh_1$

$$W_{AB} = V_A - V_B$$

where

V_A = potential energy at point A

V_B = potential energy at point B

From above equation we can say that the work done in moving the particle is just the difference of potential energy between its final and initial positions.

Case2:- 'g' is not constant.

Calculate Work done in lifting a particle from $r = r_1$ to $r = r_2$ ($r_2 > r_1$) along a vertical path,

We will get, $W = V(r_2) - V(r_1)$

In general the gravitational potential energy at a distance 'r' is given by :

where

V(r) = potential energy at distance 'r'

V_o = At this point gravitational potential energy is zero.

Gravitational potential energy is $\propto$ to the mass of the particle.

Gravitational Potential

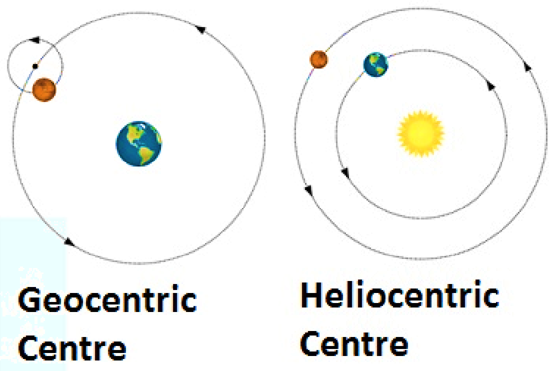
Gravitational Potential is defined as the potential energy of a particle of unit mass at that point due to the gravitational force exerted byearth.

Gravitational potential energy of a unit mass is known as gravitational potential.

Mathematically:

$$G_{potential} = -\frac{GM}{R}$$

Planetary Motion



Ptolemy was the first scientist who studied the planetary motion.

He gave geocentric model. It means all the planets, stars and sun revolve around the earth and earth is at the centre.

Heliocentric model was proposed by some Indian astronomers.

According to which all planets revolve around the sun.

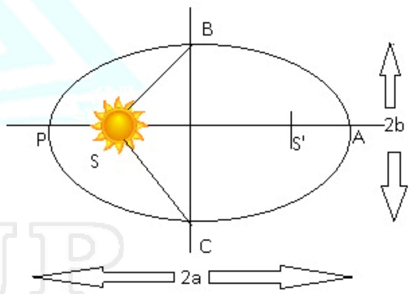
Nicholas proposed the Nicholas Copernicus model according to which all planets move in circles around the sun.

After Nicholas one more scientist named Tycho Brahe did lot of observations on planets.

Finally came Johannes Kepler who used Tycho Brahe observations, and he gave Kepler's 3 laws of Gravitation.

These 3 laws became the basis of Newton's Universal law of Gravitation.

Kepler's 1st Law: Law of Orbits



Statement: The orbit of every planet is an ellipse around the sun with sun at one of the two foci of ellipse.

Whenever a planet revolves around sun it traces an ellipse around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semi major axis is half the distance AP.

Kepler's 1st law Vs. Copernicus Model

According to Copernicus planets move in circular motion whereas according to Kepler planets revolve in elliptical orbit around the sun.

Copernicus model is based on one special case because circle is a special case of ellipse whereas Kepler's laws aremore of ageneral form.

Kepler's law also tells us about the orbits which planets follow.

To Show ellipse is a special form of Circle

Select two points F_1 and F_2.

Take a piece of string and fix its ends at F_1 and F_2.

Stretch the string taut with the help of a pencil and then draw a curve by moving the pencil keeping the string taut throughout. Fig. (a).

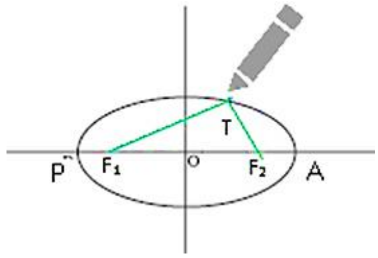
The resulting closed curve is an ellipse. For any point T on the ellipse, the sum of distances from F_1 and F_2 is a constant. F_1, F_2 are called the foci.

Join the points F_1 and F_2 and extend the line to intersect the ellipse at points P and A as shown in Fig. (a).

The centre point of the line PA is the centre of the ellipse O and the length PO = AO, which is also known as the semimajor axis of the ellipse.

For a circle, the two foci merge onto one and the semi-major axis becomes the radius of the circle.

A string has its ends fixed at F_1 and F_2. The tip of the pencil holds the string taut and is moved around and we will get an ellipse.



Kepler's 2nd law: Law of Areas

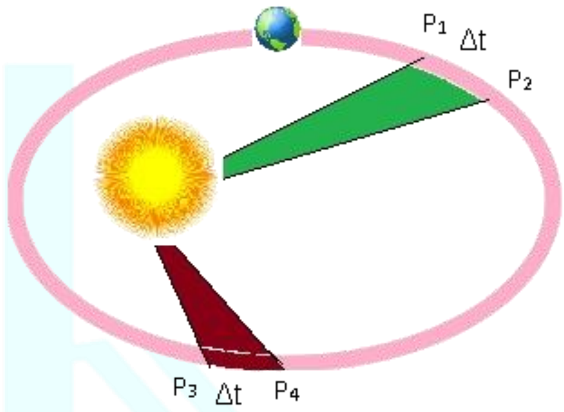
Statement: The line that joins a planet to the sun sweeps out equal areas in equal intervals of time.

Area covered by the planet while revolving around the sun will be equal in equal intervals of time. This means the rate of change of area with time is constant.

Suppose position and momentum of planet is denoted by 'r' and 'p' and the time taken will be Δt.

$\Delta A = \frac{1}{2} x r x v \Delta t$ (where vΔt is distance travelled by a planet in Δt time.)

$$\frac{\Delta A}{\Delta t} = \frac{1}{2}(r x v)$$



where

(Linear momentum) p=mv or we can write as

$v = \frac{p}{m}$

$= \frac{1}{2} m(r \times p)$

$= \frac{1}{2}\frac{L}{2} m$ where L= angular momentum (It is constant for any central force)

$\frac{\Delta A}{\Delta t}$ = constant (This means equal areas are covered in equal intervals of time).

Kepler's 3rdLaw: Law of periods

Statement:

According to this law the square of time period of a planet is ∝ to the cube of the semi-major axis of its orbit.

Suppose earth is revolving around the sun then the square of the time period (time taken to complete one revolution around sun) is ∝ to the cube of the semi major axis.

It is known as Law of Periods as it is dependent on the time period of planets.

Derivation of 3rd Law: assumption: The path of the planet is circular.

Let m=mass of planet

M= mass of sun

According to Newton's Law of Gravitation:

$$F = \frac{GMm}{r^2}$$

$$F_c = \frac{mv^2}{r}$$

where,

F_c =centripetal force which helps the planet to move around sun in elliptical order.

$F = F_c$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$ where r=radius of the circle

$\frac{GM}{r} = v^2$ (1)

$v = 2\frac{\pi r}{T}$

Squaring both the sides the above equation

$v^2 = 4\,\pi^2 r^2/T^2$

putting the value (1)

$\frac{GM}{r} = 4\,\pi r^2/T^2$

$T^2 = (4\,\pi^2\, r^3/GM)$ where $(4\,\pi^2/GM)$ = constant

$T^2 = r^3$ (In ellipse semi-major axis is same as radius of the circle)

Escape Velocity

Escape velocity is the minimum velocity that a body must attain to escape the gravitational field of the earth.

Suppose if we throw a ball, it will fall back. This is happening due to the force of gravitation exerted on the ball by the surface of the earth due to which the ball is attracted towards the surface of the earth.

If we increase the velocity to such an extent that the object which is thrown up will never fall back. This velocity is known as escape velocity.

Ball is thrown up but it falls down because of force of gravitation.

The same ball is thrown with a velocity that it escapes the force of gravitation of earth and does not come back. This velocity is known as escape velocity.

Mathematically:

Suppose we throw a ball and the initial velocity of the ball is equal to the escape velocity such that ball never comes back.

Final Position will be infinity.

At Final Position: At Infinity

Total Energy (∞) = Kinetic Energy (∞) + Potential Energy (∞)

$$\text{Kinetic Energy } (\infty) = \frac{1}{2}mv_f^2$$

where v_f = final velocity

$$\text{Potential Energy } (\infty) = -GMm/r + V_0$$

where M = mass of the earch

m = mass of the ball

V_0 = potential energy at surface of earth,

r = ∞ r = distance from the centre of the earth.

Therefore: Potential Energy (∞) = 0

Total Energy (∞) = $\frac{1}{2}mv_f^2$ (1)

At initial position:

$$E. = \frac{1}{2}mv_i^2$$

$$E = -GMm/(R_e + h) + V_0$$

where h= height of the ball from the surface of the earth.

$$\text{Total Energy (initial)} = \frac{1}{2}mv_i^2 - GMm/(R_e + h) \ (2)$$

According to law of conservation of energy

Total Energy (∞) = Total Energy (initial)

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - GMm/(R_e + h)$$

As L.H.S. = positive

$$\frac{1}{2}mv_i^2 = -GMm/(R_e + h) \geq 0$$

$$\frac{1}{2}mv_i^2 = GMm/(R_e + h)$$

By calculating

$$v_i^2 = 2GM/(R_e + h)$$

Assume Ball is thrown from earth surface h << R_e.

This implies $R_e + h$ is same as Re as we can neglect h.

Therefore, $v_i^2 = 2GM/(R_e)$

or $v_i = \sqrt{(2GM/R_e)}$

This is the initial velocity with which if the ball is thrown it will never fall back on the earth surface.

In terms of 'g'

$$g = GM/R_e^2$$

Escape velocity can be written as

$$V_e = \sqrt{2gR_e}$$

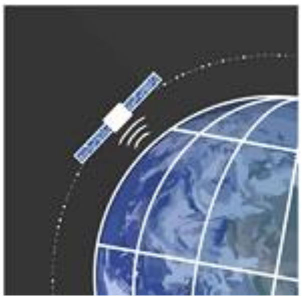
Earth Satellites

Any object revolving around the earth.

Natural Satellite

Satellite created by nature.

Example: - Moon is the only natural satellite of earth.



Earth Satellite

Natural Satellite

Artificial Satellites:

Human built objects orbiting the earth for practical uses. There are several purposes which these satellites serve.

Example:- Practical Uses of Artificial satellites

Communication

Television broadcasts

Weather observation

Military support

Navigation

Scientific research

Determining the Time Period of Earth Satellite

Time taken by the satellite to complete one rotation around the earth.

As satellites move in circular orbits there will be centripetal force acting on it.

$F_c = \frac{mv^2}{R_e+h}$ It is towards the centre.

Where,

H = distance of satellite forms the earth

F_c = centripetal force

$$F_G = \frac{GmM_e}{(R_e + h)^2}$$

Where,

F_g = Gravitation force

M = mass of the satellite

M_e = mass of the earth

$F_c = F_G$

$$mv^2 / R_e + h = GmM_e / (R_e + h)^2$$

$$v^2 = GM_e / R_e + h$$

$$v = \sqrt{GM_e / R_e + h} \ (1)$$

This is the velocity with which satellite revolve around the earth.

The satellite covers distance = 2 π (R_e + h) with velocity v.

$$T = 2\Pi (R_e + h) / v$$

$$2\Pi (R_e + h) / \sqrt{GM_e / R_e + h} \text{ From (1)}$$

$$T = 2\Pi (R_e + h)^{3/2} / \sqrt{GM_e}$$

Special Case:-

h<< R_e (satellite is very near to the surface of the earth)

Then $T = 2\Pi \sqrt{R_e^3} / GM_e$

After calculating

$$T = 2\Pi \sqrt{R_e / g}$$

Energy of an orbiting satellite

m= mass of the satellite, v=velocity of the satellite

$$E. = \frac{1}{2}mv^2$$

= ½ $m\,(GM_e/R_e + h)$ by using (1)

$$E. = \frac{1}{2}GMm_e/(R_e + h)$$

$$E = -GM_em/(R_e + h)$$

Total Energy = K.E. + P.E.

$$= \frac{1}{2}GMm_e/(R_e + h) + -GM_em/(R_e + h)$$

$$\text{E.} = GM_em/2\,(R_e + h)$$

$$\text{P.E.} = 2 \times \text{K.E.}$$

Total energy is negative. This means the satellite cannot escape from the earth's gravity.

Geostationary Satellite:

Geo means earth and stationary means at rest. This means something which is stationary.

Satellites orbiting around the Earth in equatorial plane with time period equal to 24 hours.

Appear to be stationary with respect to earth. They also rotate around earth with time period of 24 hours.

These satellites can receive telecommunication signals and broadcast them back to a wide area on earth.

Example: INSAT group of satellites.

Polar Satellites

These are low altitude satellites. This means they orbit around earth at lower heights.

They orbit around the earth in North-South direction. Whereas earth is moving from East to West.

A camera is fixed above this type of satellite so they can view small strips of earth.

As earth also moves, so at each instance different types of stripes of earth can be viewed.

Adjacent stripes of earth are viewed in subsequent orbits.

They are useful in remote sensing, meteorology and environmental studies of the earth.

In the above image we can see that the orbit of polar satellites is from north to south direction.

Weightlessness

Weightlessness is a condition of free fall, in which the effect of gravity is cancelled by the inertial (e.g., centrifugal) force resulting from orbital flight. There is no force of gravity acting on the objects.

It is the condition in which body does not feel its weight at all.

When an apple falls from a tree it won't feel its weight. This condition experienced by anybody while in free-fall is known as weightlessness.

Examples: -When we throw an object from the top of building, the object experiences free fall, that is the object is not under any force. This is weightlessness.

Weightlessness in the orbital motion of satellites

- In case of a satellite that is rotating around the earth.
- There is an acceleration which is acting towards the centre of the Earth.

- This acceleration is known as centripetal acceleration (ac).
- There is also earth's acceleration which is balancing this centripetal acceleration.
  $g = a_c$ they are equal in magnitude and they are balancing each other.
- Inside the satellites there is no acceleration which means everything is moving with uniform velocity.
- Inside an orbiting satellite weightlessness is experienced.

Top Formulae

Newton's law of gravitation	$F = \frac{Gm_1m_2}{r^2}$, $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$
Acceleration due to gravity	$g = \frac{GM}{R^2} = \frac{4}{3}\pi GR\rho$
Variation of g	(a) Altitude (height) effect $g' = g\left(1 + \frac{h}{R}\right)^{-2}$ If h << R, then $g' = g\left(1 - \frac{2h}{R}\right)$ (b) Effect of depth $g'' = \left(1 - \frac{d}{R}\right)$
Intensity of gravitational field	$\vec{E}_g = \frac{GM}{r^2}(-\vec{r})$ For the Earth, $E_g = g = 9.86\ m/s^2$
Gravitational potential	$V_g = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$ For points outside (r >R), $V_g = -\frac{GM}{r}$ For points inside it, r < R $V_g = -GM\left[\frac{3R^2 - r^2}{2R^3}\right]$
Change in potential energy in going to a height h above the surface	$\Delta U_g = mgh$ if $h << R_e$ In general, $\Delta U_g = \frac{mgh}{\left(1 + \frac{h}{R}\right)}$

Orbital velocity of a satellite	$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$ $v_0 = \sqrt{\frac{GM}{R+h}} \quad \because r = h + R$ If $h << R$ $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 8 \text{ km/sec.}$
Velocity of projection	Loss of KE = Gain in PE $\frac{1}{2}mv_p^2 = -\frac{gMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$ $v_p = \left[\frac{2GMh}{R(R+h)}\right]^{1/2} = \left[\frac{2gh}{1+\frac{h}{R}}\right]^{1/2} \quad \left(\because GM = gR^2\right)$
Period of revolution	$T = \frac{2\pi r}{v_0} = \frac{2\pi(R+h)^{3/2}}{R\sqrt{\ }}$ Or $\quad T^2 = \frac{4\pi^2 r^3}{GM}$ If $h << R$ $\quad T = \frac{2\pi R^{3/2}}{R\sqrt{\ }} = 1\frac{1}{2}\text{hr}$
Kinetic energy of a satellite	$KE = \frac{GMm}{2r} = \frac{1}{2}mv^2$
Potential energy of a satellite	$U = -\frac{GMm}{r}$
Binding energy of a satellite	$= \frac{1}{2}\frac{GMm}{\ }$
Escape velocity	$v_e = \sqrt{\frac{2GM}{\ }} = \sqrt{2gR} = R\sqrt{\frac{8\pi Gd}{3}}$ $v_e = v_0\sqrt{2}$
Effective weight in a satellite	w = 0 Satellite behaves like a freely falling body.
Kepler's laws for planetary motion	(a) Elliptical orbit with the Sun at one focus (b) Areal velocity constant dA/dt = constant (c) $T^2 \propto r^3$; $r = (r_1 + r_2)/2$

Class : 11th Physics
Chapter- 8 : Gravitation

Gravitation

Satellite

Heavenly object that revolves around a planet

Geostationary satellite
Height from surface of earth = 36000km Radius of orbit = 42,400km
Time periods = 24 hours
Orbital velocity = 3.1km/s

Polar satellite
Height from earth's surface = 880km
Time period = 84min
Orbital velocity = 7.92km/s

Escape speed $v_{esc} = \sqrt{2GM/R} = \sqrt{2gR}$

Earth 's v_{esc} = 11.2km/s

Orbitals speed $v_{orb} = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$

Total Energy of a satellite

E = K.E + P.E

$= \left(\frac{GMm}{2r}\right) + \left(\frac{-GMm}{r}\right) = -\frac{1}{2}G\frac{Mm}{r}$

Weightlessness
During free fall under gravity inside a spacecraft or satellite a body is weightless i.e., effective weight becomes zero.

Gravitational field and potential

· Gravitational field Intensity

$I = \frac{Gm}{r^2}$

· Gravitational potential

$V_g = \frac{work\ done}{mass} = \frac{-Gm}{r}$

· Gravitational Potential Energy

$U = \frac{-GMm}{r}$

Kepler's Law-of planetary motion
(i) Law of Orbit: Every planet revolves around the Sun in an elliptical orbit and Sun is at its one focus.
(ii) Law of Area: The radius vector drawn from the Sun to a planet sweeps out equal areas in equal intervals of time.

$\frac{dA}{dt} = \frac{L}{2m}$ = Constant

(iii) Law of Periods: $T^2 \propto a^3$
where, a = length of semi-major axis

Acceleration due to gravity (g)

Acceleration possessed by an object during free fall due to earth's gravitational pull,

$g = \frac{GM_e}{R_e^2}$

Variation in acceleration due to gravity (g)

(i) with height $g' = g\left(1 - \frac{2h}{R_E}\right)$,
i.e. g decreases with height

(ii) with depth $g' = g\left(1 - \frac{d}{R_E}\right)$,
i.e. g decreases with depth

(iii) with rotation of earth about its own axis
$g' = g - R\omega^2 \cos^2 \lambda$,
At poles, $\lambda = 90^0$, g maximum
At equator, $\lambda = 0^0$, g minimum

Newton's universal low of Gravitation

When two mass bodies are separted by a distance, they experience an attractive force which is directly proportional to the product of their masses and inversaly proportional to the square of their separation.

$F = \frac{Gm_1m_2}{r^2}$, $G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

Characteristics of gravitational force
(i) Weakest force in Nature.
(ii) Central as well as conservative.
(iii) Always attractive in nature.
(iv) Applicable for all bodies irrespective of their shape, size and position.

Important Questions

Multiple Choice Questions-

1. A body is projected vertically from the surface of the earth of radius R with velocity equal to half of the escape velocity. The maximum height reached by the body is
   (a) R
   (b) R/2
   (c) R/3
   (d) R/4
2. When the planet comes nearer the sun moves
   (a) fast
   (b) slow
   (c) constant at every point
   (d) none of the above
3. Keplers second law regarding constancy of arial velocity of a planet is a consequence of the law of conservation of
   (a) energy
   (b) angular momentum
   (c) linear momentum
   (d) none of these
4. The escape velocity for a body projected vertically upwards from the surface of the earth is 11km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
   (a) $11/\sqrt{2}$ km/s
   (b) $11\sqrt{2}$ km/s
   (c) 2 km/s
   (d) 11 km/s
5. The radii of the earth and the moon are in the ratio 10 : 1 while acceleration due to gravity on the earths surface and moons surface are in the ratio 6 : 1. The ratio of escape velocities from earths surface to that of moon surface is
   (a) 10 : 1
   (b) 6 : 1
   (c) 1.66 : 1
   (d) 7.74 : 1
6. The escape velocity of a body from the surface of the earth is v. It is given a velocity twice this velocity on the surface of the earth. What will be its velocity at infinity?
   (a) v
   (b) 2v
   (c) $\sqrt{2}$v
   (d) $\sqrt{3}$v
7. The period of geostationary artificial satellite is
   (a) 24 hours
   (b) 6 hours
   (c) 12 hours
   (d) 48 hours
8. If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earths surface would
   (a) decrease by 2%
   (b) remain unchanged
   (c) increase by 2%
   (d) will increase by 9.8%
9. The mean radius of the earth is R, its angular speed on its own axis is w and the acceleration due to gravity at earth's surface is g. The cube of the radius of the orbit of a geo-stationary satellite will be
   (a) r^2g / w
   (b) R^2w^2 / g
   (c) $RG\, w^2$
   (d) R^2g / w^2
10. If escape velocity from the earth's surface is 11.2 km/sec. then escape velocity from a planet of mass same as that of earth but radius one fourth as that of earth is
    (a) 11.2 km/sec
    (b) 22.4 km/sec
    (c) 5.65 km/sec
    (d) 44.8 km/sec

Very Short:

1. What velocity will you give to a donkey and what velocity to a monkey so that both escape the gravitational field of Earth?

2. How does Earth retain most of the atmosphere?
3. Earth is continuously pulling the moon towards its center. Why does not then, the moon falls on the Earth?
4. Which is greater out of the following:
   (a) The attraction of Earth for 5 kg of copper.
   (b) The attraction of 5 kg copper for Earth?
5. Where does a body weigh more – at the surface of Earth or in a mine?
6. How is it that we learn more about the shape of Earth by studying the motion of an artificial satellite than by studying the motion of the moon?
7. If the Earth is regarded as a hollow sphere, then what is the weight of an object below the surface of Earth?
8. What is the formula for escape velocity in terms of g and R?
9. What is the orbital period of revolution of an artificial satellite revolving in a geostationary orbit?
10. Can we determine the mass of a satellite by measuring its time period?

Short Questions:

1. Explain how the weight of the body varies en route from the Earth to the moon. Would its mass change?
2. Among the known type of forces in nature, the gravitational force ¡s the weakest. Why then does ¡t play a dominant role in the motion of bodies on the terrestrial, astronomical, and cosmological scale?
3. Show that the average life span of humans on a planet in terms of its natural years is 25 planet years if the average span of life on Earth is taken to be 70 years.
4. Hydrogen escapes faster from the Earth than oxygen. Why?
5. In a spaceship moving in a gravity-free region, the astronaut will not be able to distinguish between up and down. Explain why?
6. Why the space rockets are generally launched from west to east?
7. Explain why the weight of a body becomes zero at the centre of Earth.
8. We cannot move even our little fingers without disturbing the whole universe. Explain why.

Long Questions:

1. (a) Derive the expression for the orbital velocity of an artificial Earth's satellite. Also, derive its value for an orbit near Earth's surface.
   (b) Derive the expression for escape velocity of a body from the surface of Earth and show that it $\sqrt{2}$ times the orbital velocity close to the surface of the Earth. Derive its value for Earth.
2. (a) Explain Newton's law of gravitation.
   (b) Define gravitational field intensity. Derive its expression at a point at a distance x from the center of Earth. How is it related to acceleration due to gravity?
3. Discuss the variation of acceleration due to gravity with:
   (a) Altitude or height
   (b) Depth
   (c) Latitude *i.e.* due to rotation of Earth.

Assertion Reason Questions:

1. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: Gravitational potential of earth at every place on it is negative.

   Reason: Everybody on earth is bound by the attraction of earth.
2. ***Directions:*** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

Assertion: Planets appear to move slower when they are farther from the sun than when they are nearer.

Reason: All planets move in elliptical orbits with sun at one of the foci of the ellipse.

Answer Key

Multiple Choice Answers-

1. **Answer:** (c) R/3
2. **Answer:** (a) fast
3. **Answer:** (b) angular momentum
4. **Answer:** (d) 11 km/s
5. **Answer:** (d) 7.74 : 1
6. **Answer:** (d) $\sqrt{3}$v
7. **Answer:** (a) 24 hours
8. **Answer:** (c) increase by 2%
9. **Answer:** (d) R^2g / w^2
10. **Answer:** (b) 22.4 km/sec

Very Short Answers:

1. **Answer:** We will give them the same velocity as escape velocity is independent of the mass of the body.
2. **Answer:** Due to force of gravity.
3. **Answer:** The gravitational force between the Earth and the moon provides the necessary centripetal force to the moon to move around the Earth. This centripetal force avoids the moon to fall onto the Earth.
4. **Answer:** Same.
5. **Answer:** At the surface of Earth, a body weighs more.
6. **Answer:** This is because an artificial satellite is closer to the Earth than Moon.
7. **Answer:** Zero.
8. **Answer:** Ve = $\sqrt{2gR}$.
9. **Answer:** It is 24 hours.
10. **Answer:** Yes.

Short Questions Answers:

1. **Answer:** When a body is taken from Earth to the moon, then its weight slowly decreases to zero and then increases till it becomes $\frac{1}{6}$th of the weight of the body on the surface of the moon.

   We know that mgh = mg $(1 - \frac{2h}{R})$

   As h increases, gh, and hence mgh, decreases. When R = $\frac{R}{2}$ the force of attraction of Earth is equal to the force of attraction of the moon.

   Then gh = 0, so mg becomes zero, and the value of g on the moon's surface is $\frac{1}{6}$th of its value on the surface of Earth. Hence on increasing h beyond $\frac{R}{2}$, mg starts increasing due to the gravity of the moon. f ts mass remains constant.

2. **Answer:** Electrical forces are stronger than gravitational forces for a given distance, but they can be attractive as well as repulsive, unlike gravitational force which is always attractive. As a consequence, the forces between massive neutral bodies are predominantly gravitational and hence play a dominant role at long distances. The strong nuclear forces dominate only over a range of distances of the order of 10^{-14} m to 10^{-15} m.

3. **Answer:** Take the distance between Earth and Sun twice the distance between Earth and planet. According to Kepler's third law of planetary motion,

$$\left(\frac{T_e}{T_p}\right)^2 = \left(\frac{R_e}{R_p}\right)^3$$

   where T_e, T_R is the average life span on Earth and planet respectively.

   R_g = distance between Earth and Sun.

R_p = distance between Earth and planet.

Here, $R_e = 2R_p$

or $$\frac{R_e}{R_p} = 2$$

$$T_e = 70 \text{ years}$$

$$T_p = ?$$

$\therefore$ $$\left(\frac{T_e}{T_p}\right)^2 = (2)^3$$

or $$\frac{T_e}{T_p} = (2)^{\frac{3}{2}}$$

or $$T_p = \frac{T_e}{(2)^{\frac{3}{2}}} = \frac{70}{\sqrt{2^3}} = \frac{70}{\sqrt{8}} = \frac{70}{2\sqrt{2}}$$

$$= 35 \times \frac{\sqrt{2}}{2} = 17.5 \times 1.414$$

$$= 24.75 \approx 25 \text{ planet years.}$$

4. **Answer:** The thermal speed of hydrogen is much larger than oxygen. Therefore, a large number of hydrogen molecules are able to acquire escape velocity than that of oxygen molecules. Hence hydrogen escapes faster from the Earth than oxygen.

5. **Answer:** The upward and downward sense is due to the gravitational force of attraction between the body and the earth. In a spaceship, the gravitational force is counterbalanced by the centripetal force needed by the satellite to move around the Earth in a circular orbit. Hence in the absence of zero force, the astronaut will not be able to distinguish between up and down.

6. **Answer:** Since the Earth revolves from west to east around the Sun, so when the rocket is launched from west to east, the relative velocity of the rocket = launching velocity of rocket + linear velocity of Earth. Thus the velocity of the rocket increases which helps it to rise without much consumption of the fuel. Also, the linear velocity of Earth is maximum in the equatorial plane.

7. **Answer:** We know that the weight of a body at a place below Earth's surface is given by

$$W = mgd \quad \text{.... (i)}$$

Where gd = acceleration due to gravity at a place at a depth 'd' below Earth's surface and is given

$$g_d = g\left(1 - \frac{d}{R}\right) \quad \text{....(ii)}$$

At the centre of Earth, d = R,

$\therefore$ $$g_d = g\left(1 - \frac{R}{R}\right)$$

$$= g(1-1) = g \times 0 = 0$$

From Eqn. (i) W = 0 at the center of Earth.

i.e., g decreased with depth and hence becomes zero at the center of Earth, so W = 0 at Earth's center.

8. **Answer:** According to Newton's law of gravitation, every particle of this universe attracts every other particle with a force that is inversely proportional to the square of the distance between them. When we move our fingers, the distance between the particle's changes, and hence the force of attraction changes which in turn disturbs the whole universe.

Long Questions Answers:

1. **Answer:**

   1. Let m = mass of the satellite.

   M, R = mass and radius of Earth.

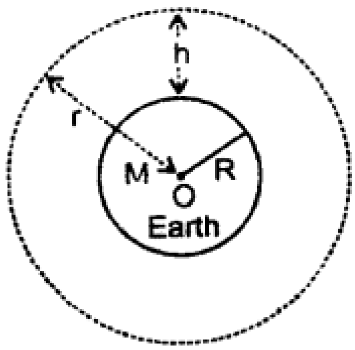
   h = height of the satellite above the surface of Earth.

   r = radius of the robot of the satellite

   = R + h.

   v_0 = orbital velocity of the satellite.

   The centripetal force $\frac{mv_0^2}{r}$ required by the satellite to move in a circular orbit is proved by the gravitational force between satellite and the Earth.



i.e. $$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

or $$v_0 = \sqrt{\frac{GM}{R+h}} \quad ....(i)$$

Also we know that

$$g = \frac{GM}{R^2}$$

or $$GM = gR^2$$

$\therefore$ $$v_0 = R\sqrt{\frac{g}{R+h}}$$

$$= R\sqrt{\frac{g}{(R+h)}} \quad ....(ii)$$

Also $$g_b = \frac{GM}{(R+h)^2} \quad ....(iii)$$

from (*i*) and (*iii*), we get

$$v_0 = \sqrt{g_h(R+h)}$$

If the satellite is close to the earth's surface, then h ≈ 0

$\therefore$ from (*ii*), $v_0 = \sqrt{gR}$

Putting,

$$g = 9.8\, ms^{-2}, R = 6.38 \times 10^6\ m$$

$\therefore$ $$v_0 = \sqrt{9.8 \times 6.38 \times 10^6} = 7.9\,\text{kms}^{-1}$$

$\therefore$ $$v_0 = \sqrt{gR} = R\sqrt{\frac{g}{R+h}}$$

$$= \sqrt{\frac{GM}{R+h}} = \sqrt{g_h(R+h)}$$

2. Escape velocity is the minimum velocity with which a body is projected from Earth's surface so as to just escape its gravitational pull or of any other planet. It is denoted by v_e.

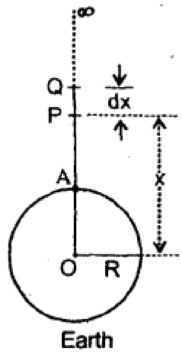
Expression: Consider the earth to be a homogenous sphere of radius R, mass M, center O, and density p.

Let m = mass of the body projected from point A on the surface of Earth with vel. v_e.

$\therefore$ K.E. of the body at point

$$A = \frac{1}{2} mv_e^2 \quad ....(i)$$

Let it reaches a point P at a distance x from O. If F be the gravitational force of attraction on the body at P, then



$$F = \frac{GMm}{x^2} \quad ....(ii)$$

Let it further moves to Q by a distance dx.

If dW be the work done in moving from P to Q, then

$$dW = Fdx = \frac{GMm}{x^2}dx \quad ....(iii)$$

If w be the total work done in moving the body from A to ∞,

Then

$$W = \int_A^\infty x^{-2}dx = GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right] = -GMm\left(0 - \frac{1}{R}\right)$$

$$= \frac{GMm}{R} \quad ....(iv)$$

$\therefore$ According to the law of conservation of energy

K.E. = RE

or $$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

or $$v_e = \sqrt{\frac{2GM}{R}} \quad ....(v)$$

Also $$g = \frac{GM}{R^2}$$

$\therefore$ $$v_e = \sqrt{\frac{2gR^2}{R}}$$

or $$v_e = \sqrt{2gR} \quad ....(vi)$$

Also $$M = \frac{4}{3}\pi R^3 \rho$$

$\therefore$ $$v_e = \sqrt{\frac{2G}{R} \cdot \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8}{3}\pi G \rho}$$

for earth

$$R = 6.38 \times 10^6\ \text{m}$$

$g = 9.8\ ms^{-2}$

$\therefore$ from (*vi*),

$$v_e = \sqrt{2 \times 9.8 \times 6.38 \times 10^6}$$

$$= 11.2\ kms^{-1}$$

Relation between v_e and v_o: Also we know that the orbital velocity around Earth close to its surface is given

by $v_0 = \sqrt{gR}$

and $v_e = \sqrt{2gR} = \sqrt{2}\sqrt{gR}$

$= \sqrt{2} v_0$

Hence proved.

2. **Answer:**

(a) We know that Newton's law of gravitation is expressed mathematically as:

$$F = \frac{Gm_1 m_2}{r^2}$$

or in vector form $F = \frac{Gm_1 m_2}{r^3}\hat{r}$

where $\hat{r}$ = unit vector along F

It was found that law is equally applicable anywhere in the universe between small and big objects like stars and galaxies. The value of G remains the same everywhere. (Some scientists have claimed that as the size of the object under consideration becomes big like a galaxy, the value of G also changes). Hence this law of Newton is also called Newton's universal law of gravitation.

The force of attraction is called the force of gravitation or gravitational force. This force is only attractive and is never repulsive. The force is both ways i.e., particle 1 attracts particle 2 and so does particle 2 attracts particle 1.

Hence $F_{12} = -F_{21}$.

The law is a direct outcome of the study of acceleration of bodies. Newton wondered how Moon revolves around the Earth or other planets revolve around the Sun. His calculations showed that the Moon is accelerated by the same amount as does any other object towards the Earth.

His famous narration of the apple falling from the tree and noticing every other object fall towards Earth led to the announcement of his famous law of gravitation about 50 years later in his book 'Principia'.

Out of the known forces in nature, the Gravitational force is the weakest, yet it is the most apparent one as it acts for long distances and between objects which are visible to us. The law of gravitation has been used to determine the mass of heavenly bodies. It has been used to study the atmosphere of planets. Man-made satellites remain in the orbits due to gravitation.

(b) The gravitational field intensity at a point is defined as the force acting on a unit mass placed at that point in the field.

Thus, the gravitational field intensity is given by:

$$E = \frac{F}{m}$$

Now at distance x from the centre of Earth, the gravitational force is

$$F = \frac{GMm}{x^2}\hat{x}$$

$$E = \frac{F}{m} = \frac{GMm}{x^2}\hat{x} = \frac{GM}{x^2}\hat{x}$$

$$|E| = \frac{GM}{x^2}$$

or $$E = \frac{GM}{x^2}$$

On Earth

$$E = \frac{F}{m} = \frac{\text{Force}}{\text{mass}} = \text{acceleration}$$

$$E = \frac{F}{m} = g$$

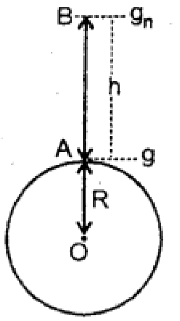
So, the intensity of the gravitational field at the surface of Earth is equal to the acceleration due to gravity.

3. **Answer:**

Let M, R be the mass and radius of the earth with centre O.

g = acceleration due to gravity at a point

An on Earth's surface.



(a) Variation of g with height: Let g0h be the acceleration due to gravity at a point B at a height h above the earth's surface

$$\therefore \quad g = \frac{GM}{R^2} \quad ....(1)$$

and
$$g = \frac{GM}{(R+h)^2} \quad ....(2)$$

$\frac{(2)}{(1)}$ gives,
$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

or
$$g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

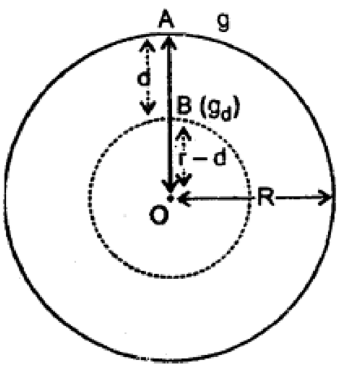
If h << R, then using Binomial Expansion, we get

$$g_h = g\left(1-\frac{2h}{R}\right)$$

$$= g - \frac{2gh}{R} \quad ....(3)$$

Thus, from Eqn. (3), we conclude that acceleration due to gravity decreases with height.

(b) With depth: Let the Earth be a uniform sphere.



Let gd = acceleration due to gravity at a depth d below earth's surface i.e., at point B.

Let ρ = density of Earth of mass M.

$$\therefore \quad g = \frac{GM}{R^2} \quad ....(i)$$

where
$$M = \frac{4}{3}\pi R^3 \rho$$

Also, let M' = mass of Earth at a depth d, then

$$M' = \frac{4}{3}\pi (R-d)^3 \rho$$

$$\therefore \quad g = \frac{G}{R^2}\frac{4}{3}\pi R^3 \rho$$

$$= \frac{4}{3}\pi G\rho R \quad ....(ii)$$

Similarly
$$g_d = \frac{GM'}{(R-d)^2}$$

$$= \frac{4}{3}\pi G\rho(R-d) \quad ....(iii)$$

$$\therefore \quad \frac{g_d}{g} = \frac{R-d}{R} = 1 - \frac{d}{R}$$

or
$$g_d = g\left(1-\frac{d}{R}\right) \quad ....(iv)$$

From equation (iv), we see that acceleration due to gravity decreases with depth.

Special case: At the centre of Earth, d = R

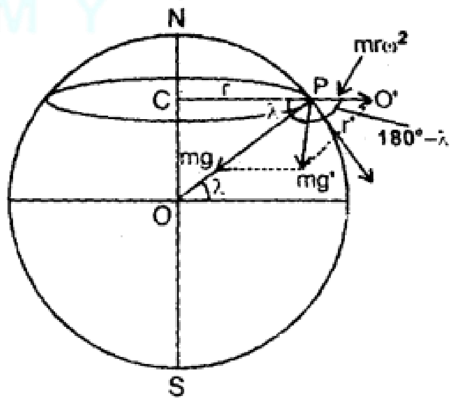
$\therefore g_d = 0$

Hence an object at the centre of Earth is in a state of weightlessness.

(c) Variation of g with latitude:

Let m = mass of a particle at a place P of latitude X.

ω = angular speed of Earth about axis NS.



As the earth rotates about the NS axis, the particle at P also rotates and describes a horizontal circle of radius r,

where r = PC = OP cos λ, = R cos λ

Let g' be the acceleration due to gravity at P when the rotation of Earth is taken into account. Now due to the rotation of the earth, two forces that act on the particle at P are:

Its weight mg, acting along with PO.

Centrifugal force mroo2 along PO'.

∴ The angle between them = 180 – λ

∴ According to the parallelogram law of vector addition

$$mg' = \sqrt{(mg)^2 + (mr\omega^2)^2 + 2(mg)(mr\omega^2)\cos(180° - \lambda)}$$

$$= \sqrt{m^2g^2 + m^2r^2\omega^4 - 2m^2gr\omega^2\cos\lambda}$$

or $$g' = g\sqrt{1 + \frac{r^2\omega^4}{g^2} - \frac{2r\omega^2}{g}\cos^2\lambda}$$

$$= g\sqrt{1 + \frac{\omega^4R^2\cos^2\lambda}{g^2} - \frac{2R\omega^2}{g}\cos^2\lambda}$$

$$= g\sqrt{1 - \frac{2R\omega^2}{g}\cos^2\lambda}$$

[As $\frac{R\omega^2}{g}$ is very small $\left(= \frac{1}{289}\right)$ so its square and higher powers are neglected.]

$$\therefore \quad g' = \left(1 - 2\frac{R\omega^2}{g}\cos^2\lambda\right)^{\frac{1}{2}}$$

Using binomial expansion, we get

$$g' = \left(g - \frac{1}{2} \times \frac{2R\omega^2}{g} \times g\cos^2\lambda\right)$$

$$g' = g - R\omega^2\cos^2\lambda$$

$\Rightarrow g$ decreases with the rotation of the earth.

At poles, $\lambda = 90°$, $\therefore g' = g_p = g$

At equator, $\lambda = 0$, $g' = g - R\omega^2\cos^2\lambda$

⇒ g decreases with the rotation of the earth.

At poles, $\lambda = 90°$, ∴ $g' = g_p = g$

At equator, $\lambda = 0$, $g' = g_e = g - R\omega^2$.

Clearly $g_p > g_e$.

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   Explanation:

   Because gravitational force is always attractive in nature, and everybody is bound by this gravitational force of attraction of earth.

2. (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

   Explanation:

   Both assertion and reason are true, but reason is not correct explanation of the assertion.

Case Study Questions-

1. If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: can we throw an object with such high initial speeds that it does not fall back to the earth ? Thus minimum speed required to throw object to infinity away from earth's gravitational field is called escape velocity.

   $$v_e = \sqrt{2gr}$$

   Where g is acceleration due to gravity and r is radius of earth and after solving v_e 11.2 km/s. This is called the escape speed, sometimes loosely called the escape velocity. This applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.

i. **Time period of moon is**
   a. 27.3 days
   b. 20 days
   c. 85 days
   d. None of these

ii. **Escape velocity from earth is given by**
   a. 20 km/s
   b. 11.2 km/s
   c. 2 km/s
   d. None of these

iii. **Define escape velocity. Give its formula**

iv. **Why moon don't Have any atmosphere?**

v. **What is satellite? Which law governs them?**

2. Satellites in a circular orbits around the earth in the equatorial plane with T = 24 hours are called Geostationary Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fails to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationery satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India is one such group of geostationary satellites widely used for telecommunications in India. Another class of satellites is called the Polar satellites. These are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions. at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

i. **Time period of geospatial satellite is**
   a. 24 hours
   b. 48 hours
   c. 72 hours
   d. None of these

ii. **Polar satellites are approximately revolving at height of**
   a. 500 to 800km
   b. 1500 to 2000 km
   c. 3000 to 4000 km
   d. None of these

iii. **Which satellite used to view polar and equatorial regions?**

iv. **Write note on polar satellites**

v. **Write a note on geostationary satellite. Give its applications.**

Case Study Answer-

1. Answer

i. (a) 27.3 days

ii. (b) 500 to 800km

iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.

iv. The escape speed for the moon turns out to be 2.3 km/s, about five times smaller than that of earth. Therefore all atmospheric gas can go easily out of atmosphere of moon. This is the reason that moon has no atmosphere.

v. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.

2. Answer

i. (a) 24 hours

ii. (a) Pascal's law

iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.

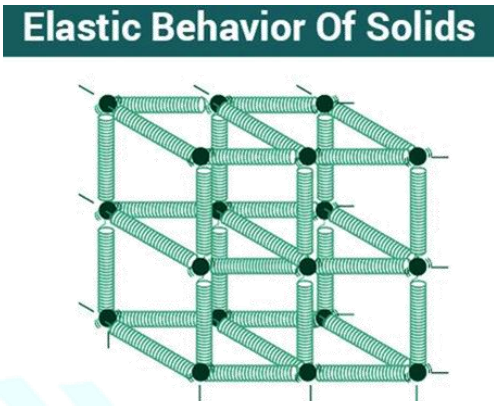
iv. Polar satellites are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction. Since its time period is around 100 minutes it crosses any altitude many times a day. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

Satellites in circular orbits around the earth in the equatorial plane with time period same as earth are called Geostationary Satellites.

Applications: Radio waves broadcast. Satellites widely used for telecommunications in India. GPS system, navigation system, defence etc.

Mechanical Properties of Solids 9

Elastic Behaviour of Solids



Elastic Behavior of Solids – What happens to a rubber band when you stretch it and let go? It deforms but regains its original nature when you stop applying a force. But say, you take an aluminium rod and try to bend it using your arm strength. You somehow do manage to bend it a little and then stop applying force. Does the rod regain its original shape? of course not. It is referred to as the Elastic Behavior of Solids

What happens to a rubber band when you stretch it and let go? It deforms but regains its original nature when you stop applying force. But suppose you try to bend an aluminium rod using your arm strength. You somehow do manage to bend it a little and then stop applying force. Does the rod regain its original shape? Of course not.

This difference in the behaviour of the materials is based on their elastic and plastic nature. The rubber band has high elasticity. Elasticity is the ability of a body to resist any permanent changes to it when stress is applied. The body regains its original shape and size when stress application ceases.

All materials have an elastic limit beyond which, if continuous stress is applied, they will start losing their ability to exhibit perfect elastic behaviour and start deforming. In contrast, plastic deformation is the non-reversible deformation of solid materials on the application of forces.

Important Points on Elastic Behaviour of Solids

An elastic body is one that regains its original shape and size when deforming forces are removed.

A plastic body is one that succumbs to deforming forces (however small) and cannot return to its original shape and size.

Elasticity is the property of a body to regain its original shape and size when deforming forces are removed. It exhibits an opposition to change.

Elasticity

This difference in the behaviour of the material is based on their elastic and plastic nature. The rubber band has high elasticity. Elasticity is the ability of a body to resist any permanent changes to it when stress is applied. the body regains its original shape and size when stress application ceases.

Difference Between Elasticity and Plasticity

All materials have an elastic limit beyond which, if continuous stress is applied, they will start losing their ability to exhibit perfect elastic behaviour and start deforming. In contrast, plasticity is the non-reversible deformation of solid materials on the application of forces.

Looking at the elasticity in the atomic level, solids are made of atoms (or molecules). They are surrounded by other such atoms which are held in a state of equilibrium by interatomic forces. When an external force is applied these particles are displaced, resulting in the deformation of the solid. When the application of the deforming force is stopped, interatomic forces drive the atoms to regain their state of equilibrium.

The concept of elasticity is an idealization as no material is perfectly elastic. For example, if you use a hair tie to groom yourself, you may have noticed that its size tends to deform after prolonged use. After a point, it may snap as well. This is because the hair tie eventually loses its elastic nature.

Stress

Stress is defined as the ratio of the internal force F, produced when the substance is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In other words,

Stress is of two types:

(i) Normal stress: It is defined as the restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types: tensile stress and compressive stress.

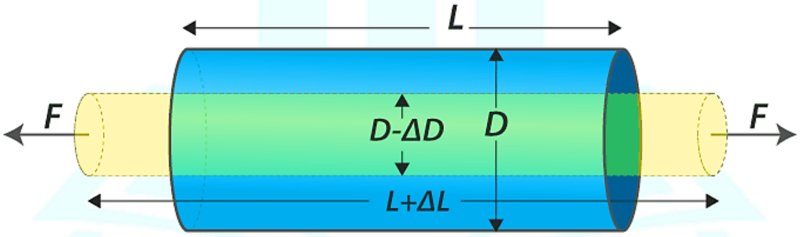
(ii) Tangential stress: When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.

Strain

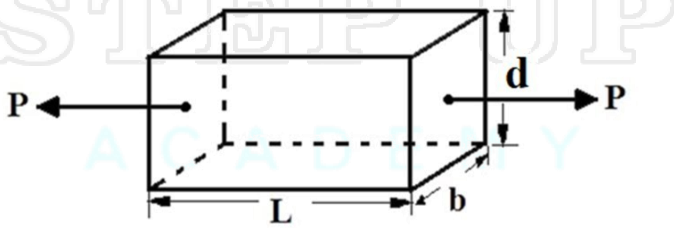
It is defined as the ratio of the change in size or shape to the original size or shape. It has no dimensions; it is just a number.

Strain is of three types:

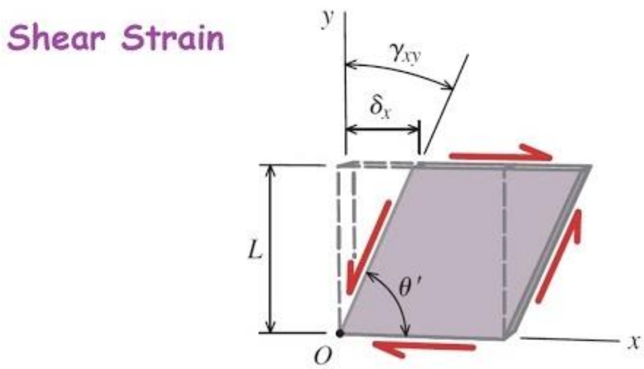
(i) Longitudinal strain: If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as:



(ii) Volumetric strain: If the deforming force produces a change in volume alone, the strain produced in the body is called volumetric strain. It is given as:



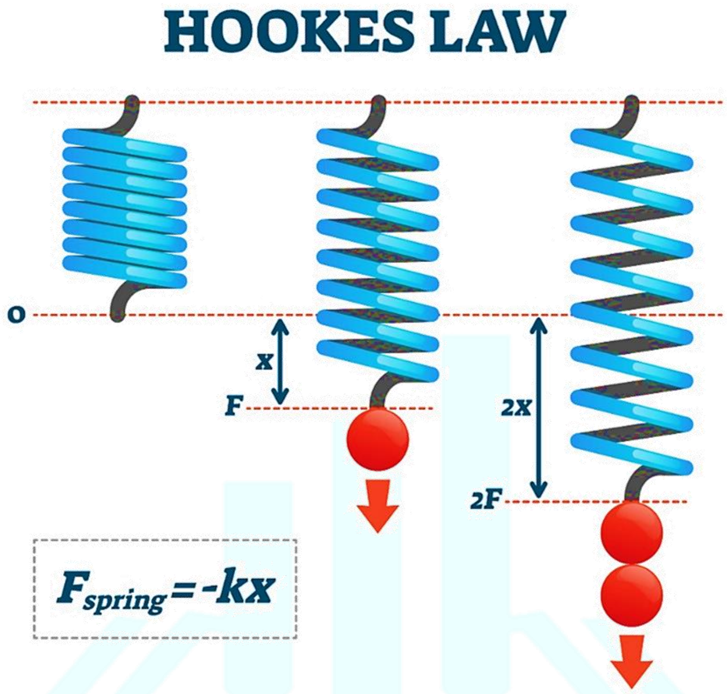
(iii) Shear strain: The angle tilt caused in the body due to tangential stress expressed is called shear strain. It is given as:



The maximum stress to which the body can regain its original status on the removal of the deforming force is called elastic limit.

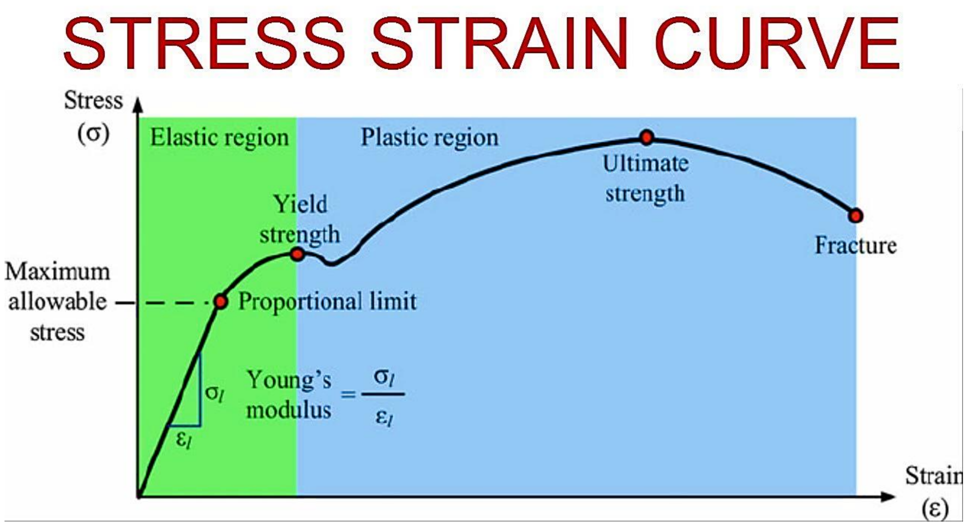
Hooke's Law

Hooke's law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity. Thus



Stress Strain Curve

Stress strain curves are useful to understand the tensile strength of a given material. The given figure shows a stress-strain curve of a given metal.

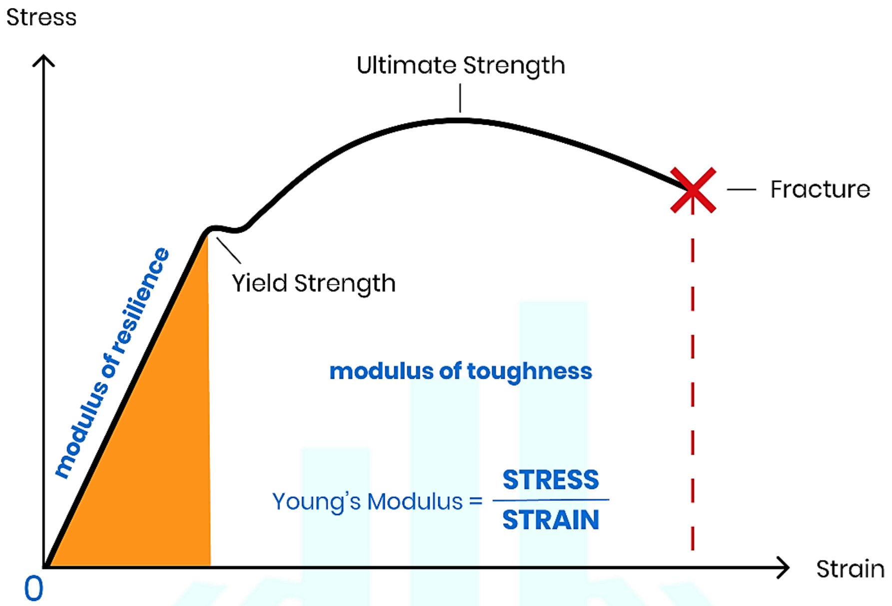


- The curve from O to A is linear. In this region Hooke's Proportional limit law is obeyed.
- In the region from A to 6 stress and strain are not . proportional. Still, the body regains its original dimension, once the load is removed.
- Point B in the curve is yield point or elastic limit and the corresponding stress is known as yield strength of the material.

- The curve beyond B shows the region of plastic deformation.
- The point D on the curve shows the tensile strength of the material. Beyond this point, additional strain leads to fracture, in the given material.

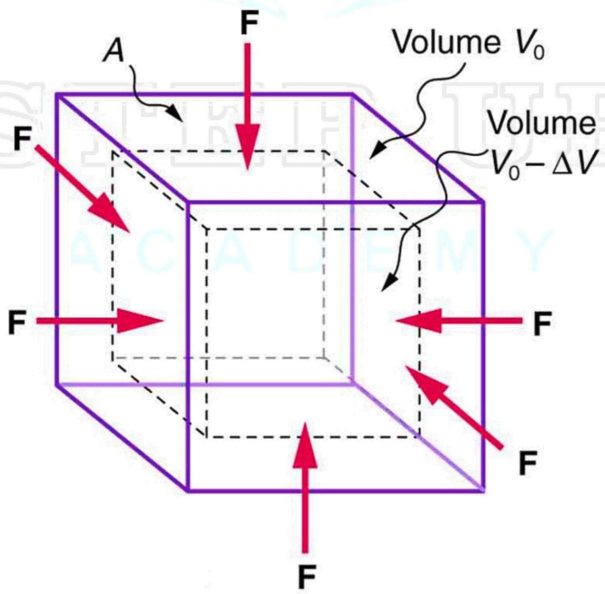
Young's Modulus

For a solid, in the form of a wire or a thin rod, Young's modulus of elasticity within elastic limit is defined as the ratio of longitudinal stress to longitudinal strain. It is given as:



Bulk Modulus

Within elastic limit the bulk modulus is defined as the ratio of longitudinal stress and volumetric strain. It is given as:



-ve indicates that the volume variation and pressure variation always negate each other.

Reciprocal of bulk modulus is commonly referred to as the "compressibility". It is defined as the fractional change in volume per unit change in pressure.

Shear Modulus or Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain.

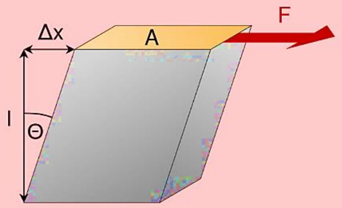
Modulus of rigidity is given by

Shear Modulus

Modulus of Rigidity

The shear modulus is the shear stiffness of a material.

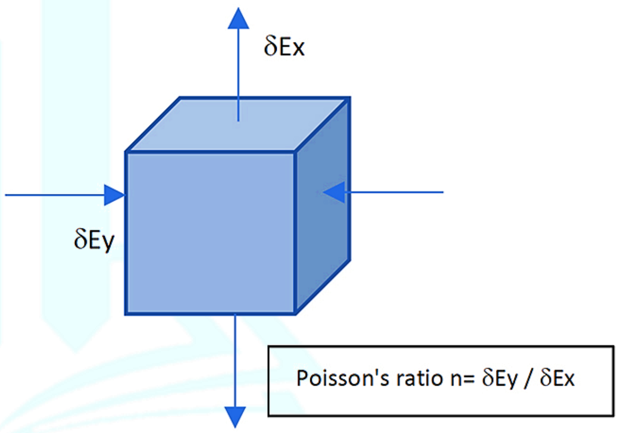
$$G = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$$



It is the ratio of shear stress to shear strain.

Poisson's Ratio

The ratio of change in diameter (ΔD) to the original diameter (D) is called lateral strain. The ratio of change in length (Δl) to the original length (l) is called longitudinal strain. The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

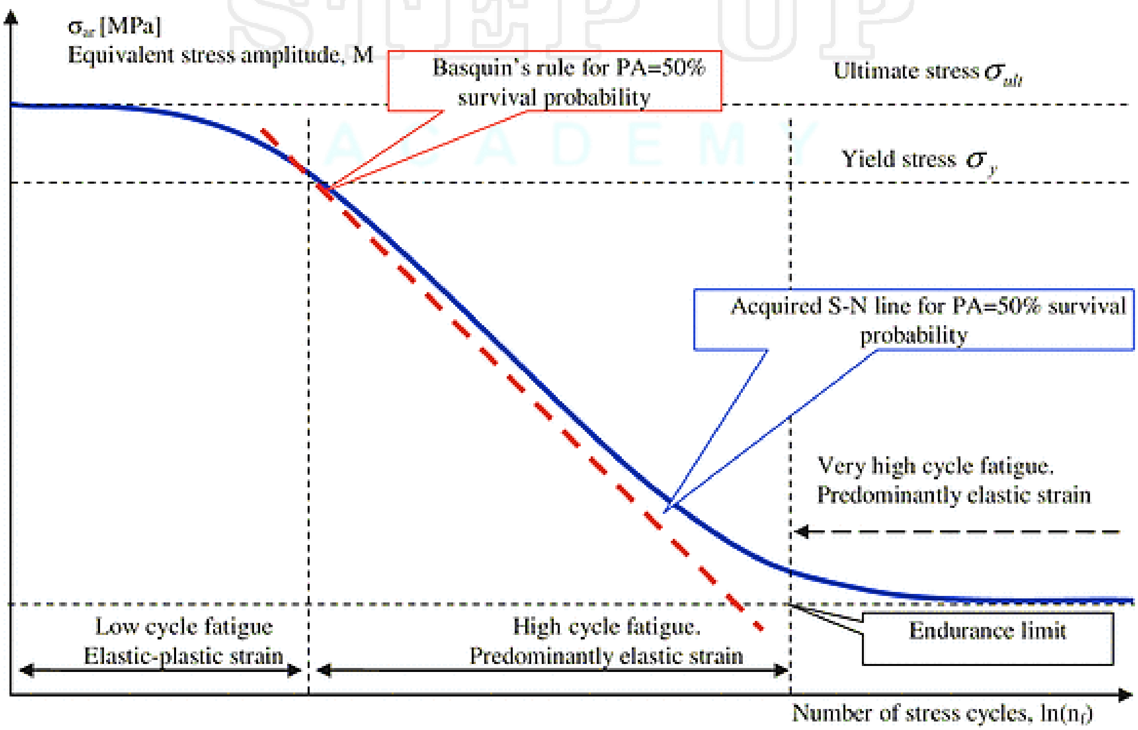


Elastic Fatigue

It is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming forces.

Relations between Elastic Moduli

For isotropic materials (i.e., materials having the same properties in all directions), only two of the three elastic constants are independent. For example, Young's modulus can be expressed in terms of the bulk and shear moduli.



Breaking Stress

The ultimate tensile strength of a material is the stress required to break a wire or a rod by pulling on it. The breaking stress of the material is the maximum stress which a material can withstand. Beyond this point breakage occurs.

$$\textit{Formula used}: \frac{Force}{Area}$$

Complete step-by-step solution -

The formula for breaking stress is given as

$$\textit{Breaking Stress} = \frac{Force}{Area}$$

Breaking stress checks for metals determine how long a single alloy is stretched until it reaches its maximum tensile strength and how much metal can be loaded until structural stability is lost. Therefore, it is a very important concept in material science and for safety considerations.

Breaking stress is also known as the ultimate tensile stress or breaking strength.

Tensile stress is the stress state induced by the load being applied that appears to elongate the material in the load-axis, that is, the force generated by the material being tensioned. The power of equivalent cross-sectional structures charged with voltage is independent of the nature of the cross-section.

Note- Breaking stress is a limit state of tensile stress that leads to tensile failure in one of two manners: Ductile failure - Some yield as the first stage of failure, some hardening in the second stage and breakage after a possible "neck" formation. Brittle failure - It is defined as the abrupt breaking of the material into two or more pieces at a low stress state.

Bulk Modulus

Bulk modulus is the ratio of hydraulic stress to the corresponding hydraulic strain.

Denoted by 'B'

$$B = -\frac{p}{\frac{\Delta V}{V}}$$

Where p = hydraulic stress, $\frac{\Delta V}{V}$ = hydraulic strain

(-) ive signs show that the increase in pressure results in decrease in volume.

S.I. Unit: N/m^2 or Pascal (Pa)

B(solids) > B(liquids) > B(gases)

Compressibility

Compressibility is the measure of compression of a substance.

Reciprocal of bulk modulus is termed as 'Compressibility'.

Mathematically:

$$k = \frac{1}{B} = -\left(\frac{1}{p}\right)\left(\frac{\Delta V}{V}\right)$$

It is denoted by 'k'.

k(solids) < k(liquids) < k(gases)

Hydraulic Stress

Hydraulic stress is the restoring force per unit area when force is applied by a fluid on the body.

For example:

Consider a rubber ball and if it is dipped in the pond. Due to the pressure of water from all directions force acts on the ball as a result, the ball seems to be slightly contracted.

Ball under the water

Because of the force exerted by the water there is restoring force which develops in the ball which is equal in magnitude to the force applied by the water but in opposite direction. This type of stress is known as hydraulic stress.

Top Formulae

Normal stress	S = F/a, where a = πr^2
Longitudinal strain	$\frac{\Delta \ell}{\ell}$
Young's modulus	$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_0} = \frac{F}{A}\frac{L_0}{\Delta L}$
Breaking force	= breaking stress × area of cross-section
Volumetric strain	$\frac{\Delta V}{V}$
Bulk modulus	$B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V/V_0} = -\frac{\Delta p V_0}{\Delta V}$
Shearing strain	$\frac{\Delta L}{L} = \theta$
Shear modulus	$\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F/A}{x/h} = \frac{F}{A}\frac{h}{x}$
Modulus of rigidity	$G = \frac{F}{a\theta}$
Elastic potential energy of a stretched wire	=(1/2) × stress × strain × volume

Class : 11th Physics
Chapter- 9 : Mechanical Properties of Solids

Mechanical Properties of Solids

Poision's ratio

$$\text{Poision's ratio}(\sigma) = \frac{\text{Laternal strain }(\beta)}{\text{Longitudinal strain }(\alpha)} = \frac{\Delta d / d}{\Delta l / l}$$

Value of σ lies between 0 and 0.5

Elastic potential energy

Elastic potential energy in a stretched wire $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume of the wire}$

Elasticity

Property of material by virtue of which it regains it original shape & size after the removal of deforming force

Relation between Y, B, η & σ

(i) $Y = 3B(1 - 2\sigma)$

(ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \frac{3B - 2\eta}{2\eta + 6B}$

(iv) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ or $Y = \frac{9B\eta}{\eta + 3B}$

Types

(i) Longitudinal stress

$$= \frac{\text{Deforming force applied normal}}{\text{Area}} = \frac{F}{A}$$

(ii) Volumetric stress

$$= \frac{\text{Force Changing Volume}}{\text{Area}} = \frac{F_V}{A}$$

(iii) Shearing area or tangential stress

$$= \frac{\text{Tangential Force}}{\text{Area}} = \frac{F_t}{A}$$

Hooke's low

Within the elastic limit, stress is directly proportional to strain. i.e., stress ∝ strain

Stress

Restoring force per unit area

i.e., stress $= \frac{F}{A}$

Types of Modulus of Elasticity

Young's Modulus

Young's modulus of elasticity

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F / A}{l / \Delta l}$$

$$Y = \frac{F\Delta l}{Al} = \frac{Mg\Delta l}{\pi r^2 l}$$

Modulus of Rigidity

Rigidity or shear modulus of elasticity

$$\eta = \frac{\text{tangential stress}}{\text{shearing strain}}$$

$$\eta = \frac{\sigma_t}{\varepsilon_s} = \frac{\frac{F}{A}}{\theta} = \frac{F}{A\theta}$$

Strain

Ratio of change in configuration to original configuration

$$\text{Strain} = \frac{\text{change in configuration}}{\text{original configuration}}$$

Bulk Modulus

Bulk modulus or volume modulus of elasticity

$$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$$

$$B = -\frac{\Delta P}{\frac{\Delta V}{V}}; \frac{1}{B} = \text{compressibility}$$

Types

(i) Longitudinal strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l_o}$

(ii) Volumetric strain $\varepsilon_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V_o}$

(iii) Shearing strain = angular displacement of the plane perpendicular to the fixed surface = θ

Important Questions

Multiple Choice Questions-

1. The ratio of the change in dimension at right angles to the applied force to the initial dimension is known as
   (a) Youngs modulus
   (b) Poissions ratio
   (c) Lateral strain
   (d) Shearing strain
2. Hookes law essentially defines
   (a) Stress
   (b) Strain
   (c) Yield point
   (d) Elastic limit
3. Theoretical value of Poissions ratio lies between
   (a) -1 to 0.5
   (b) -1 to -2
   (c) 0.5 to 1
   (d) None
4. A wire suspended vertically from one of its ends is stretched by attaching a weight of 100N to its lower end. What is the elastic potential energy stored in the wire, if the weight stretches the wire by 1.5 mm?
   (a) 5×10^{-2} J
   (b) 10^{-3} J
   (c) 2.5×10^{-3} J
   (d) 7.5×10^{-2} J
5. An iron bar of length l m and cross section A m^2 is pulled by a force of F Newton from both ends so as to produce and elongation in meters. Which of the following statement statements is correct?
   (a) Elongation is inversely proportional to length l
   (b) Elongation is directly proportional to cross section A
   (c) Elongation is inversely proportional to A
   (d) Elongation is directly proportional to Youngs modulus
6. temperature?
   (a) Copper
   (b) Invar steel
   (c) Brass
   (d) Silver
7. Longitudinal strain is possible in the case of
   (a) Gases
   (b) Liquid
   (c) Only solids
   (d) Only gases & liquids
8. Two wires A and B are of the same length. The diameters are in the ratio 1 : 2 and the Youngs modulus are in ratio 2 : 1. if they are pulled by the same force, then their elongations will be in ratio
   (a) 4 : 1
   (b) 1 : 4
   (c) 1 : 2
   (d) 2 : 1
9. A body of mass 500 g is fastened to one end of a steel wire of length 2 m and area of cross-section 2 mm^2. if the breaking stress of he wire is 1.25 × 107 N/m^2, then the maximum angular velocity with which the body can be rotated in a horizontal circle is
   (a) 2 rad/s
   (b) 3 rad/s
   (c) 4 rad/s
   (d) 5 rad/s
10. If a material is heated and annealed, then its elasticity is
   (a) Increased
   (b) Decreased
   (c) Not change
   (d) Becomes zero

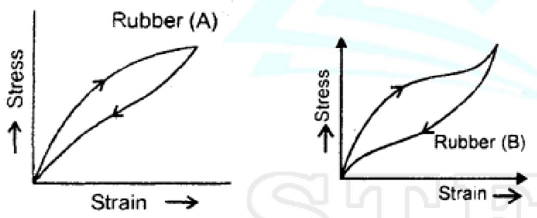
Very Short:

1. Give an example of pure shear.
2. What is an elastomer?
3. What is breaking stress?
4. What is the:
   (a) value of modulus of rigidity of a liquid?
   (b) order of strain within the elastic limit?
5. A wire is stretched to double its length. What is the value of longitudinal strain?
6. Mention a situation where the restoring force is not equal and opposite to the applied force.

7. What is a Cantilever?
8. A wire is suspended from a roof but no weight is attached to the wire. Is the wire under stress?
9. Why strain has no units?
10. What is Poisson's ratio?

Short Questions:

1. What are the factors due to which three states of matter differ from one's Other?
2. When we stretch a wire, we have to perform work Why? What happens to the energy given to the wire in this process?
3. Why are the bridges declared unsafe after long use?
4. Why are the springs made of steel and not of copper?
5. A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers (A) and (B) of Figure would you prefer to use for this purpose? Why?



6. Metal wires after being heavily loaded dop'\ regain their lengths completely explain why?
7. Explain. Why spring balances show wrong readings after they have been, Used for a long time?
8. Elasticity is said to be the internal property of matter. Explain.

Long Questions:

1. (a) Derive the expression for the orbital velocity of an artificial Earth's satellite. Also, derive its value for an orbit near Earth's surface.

   (b) Derive the expression for escape velocity of a body from the surface of Earth and show that it $\sqrt{2}$ times the orbital velocity close to the surface of the Earth. Derive its value for Earth.
2. (a) Explain Newton's law of gravitation.

   (b) Define gravitational field intensity. Derive its expression at a point at a distance x from the center of Earth. How is it related to acceleration due to gravity?
3. Discuss the variation of acceleration due to gravity with:

   (a) Altitude or height

   (b) Depth

   (c) Latitude i.e. due to rotation of Earth.

Assertion Reason Questions:

1. ***Directions:***

   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

   (c) If assertion is true but reason is false.

   (d) If the assertion and reason both are false.

   Assertion: Steel is more elastic than rubber.

   Reason: Under given deforming force, steel is deformed less than rubber.
2. ***Directions:***

   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

   (c) If assertion is true but reason is false.

   (d) If the assertion and reason both are false.

   Assertion: Glassy solids have sharp melting point.

   Reason: The bonds between the atoms of glassy solids get broken at the same temperature.

Answer Key

Multiple Choice Answers-

1. **Answer:** (c) Lateral strain
2. **Answer:** (d) Elastic limit
3. **Answer:** (a) -1 to 0.5
4. **Answer:** (d) 7.5×10^{-2} J
5. **Answer:** (c) Elongation is inversely proportional to A
6. **Answer:** (b) Invar steel
7. **Answer:** (c) Only solids
8. **Answer:** (d) 2 : 1
9. **Answer:** (d) 5 rad/s
10. **Answer:** (b) Decreased

Very Short Answers:

1. **Answer:** The twisting of a cylinder produces pure shear.
2. **Answer:** It is a substance that can be elastically stretched to large values t of strain.
3. **Answer:** It is defined as the ratio of maximum load to which the wire is < subjected to the original cross-sectional area.
4. **Answer:**

   (a) zero.

   (b) 10^{-3} cm per cm = 10^{-3} cm/cm.
5. **Answer:** Unity.
6. **Answer:** This happens when the body is deformed beyond the elastic limit.
7. **Answer:** It is a beam loaded at one end and free at the other end.
8. **Answer:** Yes, the weight of the wire itself acts as the deforming force.
9. **Answer:** As it is the ratio of two similar quantities.
10. **Answer:** It is the ratio of lateral strain to linear strain.

Short Questions Answers:

1. **Answer:**

   Three states of-matter differ from each other due to the following two factors:

   (a) The different magnitudes of tester atomic and intermolecular forces.

   (b) The degree of random thermal motion of the atoms and molecules of a substance depends upon the temperature.
2. **Answer:** In a normal situation, the atoms of a solid are at the locations of minimum potential energy. When we stretch a wire, the work has to be done against interatomic forces. This work is stored in the wire in the form of elastic potential energy.
3. **Answer:** A bridge during its use undergoes alternative strains a large number of times each day, depending upon the movement of vehicles on it. When a bridge is used for a long time it loses its elastic strength, due to which the number of strains in the bridge for given stress will become large and ultimately the bridge may collapse. Thus, !» to avoid this, the bridges are declared unsafe after long use.
4. **Answer:** Spring will be a better one if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of the material of the spring. Since Young's modulus of elasticity of steel is more than that of copper, hence steel is preferred in making the springs.
5. **Answer:** The area of this hysteresis loop measures the amount of heat energy dissipated by the material. Since the area of the loop B is more than that of A, therefore B can absorb more vibrations than that of Av Hence B is preferred.
6. **Answer:** A material regains its original Configuration (length, shape dr volume) only when the deforming force is within the elastic limit. Beyond the elastic limit, the bodies lose the property of elasticity and hence don't completely regain the length of being heavily loaded.
7. **Answer:** When spring balances are used for a long time, they get fatigued. So the springs of such balances will take time to recover their original configuration. Hence the readings shown by such spring balances will be wrong.
8. **Answer:** When a deforming force acts on a body, the atoms of the substances get displaced from their original positions. Due to this the configuration of the matter (substance) changes. The moment, the deforming force is removed, the

atoms return to their original positions and hence the substance or matter regains its original configuration. Hence elasticity is said to be the internal property of matter.

Long Questions Answers:

1. **Answer:**

(a) The following factors affect the elasticity of a material:

- **Effect of hammering and rolling:** It causes a decrease in the plasticity of the material due to break-up of crystal grains into smaller units and hence – the elasticity of the material increases.
- **Effect of Annealing:** Annealing results in the increases in the plasticity of the material due to, the formation of large crystal grains. Hence the elasticity of the material decreases.
- **Effect of the presence of impurities:** The effect of the presence of impurities in a material can be both ways i.e. it can increase as well as decrease the elasticity r of the material. The type of effect depends upon the nature of the impurity present in the material.
- **Effect of temperature:** The increase in the temperature of the material in most cases causes a decrease in the elasticity of the material. The elasticity of invar does not change with the change of temperature.

(b) **Poisson's Ratio (σ):** Within elastic limits, it is defined; as the ratio of lateral strain (β) to the linear strain i.e.

$$\sigma = \frac{\beta}{\alpha}$$

(c) **Breaking Load:** It is defined as the product of the breaking stress and area of cross-section of the given object. It is also called maximum load a body (cable/ wire) can support

i.e., breaking load = Breaking stress × area of cross-section. It should be noted that breaking stress is a constant for the given material.

2. **Answer:**

Let L, A be the length and area of the cross-section of the wire.

Also, let l be the extension produced on applying a force F, then

using the relation,

$$Y = \frac{\text{Stress}}{\text{Strain}}, \text{ we get}$$

$$Y = \frac{F/A}{l/L} = \frac{FL}{Al} \quad \text{....(1)}$$

where Y = Young's modulus.

Now when $F = T_1$ and $l = L_1 - L$.

Then $$Y = \frac{T_1 L}{A(L_1 - L)} \quad \text{....(2)}$$

and when $F = T_2$ and $l = L_2 - L$,

Then $$Y = \frac{T_2 L}{A(L_2 - L)} \quad \text{....(3)}$$

∴ From (2) and (3), we get

$$\frac{T_1 L}{A(L_1 - L)} = \frac{T_2 L}{A(L_2 - L)}$$

or $$T_1(L_1 - L) = T_2(L_1 - L)$$

or $$(T_2 - T)L = T_2 L_1 - T_1 L_2$$

or $$L = \frac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$$

Assertion Reason Answer:

1. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

Explanation:

Elasticity is a measure of tendency of the body to regain its original configuration. As steel is deformed less than rubber therefore steel is more elastic than rubber.

2. (d) If the assertion and reason both are false.

Explanation:

In a glassy solid (i.e., amorphous solid) the various bonds between the atoms or ions or molecules of a solid are not equally strong. Different bonds are broken at different temperatures. Hence there is no sharp melting point for a glassy solid.

Case Study Questions-

1. The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere. At sea level, it is 1.013 × 10^5 Pa (1 atm). Italian scientist Evangelista Torricelli (1608–1647) devised for the first time a method for measuring atmospheric pressure.

$$p = p_a + pgh$$

Where r is the density of mercury and h is the of the mercury column in the tube In the experiment it is found that the mercury column in the barometer has a height of about 76 cm at sea level equivalent to one atmosphere (1 atm). This can also be obtained using the value of r. A common way of stating pressure is in terms of cm or mm of mercury (Hg). A pressure equivalent of 1 mm is called a torr (after Torricelli). 1 torr = 133 Pa. The mm of Hg and torr are used in medicine and physiology. In meteorology, a common unit is the bar and millibar.1 bar = 10^5 Pa. An open tube manometer is a useful instrument for measuring pressure differences.

i. **Who gave for the first time a method for measuring atmospheric pressure?**
   a. Newton
   b. Pascal
   c. Torricelli
   d. None of the above

ii. **1 torr is equal to**
   a. 1000 pa
   b. 133 pa
   c. 50 pa
   d. None of these

iii. **What is 1 torr? Where it is used?**

iv. **Which device is used for measurement of pressure difference?**

v. **What is atmospheric pressure?**

2. Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions. This is another form of the Pascal's law and it has many applications in daily life. A number of devices, such as hydraulic lift and hydraulic brakes, are based on the Pascal's law. In these devices, fluids are used for transmitting pressure. Fluid flow is a complex phenomenon. Bernoulli's principle helps in explaining blood flow in artery. The artery may get constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction a greater demand is placed on the activity of the heart. The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This may result in heart attack. Dynamic lift is the force that acts on a body, such as airplane wing, a hydrofoil or a spinning ball, by virtue of its motion through a fluid. In many games such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle. A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to the ball is larger and below it is smaller .The stream lines, thus, get crowded above and rarified below. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spining is called Magnus effect. The Venturi-meter is a device to measure the flow speed of incompressible fluid. The principle behind this meter has many applications. The carburetor of automobile has a Venturi channel (nozzle) through which air flows with a high speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion. Filter pumps or aspirators, Bunsen burner, atomisers and sprayers used for perfumes or to spray insecticides work on the same principle.

i. **The Venturi-meter is a device used to measure the**
   a. Flow speed of incompressible fluid.
   b. Area occupied by fluid.

ii. **hydraulic brakes works on principle of**
   a. Pascal's law
   b. Newton's law
   c. Bernoulli's principle
   d. None of these

iii. **With the help of Bernoulli's principle. How heart attack happens?**

iv. **Explain Magnus effect with example of ball with spin in air.**

v. **What is dynamic lift?**

Case Study Answer

1. Answer

i. (c) Torricelli

ii. (b) 133 pa

iii. A pressure equivalent of 1 mm is called a torr 1torr = 133 Pa.

The mm of Hg and torr are used in medicine and physiology.

iv. An open tube manometer is a useful instrument for measuring pressure differences.

v. The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere. At sea level, it is 1.013×10^5 Pa (1 atm). 76 cm at sea level equivalent to one atmosphere (1 atm).

2. Answer

i. (a) Flow speed of incompressible fluid.

ii. (a) Pascal's law

iii. With the help of Bernoulli's principle we can explain heart attack phenomenon. The artery may get constricted due to the accumulation of plaque on its inner walls. In order to flow the blood through this constriction a large pressure is exerted on heart. The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood flows fast trough the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This result in heart attack.

iv. A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. When ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to the ball is larger and below it is smaller. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spining is called Magnus effect.

A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. When ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to the ball is larger and below it is smaller. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spinning is called Magnus effect.

Mechanical Properties of Fluids 10

Fluids

Fluids can be defined as any substance which is capable of flowing.

They don't have any shape of their own.

For example: water which does not have its own shape but it takes the shape of the container in which it is poured.

But when we pour water in a tumbler it takes the shape of the tumbler

Both liquids and gases can be categorised as fluids as they are capable of flowing.

Volume of solids, liquids and gas depends on the stress or pressure acting on it.

In this chapter we will study if we apply force on the fluid how does it affects the internal properties of fluids.

Fluids offer very little resistance to shear stress.

We will also study some characteristic properties of fluids.

Pressure

Pressure is defined as force per unit area.

$$Pressure = \frac{Force}{Area}$$

For **Example:**

Consider a very sharp needle which has a small surface area and consider a pencil whose back is very blunt and has more surface area than the needle. If we poke needle in our palm it will hurt as needle gets pierced inside our skin. Whereas if we poke the blunt side of the pencil into our hand it won't pain so much.

This is because area of contact between the palm and the needle is very small therefore the pressure is large. Whereas the area of contact between the pencil and the palm is more therefore the pressure is less.

Two factors which determine the magnitude of the pressure are:-

Force – greater the force greater is the pressure and vice-versa.

Coverage area – greater the area less is the pressure and vice-versa.

Example:

Consider a stuntman lying on the bed of nails which means there are large numbers of nails on any rectangular slab. All the nails are identical and equal in height.

We can see that the man is not feeling any pain and he is lying comfortably on the bed. This is because there isa large number of nails and all the nails are closely spaced with each other.

All the small, pointed nails make large surface area therefore the weight of the body is compensated by the entire area of all the nails.

The surface area increases therefore pressure is reduced.

But even if one nail is greater than the others then it will hurt. Because then the surface area will be less as a result pressure will be more.

Stuntman lying on bed of nails.

Pressure in Fluids:

Normal force exerted by fluid per unit area.

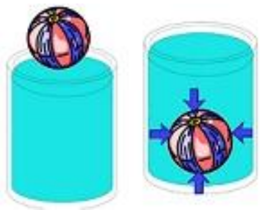
This means force is acting perpendicular to the surface of contact.

Consider a body submerged in the water, force is exerted by the water perpendicular to the surface of the body.

If there is no force applied perpendicularly but in the parallel direction then there will be motion along the horizontal direction.

Since fluid is at rest and body is submerged in the fluid. Therefore, there cannot be motion along the horizontal direction.

Therefore, we always say the force is applied perpendicularly.



Pressure is a scalar quantity. Because the force here is not a vector quantity but it is the component of force normal to the area.

Dimensional Formula $[ML^{-1}T^{-2}]$

I Unit: N/m^2 or Pascal (Pa).

Atmosphere unit (atm) is defined as pressure exerted by the atmosphere at sea level. It is a common unit of pressure.

$1atm = 1.013 \times 10^5$ Pa

Pascal's Law

Pascal's law states that if the pressure is applied to uniform fluids that are confined, the fluids will then transmit the same pressure in all directions at the same rate.

Pascal's law holds good only for uniform fluids.

For example:

Consider a vessel filled with water which is uniform throughout as there is only one type of fluid which is water.

Consider a vessel which has oil and water then it is not uniform. As it have two different fluids.

Fluid should be confined meaning fluid is present within region in space. It is not allowed to spread.

For example, 1:

A balloon filled with water and when we press it hard against the wall.

We will see the shape of the balloon changes. This is because if we apply force on balloon, pressure is exerted on the water.

Water is uniform fluid, and it is confined with in this balloon and is not allowed to spread.

On applying pressure, it is transmitted in all other directions.

Variation of pressure with depth

Consider a cylindrical object inside a fluid; consider 2 different positions for this object.

Fluid is at rest therefore the force along the horizontal direction is 0.

Forces along the vertical direction:

Consider two positions 1 and 2.

Force at position 1 is perpendicular to cross sectional area A, $F_1 = P_1$

Similarly, $F_2 = P_2$

Total force $F_{net} = F_1 + F_2$ as F_1 is along negative y axis therefore it is –ive and F_2 is along +ive y-axis.

$$F_{net} = (P_2 - P_1)A$$

This net force will be balanced by the weight of the cylinder(m).

Therefore, under equilibrium condition

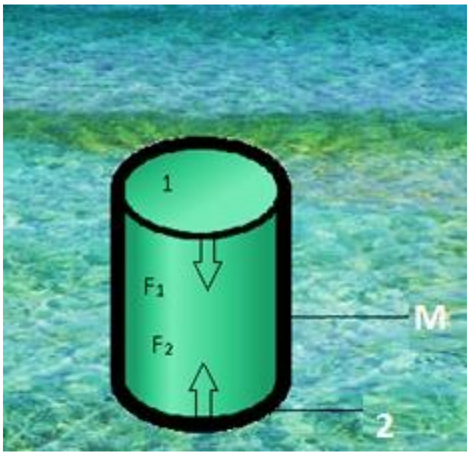
F_{net} = mg = weight of the cylinder = weight of the fluid displaced.

= ρ Vg where ρ = density = volume of the fluid

= ρhAg where V = hA (h = height and A = area)

Therefore $(P_2 - P_1) A = \rho hAg$

$P_2 - P_1 = \rho hg$, Therefore the difference in the pressure is dependent on height of the cylinder.



Cylinder is inside the fluid.

Consider the top of the cylinder exposed to air therefore $P_1 = P_a$ (where $P_a = P_1$ is equal to atmospheric pressure.)

Then $P_2 = P_a + \rho hg$

The pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount ρhg.

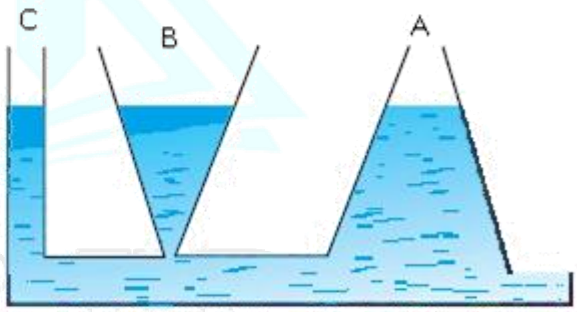
The pressure is independent of the cross sectional or base area or the shape of the container.

Hydrostatic Paradox

Hydrostatic Paradox means: - hydro = water, static =at rest

Paradox means that something taking place surprisingly.

Consider 3 vessels of very different shapes (like thin rectangular shape, triangular and some filter shape) and we have a source from which water enters into these 3 vessels.



The three vessels A, B and C contain different amounts of liquids, all up to the same height

Water enters through the horizontal base which is the base of these 3 vessels we observe that the level of water in all the 3 vessels is same irrespective of their different shapes.

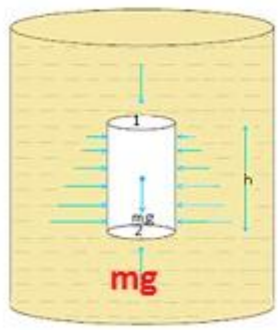
This is because pressure at some point at the base of these 3 vessels is same.

The water will rise in all these 3 vessels till the pressure at the top is same as the pressure at the bottom.

As pressure is dependent only on height therefore in all the 3 vessels the height reached by the water is same irrespective of difference in their shapes.

This experiment is known as Hydrostatic Paradox.

Atmospheric Pressure



Fluid is under gravity. The effect of gravity is illustrated through pressure on a vertical cylindrical column

Pressure exerted by the weight of the atmosphere.

Atmosphere is a mixture of different gases. All these gas molecules together constitute some weight. By virtue of this weight there is some pressure exerted by the atmosphere on all the objects.

This pressure is known as atmospheric pressure.

Value of atmospheric pressure at sea level is $1.01*10^5$

1atm = $1.01*10^5$Pa

Gauge Pressure

Pressure difference between the system and the atmosphere.

From relation P = Pa + ρgh where P = pressure at any point, Pa = atmospheric pressure.

We can say that Pressure at any point is always greater than the atmospheric pressure by the amount ρgh.

P – Pa = ρgh where

P = pressure of the system, Pa = atmospheric pressure,

(P - Pa) = pressure difference between the system and atmosphere.

hρg = Gauge pressure.

How to measure Gauge pressure

Gauge pressure is measured by Open Tube Manometer.

Open Tube Manometer is a U-shaped tube which is partially filled with mercury (Hg).

One end is open and other end is connected to some device where pressure is to be determined. This means it is like a system.

The height to which the mercury column will rise depends on the atmospheric pressure. Similarly depending on the pressure of the system the height of mercury in another tube rises.

The pressure difference between these two heights is the difference between the atmospheric pressure and system.

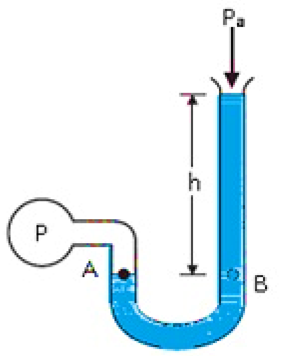
This difference in pressure is the gauge pressure.

Consider if the level of mercury column is same in both the U-tubes.

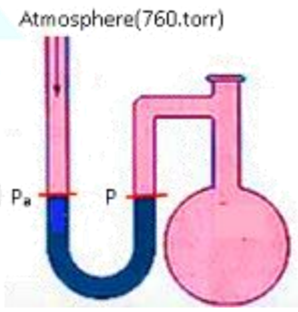
P_{atm} = P, therefore the difference between the atmospheric pressure and the pressure of the system is 0.

Gauge Pressure is 0.

P_{atm} = 760torr.



Open tube manometer



Closed end manometer

Absolute Pressure

Absolute pressure is defined as the pressure above the zero value of pressure.

It is the actual pressure which a substance has.

It is measured against the vacuum.

Absolute pressure is measured relative to absolute zero pressure.

It is sum of atmospheric pressure and gauge pressure.

P = P_a + hρg where P = pressure at any point, P_a = atmospheric pressure and hρg = gauge pressure.

Therefore P = P_a + Gauge Pressure. Where P = absolute pressure.

It is measured with the help of barometer.

Pascal's law for transmission of fluid pressure

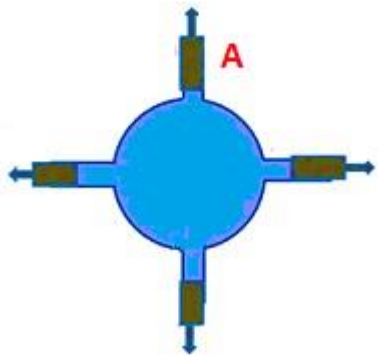
Pascal's law for transmission of fluid pressure states that the pressure exerted anywhere in a confined incompressible fluid is transmitted undiminished and equally in all directions throughout the fluid.

The above law means that if we consider a fluid which is restricted within a specific region in space and if the volume of the fluid doesn't change with the pressure, then the amount of pressure exerted will be same as the amount of pressure transmitted.

Consider a circular vessel which have 4 openings and along these 4 openings 4 pistons are attached.

When piston A is moved downwards pressure is exerted on the liquid in the downward direction, this pressure gets transmitted equally along all the directions. As a result, all the other 3 pistons move equal distance outwards.

A circular vessel fitted with movable piston at all the four ends and when piston A is moved downward a pressure is exerted downward. Equal amount of pressure is exerted along all the directions as a result they will move equal distances outward.



Applications: Pascal's law for transmission of fluid pressure

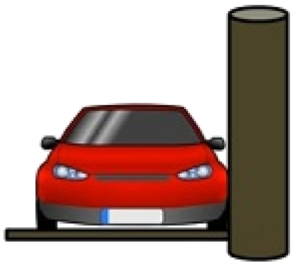
Hydraulic lift:

Hydraulic lift is a lift which makes use of a fluid.

For example: Hydraulic lifts that are used in car service stations to lift the cars.

Principle:

Inside a hydraulic lift there are 2 platforms, one has a smaller area and the other one has a larger area.



It is a tube-like structure which is filled with uniform fluid.

There are 2 pistons (P_1 and P_2) which are attached at both the ends of the tube.

Cross-sectional area of piston P_1 is A_1 and of piston P_2 is A_2.

If we apply force F_1 on P_1, pressure gets exerted and according to Pascal's law the pressure gets transmitted in all the directions and same pressure gets exerted on the other end. As a result the Piston P_2 moves upwards.

Advantage of using hydraulic lift is that by applying small force on the small area we are able to generate a larger force.

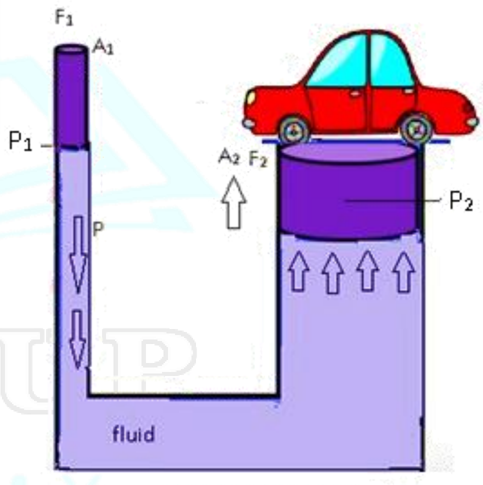
Mathematically: $F_2 = PA_2$

where F_2 = Resultant Force, A_2 = area of cross-section

$F_2 = \left(\frac{F_1}{A_1}\right) A_2$ where $P = \frac{F_1}{A_1}$ (Pressure P is due to force F_1 on the area A_1)

$F_2 = \left(\frac{A_2}{A_1}\right) F_1$. This shows that the applied force has increased by $\frac{A_2}{A_1}$

Because of Pascal's law the input gets magnified.



The above figure shows the internal structure of the hydraulic lift.

Hydraulic Brakes

Hydraulic brakes work on the principle of Pascal's law.

According to this law whenever pressure is applied on a fluid it travels uniformly in all the directions.

Therefore, when we apply force on a small piston, pressure gets created which is transmitted through the fluid to a larger piston. As a result of this larger force, uniform braking is applied on all four wheels.

As braking force is generated due to hydraulic pressure, they are known as hydraulic brakes.

Liquids are used instead of gas as liquids are incompressible.

Construction

The fluid in the hydraulic brake is known as brake fluid.

It consists of a master cylinder, four-wheel cylinders and pipes carrying brake fluid from master cylinder to wheel cylinders.

Master cylinder consists of a piston which is connected to pedal through connecting rod.

The wheel cylinders consist of two pistons between which fluid is filled.

Each wheel brake consists of a cylinder brake drum. This drum is mounted on the inner side of wheel. The drum revolves with the wheel.

Two brake shoes which are mounted inside the drum remain stationary.

Working

When we press the brake pedal, piston in the master cylinder forces the brake fluid through a linkage.

As a result, pressure increases and gets transmitted to all the pipes and to all the wheel cylinders according to Pascal's law.

Because of this pressure, both the pistons move out and transmit the braking force on all the wheels.

Advantages:

Equal braking effort to all the four wheels.

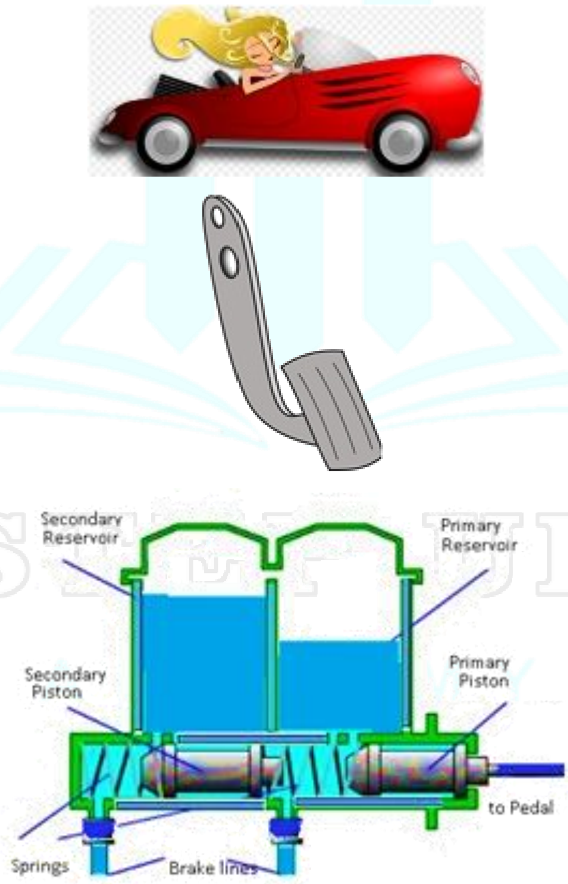
Less rate of wear due to absence of joints.

By just changing the size of one piston and cylinder, force can be increased or decreased.

Disadvantages:

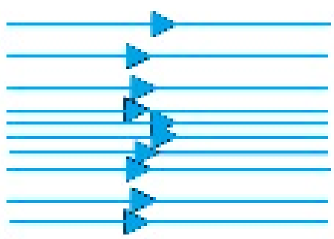
Leakage of brake fluid spoils the brake shoes.

Even the slightest presence of air pockets can spoil the whole system.

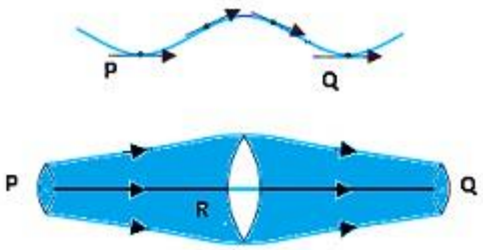


Inside of the cylinder

Types of Fluid flow: Steady Flow



Some streamlines for fluid flow



The flow of a fluid is said to be steady, if at any point, the velocity of each passing fluid particle remains constant within that interval of time.

Streamline is the path followed by the fluid particle.

It means that at any particular instant the velocities of all the particles at any point are same. But the velocity of all the particles won't be same across all the points in the space.

Steady flow is termed as 'Streamline flow' and 'Laminar flow'.

Consider a case when all the particles of fluid passing point A have the same velocity. This means that the first particle will have velocity V_1 and second will have velocity V_1 and so on. All the particles will have the same velocity V_1 at point A.

At point B, all particles will have velocity V_2.

Similarly at point C the velocity of all the particles is V_3.

We can see that the velocity is changing from point to point but at one particular point it is same.

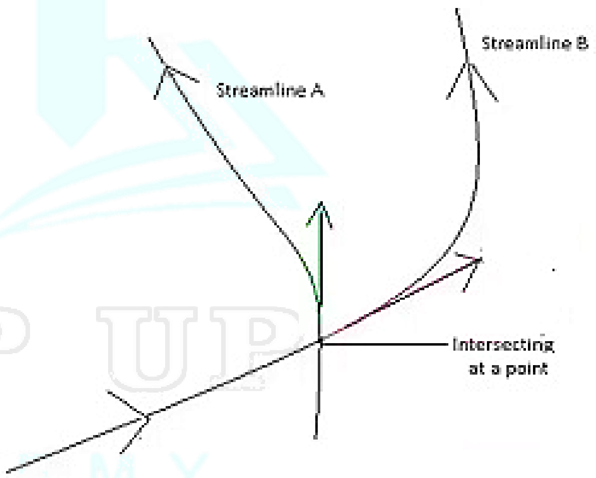
No two streamlines can intersect.

If two streamlines intersect each other, the particles won't know which path to follow and what velocity to attain. That is why no two streamlines intersect.

The meaning of streamlines:

(a) A typical trajectory of a fluid particle.

(b) A region of streamline flow.



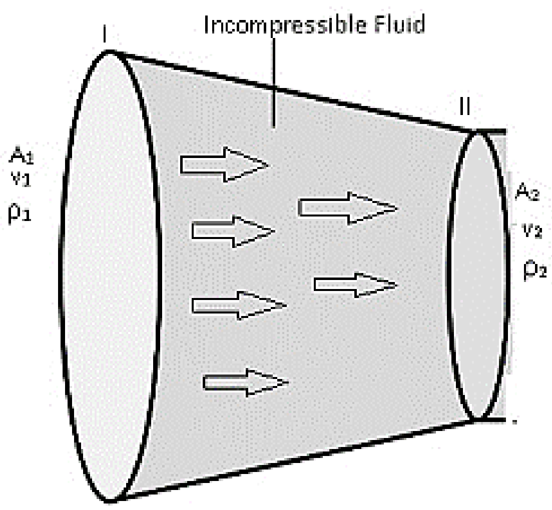
Equation of Continuity

According to the equation of continuity Av = constant. Where A = cross-sectional area and v = velocity with which the fluid flows.

It means that if any liquid is flowing in streamline flow in a pipe of non-uniform cross-section area, then rate of flow of liquid across any cross-section remains constant.

Consider a fluid flowing through a tube of varying thickness.

Let the cross-sectional area at one end (I) = A_1 and cross-sectional area of other end (II) = A_2.



The velocity and density of the fluid at one end (I) = v_1, ρ_1 respectively, velocity and density of fluid at other end (II) = v_2, ρ_2

Volume covered by the fluid in a small interval of time Δt, across left cross-sectional is Area (I) = $A_1 \times v_1 \times \Delta t$

Volume covered by the fluid in a small interval of time Δt across right cross-sectional Area (II) = $A_2 \times v_2 \times \Delta t$

Fluid inside is incompressible (volume of fluid does not change by applying pressure) that is density remains same $\rho_1 = \rho_2$. (Equation 1)

Along(I) mass = $\rho_1 A_1 v_1 \Delta t$ and along second point (II) mass = $\rho_2 A_2 v_2 \Delta t$

By using equation (1). We can conclude that $A_1 v_1 = A_2 v_2$. This is the equation of continuity.

From Equation of continuity, we can say that Av = constant.

This equation is also termed as "Conservation of mass of incompressible fluids".

Conclusion:

Volume flux/Flow rate remains constant throughout the pipe. This means rate of flow of fluid of liquid is more if cross-sectional area is more, then the velocity will be less, and vice-versa.

But the Av will remain constant.

So, the volume which is covered by the fluid at any cross-sectional area is constant throughout the pipe even if pipe has different cross-sectional areas.

The fluid is accelerated while passing from the wider cross-sectional area towards the narrower area. This means if area is more the velocity is less and vice-versa.

Turbulent Flow:

A fluid flow is said to be turbulent if the velocity of the particles vary at any point erratically.

This means fluid particles are moving here and there, they are not moving in organized manner. They all will have different velocities.

Eddies are generated by this flow. Eddies are same as ripples.

All the particles are moving here and there randomly.

Bernoulli's Principle

For a streamline fluid flow, the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho v^2}{2}\right)$ and the potential energy per unit volume (ρgh) remain constant.

Mathematically: $P + \frac{\rho v^2}{2} + \rho g h$ = constant

where P = pressure,

$$\frac{E.}{Volume} = \frac{\frac{1}{2}mv^2}{V} = \frac{\frac{1}{2}v^2}{\left(\frac{m}{V}\right)} = \frac{1}{2}\rho v^2$$

$$\frac{E.}{Volume} = \frac{mgh}{V} = \left(\frac{m}{V}\right)gh = \rho g h$$

Derive: Bernoulli's equation

Assumptions:

Fluid flow through a pipe of varying width.

Pipe is located at changing heights.

Fluid is incompressible.

Flow is laminar.

No energy is lost due to friction: applicable only to non-viscous fluids.

Mathematically:

Consider the fluid initially lying between *B* and *D*. In an infinitesimal time interval Δt, this fluid would have moved.

Suppose v_1 = speed at *B* and v_2 = speed at *D*, initial distance moved by fluid from to $C = v_1 \Delta t$.

In the same interval Δt fluid distance moved by *D* to $E = v_2 \Delta t$.

P_1 = Pressure at A_1, P_2 = Pressure at A_2.

Work done on the fluid at left end $(BC)\, W_1 = P_1 A_1 (v_1 \Delta t)$.

Work done by the fluid at the other end $(DE)\, W_2 = P_2 A_2 (v_2 \Delta t)$.

Net work done on the fluid is $W_1 - W_2 = (P_1 A_1 v_1 \Delta t - P_2 A_2 \Delta t)$

By the Equation of continuity Av = constant.

- $P_1 A_1 v_1 \Delta t - P_2 A_2 \Delta t$ where $A_1 v_1 \Delta t - P_1 \Delta V$ and $A_2 v_2 \Delta t - P_2 \Delta V$.

Therefore Word done = $(P_1 - P_2)\,\Delta V$ equation (a)

- Part of this work goes in changing Kinetic energy, $\Delta K = (½) m (v_2^2 - v_1^2)$ and part in gravitational potential energy, $\Delta U = mg\,(h_2 - h_1)$.

The total change in energy $\Delta E = \Delta K + \Delta U = (½) m (v_2^2 - v_1^2) + mg(h_2 - h_1)$. (i)

Density of the fluid $\rho = m / V$ or $m = \rho V$

Therefore, in small interval of time Δt, small change in mass Δm

- $\Delta m = \rho \Delta V$ (ii)

Putting the value from equation (ii) to (i)

$\Delta E = ½\, \rho \Delta V (v_2^2 - v_1^2) + \rho g \Delta V (h_2 - h_1)$ equation (b)

By using work-energy theorem: $W = \Delta E$

From (a) and (b)

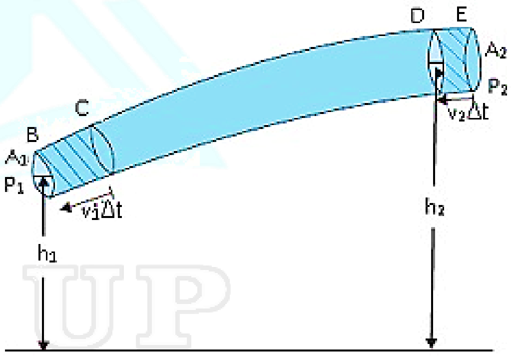
$(P_1 - P_2)\, \Delta V (½)\, \rho \Delta V (v_2^2 - v_1^2) + \rho g \Delta V (h_2 - h_1)$

$P_1 - P_2 = ½ \rho v_2^2 - ½ \rho v_1^2 + \rho g h_2 - \rho g h_1$ (By cancelling ΔV from both the sides).

After rearranging we get, $P_1 + (1/2)\rho v_1^2 + \rho g h_1 = (1/2)\rho v_2^2 + \rho g h_2$ $P + (1/2)\rho v^2 + \rho g h = \text{constant}$

This is the Bernoulli's equation.

The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length v1Δt moves to the section of length v2Δt in time Δt.



Bernoulli's equation: Special Cases

When a fluid is at rest. This means $v_1 = v_2 = 0$.

From Bernoulli's equation

$P_1 + (1/2)\rho v_1^2 + \rho g\, h_1 = (1/2)\rho v_2^2 + \rho g\, h_2$

By putting $v_1 = v_2 = 0$ in the above equation changes to

$P_1 - P_2 = \rho g (h_2 - h_1)$. This equation is same as when the fluids are at rest.

When the pipe is horizontal $h_1 = h_2$. This means there is no Potential energy by the virtue of height.

Therefore from Bernoulli's equation $(P_1 + (1/2)\rho v_1^2 + \rho g h_1 = (1/2)\rho v_2^2 + \rho g h_2)$

By simplifying. $P + (1/2)\rho v^2$ = constant.

Torricelli's law

Torricelli law states that the speed of flow of fluid from an orifice is equal to the speed that it would attain if falling freely for a distance equal to the height of the free surface of the liquid above the orifice.

Consider any vessel which has an orifice (slit)filled with some fluid.

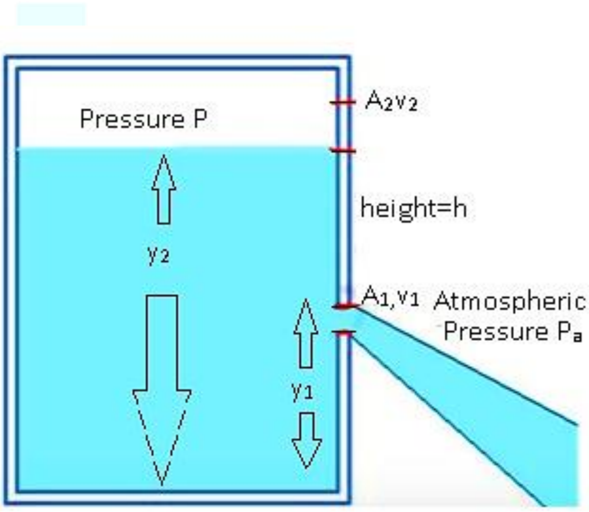
The fluid will start flowing through the slit and according to Torricelli law the speed with which the fluid will flow is equal to the speed with which a freely falling body attains such that the height from which the body falls is equal to the height of the slit from the free surface of the fluid.

Let the distance between the free surface and the slit = h

Velocity with which the fluid flows is equal to the velocity with which a freely falling body attains if it is falling from a height h.

Derivation of the Law:

- Let A_1 = area of the slit (it is very small), v_1 = Velocity with which fluid is flowing out.
  - A_2 = Area of the free surface of the fluid, v_2 = velocity of the fluid at the free surface.
- From Equation of Continuity, Av = constant. Therefore $A_1v_1 = A_2v_2$.
  - From the figure, $A_2 >>> A_1$, This implies $v_2 << v_1$ (This means fluid is at rest on the free surface), Therefore $v_2 \sim 0$.
- Using Bernoulli's equation,
  - $P + (1/2)\, pv^2 + \rho gh$ = constant.
- Applying Bernoulli's equation at the slit:
  - $Pa + (1/2)\rho v_1^2 + \rho g y_1$ (Equation 1) where P_a = atmospheric pressure, y_1 = height of the slit from the base.
- Applying Bernoulli's equation at the surface:
  - $P + \rho g y_2$ (Equation 2) where as $v_2 = 0$ therefore $(1/2)\ \rho v_1^2 = 0$, y_2 = height of the free surface from the base.
- By equating (1) and (2)
- $P_a + (1/2)\rho v_1^2 + \rho g y_1 = P + \rho g y_2$
- $(1/2)\rho v_1^2 = (P - P_a) + \rho g(y_2 - y_1)$
- $= (P - P_a)\ \rho gh$ (where $h = (y_2 - y_1)$)
- $v_1^2 = 2/\rho[(P - P_a) + \rho gh]$
- Therefore $v_1 = \sqrt{2/p}\,[(P - P_a) + \rho gh]$. This is the velocity by which the fluid will come out of the small slit.
- v_1 is known as Speed of Efflux. This means the speed of the fluid outflow.



Torricelli's law. The speed of efflux, v1,from the side of the container is given by the application of Bernoulli's equation.

Case1: The vessel is not closed it is open to atmosphere that means P = Pa.

Therefore $v_1 = \sqrt{2gh}$.This is the speed of a freely falling body.

This is accordance to Torricelli's law which states that the speed by which the fluid is flowing out of a small slit of a container is same as the velocity of a freely falling body.

Case2: Tank is not open to atmosphere but P>>Pa.

Therefore, 2gh is ignored as it is very very large, hence $v_1 = \sqrt{2P/\rho}$.

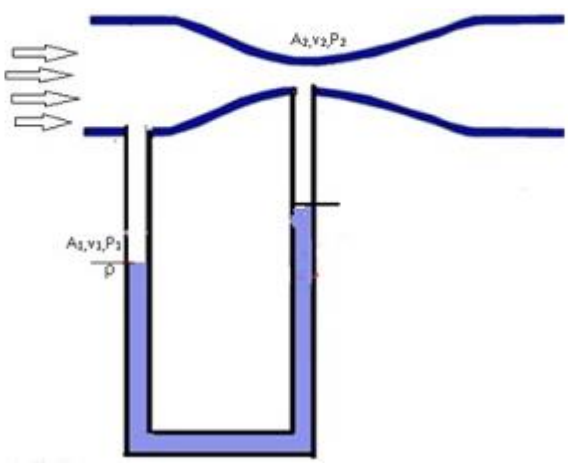
The velocity with which the fluid will come out of the container is determined by the Pressure at the free surface of the fluid alone.

Venturimeter

Venturimeter is a device to measure the flow of incompressible liquid.

It consists of a tube with a broad diameter having a larger cross-sectional area but there is a small constriction in the middle.

It is attached to U-tube manometer. One end of the manometer is connected to the constriction and the other end is connected to the broader end of the Venturimeter.



A schematic diagram of Venturimeter

The U-tube is filled with fluid whose density is ρ.

A_1 = cross-sectional area at the broader end, v_1 = velocity of the fluid.

A_2 =cross-sectional area at constriction, v_2 = velocity of the fluid.

By the equation of continuity, wherever the area is more velocity is less and vice-versa. As A_1 is more this implies v_1 is less and vice-versa.

Pressure is inversely α to Therefore at A_1 pressure P_1 is less as compared to pressure P_2 at A_2.

This implies $P_1 < P_2$ as $v_1 > v_2$.

As there is difference in the pressure the fluid moves, this movement of the fluid is marked by the level of the fluid increase at one end of the U-tube.

Venturimeter: determining the fluid speed

- By Equation of Continuity: $A_1v_1 = A_2v_2$.
- This implies $v_2 = (A_1/A_2)\ v_1$ (Equation (1))
- By Bernoulli's equation: $P_1 + (1/2)\rho v_1^2 + \rho gh = (1/2)\rho v_2^2 + \rho gh$
    - As height is same we can ignore the term ρg
    - This implies $P_1 - P_2 = (1/2)\rho(v_2^2 + v_1^2)$
    - $= 1/2\,\rho(A_1^2 / A_2^2 v_1^2 - v_1^2)$ (Using equation (1))
    - $= 1/2\,\rho v_1^2(A_1^2 / A_2^2 - 1)$
    - $= 1/2\,\rho v_1^2(A_1^2 / A_2^2 - 1)$
- As there is pressure difference the level of the fluid in the U-tube changes.
- $(P_1 + P_2) = h\rho_m g$ where rm (density of the fluid inside the manometer).
    - $1/2\rho v_1^2\,(A_1^2 / A_2^2 - 1) = h\rho_m g$
    - $v_1 = 2h\rho_m g / \rho\,[A_1^2 / A_2^2 - 1]^{-1/2}$

Practical Application of Venturimeter:

Spray Gun or perfume bottle- They are based on the principle of Venturimeter.

- Consider a bottle filled with fluid and having a pipe which goes straight till constriction. There is a narrow end of pipe which has a greater cross sectional area.
- The cross sectional area of constriction which is at middle is less.
- There is pressure difference when we spray as a result some air goes in, velocity of the air changes depending on the cross sectional area.
- Also because of difference in cross sectional area there is pressure difference, the level of the fluid rises, and it comes out.

Dynamic Lift

Dynamic lift is the normal force that acts on a body by virtue of its motion through a fluid.

Consider an object which is moving through the fluid, and due to the motion of the object through the fluid there is a normal force which acts on the body.

This force is known as dynamic lift.

Dynamic lift is most popularly observed in aeroplanes.

Whenever an aeroplane is flying in the air, due to its motion through the fluid here fluid is air in the atmosphere. Due to its motion through this fluid, there is a normal force which acts on the body in the vertically upward direction.

This force is known as Dynamic lift.

Examples:

Airplane wings

Spinning ball in air

Dynamic lift on airplane wings:

Consider an aeroplane whose body is streamline. Below the wings of the aeroplane there is air which exerts an upward force on the wings. As a result aeroplane experiences dynamic lift.

Magnus Effect

Dynamic lift by virtue of spinning is known as Magnus effect.

Magnus effect is a special name given to dynamic lift by virtue of spinning.

Example: Spinning of a ball.

Case1: When the ball is not spinning.

The ball moves in the air it does not spin, the velocity of the ball above and below the ball is same.

As a result, there is no pressure difference. ($\Delta P = 0$).

Therefore, there is no dynamic lift.

Case2: When the ball is moving in the air as well as spinning.

When the ball spins it drags the air above it therefore the velocity above the ball is more as compared to the velocity below the ball.

As a result there is a pressure difference; the pressure is more below the ball.

Because of pressure difference there is an upward force which is the dynamic lift.

Viscosity

Viscosity is the property of a fluid that resists the force tending to cause the fluid to flow.

It is analogous to friction in solids.

Example:

Consider 2 glasses one filled with water and the other filled with honey.

Water will flow down the glass very rapidly whereas honey won't. This is because honey is more viscous than water.

Therefore, in order to make honey flow we need to apply greater amount of force. Because honey has the property to resist the motion.

Viscosity comes into play when there is relative motion between the layers of the fluid. The different layers are not moving at the same pace.

Coefficient of Viscosity

Coefficient of viscosity is the measure of degree to which a fluid resists flow under an applied force.

This means how much resistance does a fluid have to its motion.

Ratio of shearing stress to the strain rate.

It is denoted by 'η'.

Mathematically

Δt=time, displacement = Δx

Therefore,

shearing stress = $\frac{\Delta x}{l}$ where l = length

Strain rate = $\frac{\Delta x}{l\Delta t}$

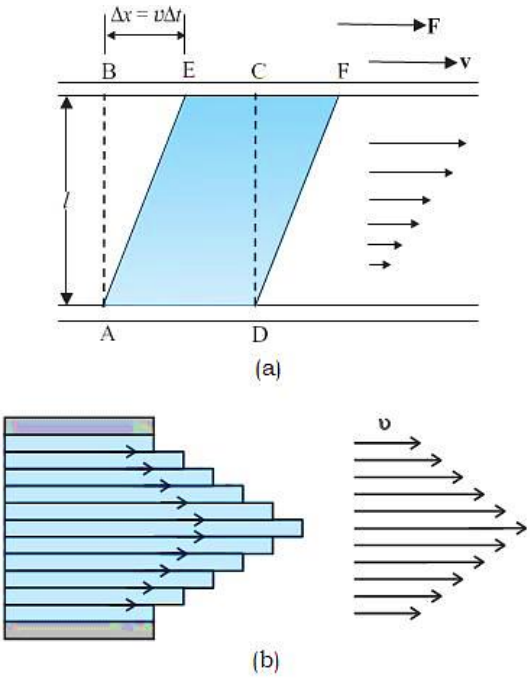
$$\eta = \frac{shearing\ stress}{strain\ rate}$$

$$\frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta x}{l\Delta t}\right)} = \frac{Fl}{vA}\ where\ \frac{\Delta x}{t} = v$$

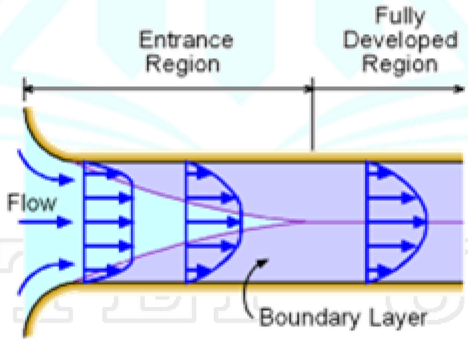
Therefore $\eta = \frac{Fl}{vA}$

I. Unit: Poiseiulle $\frac{\frac{PI}{Pa}}{Nsm^{-2}}$

Dimensional Formula: $[ML^{-1}T^{-1}]$



(a) A layer of liquid sandwiched between two parallel glass plates in which the lower plate is fixed and the upper one is moving to the right with velocity v



Velocity distribution for viscous flow in a pipe.

Stokes Law

The force that retards a sphere moving through a viscous fluid is directly ∝to the velocity and the radius of the sphere, and the viscosity of the fluid.

Mathematically: F =6πηrv where

Let retarding force F∝v where v =velocity of the sphere

F ∝ r where r=radius of the sphere

F∝η where η=coefficient of viscosity

6π=constant

Stokes law is applicable only to laminar flow of liquids. It is not applicable to turbulent law.

Example: Falling raindrops

Consider a single rain drop, when raindrop is falling it is passing through air.

The air has some viscosity; there will be some force which will try to stop the motion of the rain drop.

Initially the rain drop accelerates but after some time it falls with constant velocity.

As the velocity increases the retarding force also increases.

There will be viscous force F_v and bind force F_b acting in the upward direction. There will also be Gravitational force acting downwards.

After some time $F_g = F_r (F_v + F_b)$

Net Force is 0. If force is 0 as a result acceleration also becomes 0.

Terminal Velocity

Terminal velocity is the maximum velocity of a body moving through a viscous fluid.

It is attained when force of resistance of the medium is equal and opposite to the force of gravity.

As the velocity is increasing the retarding force will also increase and a stage will come when the force of gravity becomes equal to resistance force.

After that point velocity won't increase and this velocity is known as terminal velocity.

It is denoted by 'vt'. Where$_t$ = terminal.

Mathematically:

Terminal velocity is attained when Force of resistance = force due to gravitational attraction.

$6\pi\eta rv = mg$

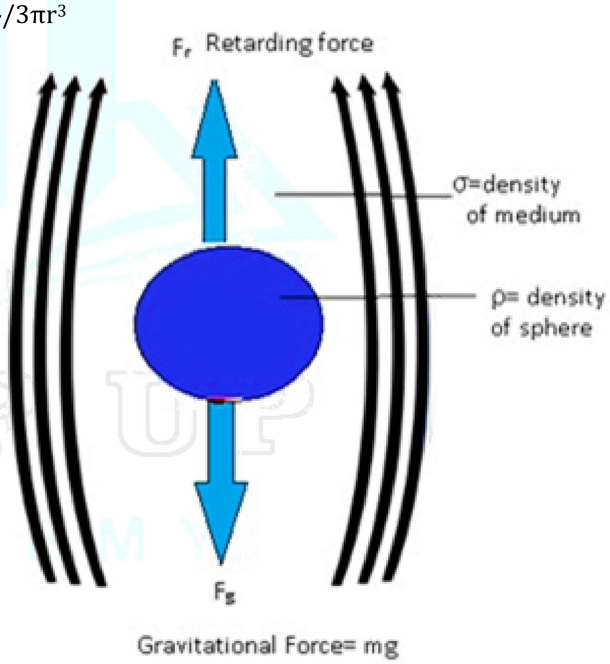
$6\pi\eta rv$ = densityxVg (Because density=m/V), density=$\rho - \sigma$ where ρ and σ are the densities of the sphere and the viscous medium resp.

$6\pi\eta rv = (\rho - \sigma)x4/3\pi r^3 g$ where Volume of the sphere(V) $=4/3\pi r^3$

By simplifying

$= (\rho - \sigma)gx4/3r^2x1/(6\eta)$

$v_t = 2r^2(\rho - \sigma)g/9\ \eta$ .This is the terminal velocity. Where($v=v_t$)



Reynolds Number

Reynolds number is a dimensionless number, whose value gives an idea whether the flow would be turbulent or laminar.

Types of flow are classified as 2 types: laminar flow and turbulent flow.

Reynolds number helps us to determine whether the flow is laminar or turbulent.

It is denoted by Re. where 'e' shows Reynolds.

Expression: $Re=\rho vd/\eta$;

where ρ = density of the fluid,

v=velocity of the fluid,

d=diameter of the pipe through which the fluid flows

η=viscosity of the fluid.

Liquid Surfaces

Certain properties of free surfaces:

Whenever liquids are poured in any container they take the shape of that container in which they are poured and they acquire a free surface.

Consider a case if we pour water inside the glass it takes the shape of the glass with a free surface at the top.

Top surface of the glass is a free surface. Water is not in contact with anything else, it is in contact with the air only.

This is known as free surfaces.

Liquids have free surfaces. As liquids don't have fixed shape they have only fixed volume.

Free surfaces have additional energy as compared to inner surfaces of the liquid.

Surface Energy

Surface energy is the excess energy exhibited by the liquid molecules on the surface compared to those inside the liquid.

This means liquid molecules at the surface have greater energy as compared to molecules inside it.

Suppose there is a tumbler and when we pour water in the tumbler, it takes the shape of the tumbler.

It acquires free surface.

Case 1: When molecules are inside the liquid:

Suppose there is a molecule inside the water, there will be several other molecules that will attract that molecule in all the directions.

As a result this attraction will bind all the molecules together.

This results in negative potential energy of the molecule as it binds the molecule.

To separate this molecule huge amount of energy is required to overcome potential energy.

Some external energy is required to move this molecule and it should be greater than the potential energy.

Therefore in order to separate this molecule a huge amount of energy is required.

Therefore a large amount of energy is required by the molecules which are inside the liquid.

Case2: When the molecules are at the surface:

When the molecule is at the surface, half of it will be inside and half of it is exposed to the atmosphere.

For the lower half of the molecule it will be attracted by the other molecules inside the liquid.

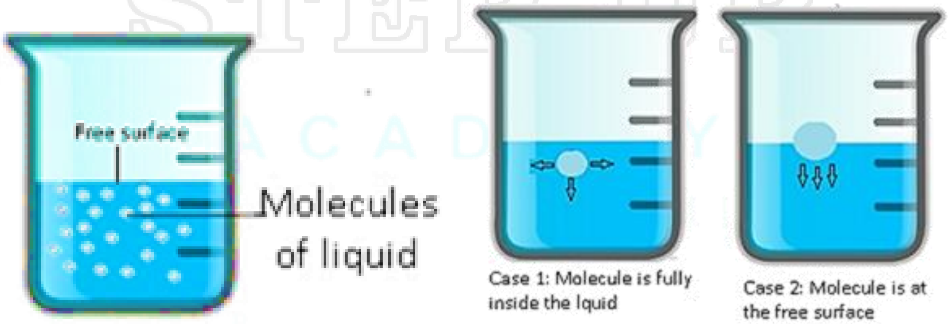
But the upper half is free. The negative potential energy is only because of lower half.

But the magnitude is half as compared to the potential energy of the molecule which is fully inside the liquid.

So the molecule has some excess energy, because of this additional energy which the molecules have at the surface different phenomenon happen like surface energy, surface tension.

Liquids always tend to have least surface are when left to itself.

As more surface area will require more energy as a result liquids tend to have least surface area.



Surface energy for two fluids in contact

Whenever there are two fluids, in contact, surface energy depends on materials of the surfaces in contact.

Surface energy decreases if the molecules of the two fluids attract.

Surface energy increases if molecules of the two fluids repel.

Surface Tension

Surface tension is the property of the liquid surface which arises due to the fact that surface molecules have extra energy.

Surface energy is the extra energy which the molecules at the surface have.

Surface tension is the property of the liquid surface because the molecules have extra energy.

Surface energy is defined as surface energy per unit area of the liquid surface.

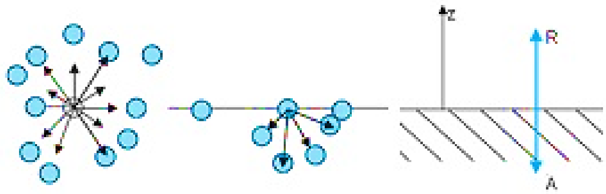
Denoted by 'S'.

Mathematically:

Consider a case in which liquid is enclosed in a movable bar.

Slide the bar slightly and it moves some distance ('d').

There will be increase in the area, (dl) where l=length of the bar.



Liquids have two surfaces one on the bar and other above the bar. Therefore area=2(dl)

Work done for this change = F x displacement.

Surface tension(S)=Surface Energy/area

Or Surface Energy=S x area

=Sx2dl

Therefore S x 2dl =F x d

$$S = F/2d$$

Surface tension is the surface energy per unit area of the liquid surface.

It can be also defined as Force per unit length on the liquid surface.

Important: At any interface (it is a line which separates two different medium) the surface tension always acts in equal and opposite direction and it is always perpendicular to the line at the interface.

Schematic picture of molecules in a liquid, at the surface and balance of forces

(a) Molecule inside a liquid. Forces on a molecule due to others are shown. Direction of arrows indicates attraction of repulsion. (b) Same, for a molecule at a surface. (c) Balance of attractive (A) and repulsive (R) forces.

Surface tension and Surface energy: practical applications

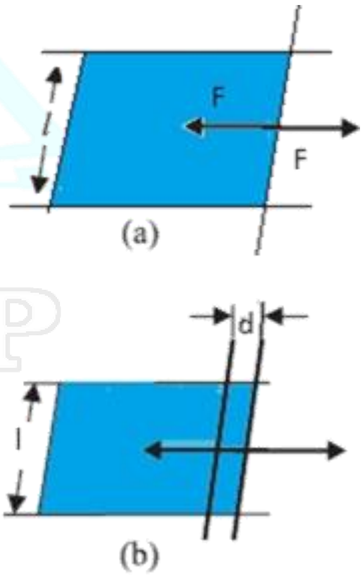
Consider a molecule which is present completely inside the liquid and if it is strongly attracted by the neighbouring molecules then the surface energy is less.

Consider a molecule which is present partially inside the liquid the force of attraction by the neighbouring molecules is lesser as a result surface energy is more.

Consider a molecule whose very little part is inside the water so very small force of attraction by the neighbouring molecules as a result more surface energy.

Conclusion: A fluid will stick to a solid surface if the surface energy between fluid and solid is smaller than the sum of energies between solid-air and fluid-air.

This means S_{sf}(solid fluid) < S_{fa}(fluid air) + S_{sa}(Solid air)



Stretching a film (a) A film in equilibrium;(b) The film stretched an extra distance.

How detergents work

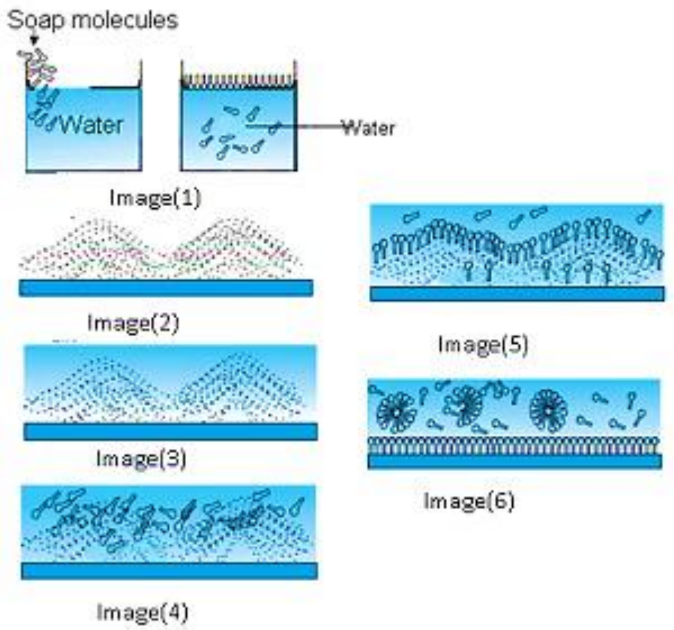
Washing alone with the water can remove some of the dirt but it does not remove the grease stains. This is because water does not wet greasy dirt.

We need detergent which mixes water with dirt to remove it from the clothes.

Detergent molecules look like hairpin shape. When we add detergents to the water one end stick to water and the other end sticks to the dirt.

As a result dirt is getting attracted to the detergent molecules and they get detached from the clothes and they are suspended in the water.

Detergent molecules get attracted to water and when water is removed the dirt also gets removed from the clothes.



Detergent action in terms of what detergent molecules do.

In image (1) Soap molecules with head attracted to water

In image (2) greasy dirt

In image (3) water is added but dirt does not get removed

In image (4) when detergent is added, other end of the molecules get attracted to the boundary where water meets dirt.

In image (5) Dirt gets surrounded by inert end and dirt from the clothes can be removed by moving water.

In image (6) dirt is held suspended, surrounded by soap molecule,

Angle of Contact

Angle of contact is the angle at which a liquid interface meets a solid surface.

It is denoted by θ.

It is different at interfaces of different pairs of liquids and solids.

For **example: -** Droplet of water on louts leaf. The droplet of water (Liquid) is in contact with the solid surface which is leaf.

This liquid surface makes some angle with the solid surface. This angle is known as angle of contact.

Water form a spherical shape on lotus leaf

Water spilt on the table.

Significance of Angle of Contact

Angle of contact determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

If the Angle of contact is obtuse: then droplet will be formed.

If the Angle of contact is acute: then the water will spread.

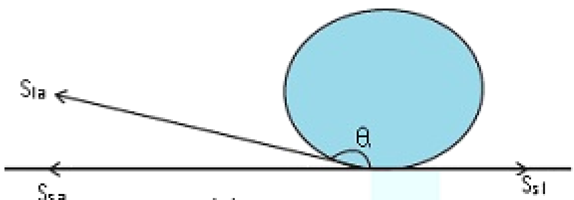
Case1: When droplet is formed

Consider we have a solid surface, droplet of water which is liquid and air.

The solid liquid interface denoted by S_{sl}, solid air interface denoted by S_{sa} and liquid air interface denoted by S_{la}.

The angle which S_{sl} makes with S_{la}. It is greater than the 900.

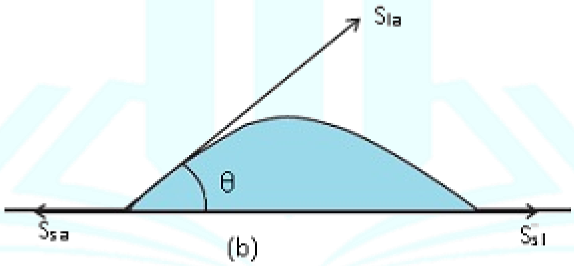
Therefore droplet is formed.



(a)

Case 2: When water just spreads

The angle which liquid forms with solid surface is less than 90^0.



(b)

Drops and Bubbles

Why water and bubbles are drops

- Whenever liquid is left to itself it tends to acquire the least possible surface area so that it has least surface energy so it has most stability.
- Therefore for more stability they acquire the shape of sphere, as sphere has least possible area.

Spherical Shape

Distinction between Drop, Cavity and Bubble

Drop: Drop is a spherical structure filled with water.

There is only one interface in the drop.

The interface separates water and air.

Example: Water droplet.

Cavity: Cavity is a spherical shape filled with air.

In the surroundings there is water and in middle there is cavity filled with air.

There is only one interface which separates air and water.

Example: bubble inside the aquarium.

Bubble: In a bubble there are two interfaces. One is air water and another is water and air.

Water droplets

Inside a bubble there is air and there is air outside.

But it consists of thin film of water.

Capillary Rise

In Latin the word capilla means hair.

Due to the pressure difference across a curved liquid-air interface the water rises up in a narrow tube in spite of gravity.

Consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water.

The contact angle between water and glass is acute. Thus the surface of water in the capillary is concave. As a result there is a pressure difference between the two sides of the top surface. This is given by:

$$(P_i - P_o) = (2S/r) = 2S/(a\sec\theta) = (2S/a)\cos\theta \ (i)$$

Thus the pressure of the water inside thetube, just at the meniscus (air-water interface) is less than the atmospheric pressure.

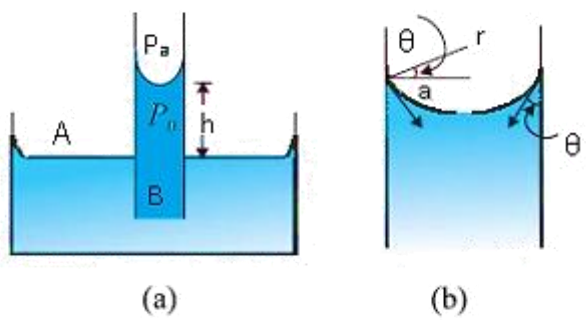
Consider the two points A and B. They must be at the same pressure,

$$P_0 + h\rho g = P_i = P_A \ (ii)$$

where ρ is the density of water and h is called the capillary

$h\rho g = (P_i - P_0) = (2S\cos\theta)/a$ (By using equations (i) and (ii))

Therefore the capillary rise is due to surface tension. It is larger, for a smaller radius.



Capillary rise, (a) Schematic picture of a narrow tube immersed water.
(b) Enlarged picture near interface.

Top Formulae

Pressure of a fluid having density ρ at height h	$P = h\rho g$
Gauge pressure	= total pressure – atmospheric pressure
Pascal's law: (*in hydraulic lift*)	$\frac{F_1}{a_1} = \frac{F_2}{\ _2}$
Surface tension and surface energy are related as	$S = F/2\ell$
Work done	= surface tension × increase in area
Excess of pressure inside the liquid drop	$p = P - P_o = \frac{2S}{r}$
Excess of pressure inside the soap bubble	$p = P - P_o = \frac{4S}{r}$
Total pressure in the air bubble at a depth h below the surface of liquid of density ρ	$P = P_o + h\rho g + \frac{2S}{r}$
In case of capillary action	Ascent / descent formula, $h = \frac{2S\cos\theta}{r\rho g}$, where θ is the angle of contact.
Newton's viscous dragging force	$F = \eta A \frac{dv}{dx}$, where η is coefficient of viscosity, A is the area of layer of liquid and $\frac{dv}{dx}$ is the velocity gradient.
Stoke's law	$F = 6\pi\eta rv$
Terminal velocity	$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$, where ρ and σ are the densities of the spherical body and medium, respectively; r is the radius of the spherical body.
Reynold's number	$R_N = \frac{\rho Dv}{\eta}$, where D is the diameter of the tube and v is the velocity of liquid flow through the tube.
Bernoulli's theorem	Pressure energy per unit mass + potential energy per unit mass + kinetic energy per unit mass = constant $\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$
Venturi meter, volume of liquid flowing per second	$V = a_1 a_2 \sqrt{\frac{2\rho_m gh}{\rho(a_1 \quad a^2)}}$, where a_1 and a_2 are the areas of cross-section of bigger and smaller tube; h is the difference of pressure head at the two tubes of a Venturi meter.
Velocity of efflux: Torricelli's law	$v = \sqrt{2gh}$

Class : 11th Physics

Chapter- 10 : Mechanical Properties of Fluids

Venturimeter
Device used to measure the rate of flow of liquid. Volume of liquid flowing per second.

$$Q = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho\left(a_1^2 - a_2^2\right)}}$$

Toricell's Low
Velocity of efflux of liquid through an orifice

$$V = \sqrt{2gh}$$

Applications
- o Lift of an aircraft wing.
- o Sprayer or atomizer
- o Blowing off the roofs during windstorm.

Stroke's law $F = 6\eta vr$

For an incompressible non-viscous streamline, irrotational flow of fluid,

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Opposing force between different layers of fluid in relative motion

Viscous drag $F = -\eta A \frac{dv}{dx}$

η = coefficient of viscosity

Streamline : In liquid flow when the Velocity is less than critical velocity, each particle of the liquid passing through a point travels along the same path and same velocity as the preceding particles.
Turbulent : When velocity of liquid flow is greater than critical velocity and particles follow zig-zag path.

Bernoulli's principal

Viscosity

Flow of fluids

Mechanical Properties of Fluids

Formulae

Laws of Fluids

- Surface tension $S = \frac{F}{l}$
- Surface Energy $= \frac{\text{work done in increasing area}}{\text{increase in surface area}} = \frac{W}{\Delta A}$
- Capillary rise or fall, $h = \frac{2S\cos\theta}{r\rho g}$
- Excess Pressure inside a drop (liquid)
  $P_{excess} = \frac{2S}{R}$
- Excess Pressure inside a bubble (soap)
  $P_{excess} = \frac{4S}{R}$

That can flow like liquids and gases

Equation of continuity
$m = a_1 v_1 \rho_1 = a_2 v_2 \rho_2$
for an incompressible liquid,
$\rho_1 = \rho_2$ then $a_1 v_1 = a_2 v_2$
or $av = \text{constant}$

Pascal's law : The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction.

$$\text{Pressure}(P) = \frac{\text{thrust}(F)}{\text{area}(A)} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Pressure exerted by a liquid column of height h, $(p) = h\rho g$

Atmospheric Pressure(Pa)
Pressure (atm) exerted by the atmosphere. At sea level, 1 atm = pressure exerted by 0.76m of Hg = hρg = 0.76 × 13.6 × 10 × 9.8 = 1.013 × 10 Nm2 = 101.3kPa

Absolute Pressure(P)
Total or actual pressure at a point. Absolute pressure = atmospheric pressure + gauge pressure

Gauge Pressure(Pg)
Difference between the absolute pressure at a point and the atmospheric pressure.
Pg = absolute pressure (P) – atmospheric pressure (Pa)

Important Questions

Multiple Choice Questions

1. Plants get water through the roots because of
   (a) Capillarity
   (b) Viscosity
   (c) Gravity
   (d) Elasticity
2. Water rises up to a height h1 in a capillary tube of radius r. the mass of the water lifted in the capillary tube is M. if the radius of the capillary tube is doubled, the mass of water that will rise in the capillary tube will be
   (a) M
   (b) 2M
   (c) M/2
   (d) 4M
3. A number of small drops of mercury coalesce adiabatically to form a single drop. The temperature of drop
   (a) Increases
   (b) Is infinite
   (c) Remains unchanged
   (d) May decrease or increase depending upon size
4. When a soap bubble is charged
   (a) It contracts
   (b) It expands
   (c) It does not undergo any change in size
   (d) None of these
5. A liquid is kept in a glass vessel. If the liquid solid adhesive force between the liquid and the vessel is very weak as compared to the cohesive force in the liquid, then the shape of the liquid surface near the solid should be
   (a) Concave
   (b) Convex
   (c) Horizontal
   (d) Almost vertical
6. A capillary tube is placed vertically in a liquid. If the cohesive force is less than the adhesive force, then
   (a) The meniscus will be convex upwards
   (b) The liquid will wet the solid
   (c) The angle of contact will be obtuse
   (d) The liquid will drip in the capillary tube
7. When there are no external forces, the shape of a liquid drop is determined by
   (a) Surface tension of the liquid
   (b) Density of liquid
   (c) Viscosity of liquid
   (d) Temperature of air only
8. Water can rise up to a height of 12 cm in a capillary tube. If the tube is lowered to keep only 9 cm above the water level then the water at the upper end of the capillary will
   (a) Overflow
   (b) From a convex surface
   (c) From a flat surface
   (d) From a concave surface
9. Rain drops are spherical in shape because of
   (a) Surface tension
   (b) Capillary
   (c) Downward motion
   (d) Acceleration due to gravity
10. When the angle of contact between a solid and a liquid is 90°, then
   (a) Cohesive force > Adhesive force
   (b) Cohesive force < Adhesive force
   (c) Cohesive force = Adhesive force
   (d) Cohesive force >> Adhesive force

Very Short:

1. State the law of floatation?
2. The blood pressure of humans is greater at the feet than at the brain?
3. Define surface tension?
4. Define surface tension?
5. Oil is sprinkled on sea waves to calm them. Why?
6. Oil is sprinkled on sea waves to calm them. Why?
7. The diameter of ball A is half that of ball B. What will be their ratio of their terminal velocities in water?
8. Define viscosity?
9. Give two areas where Bernoulli's theorem is applied?
10. What is conserved in Bernoulli's theorem?

Short Questions:

1. A glass bulb is balanced by an iron weight in an extremely sensitive beam balance covered by a bell jar. What shall happen when the bell jar is evacuated?
2. It is easier to swim in seawater than in river water. Why?
3. Does Archimedes' Principle hold in a vessel in free fall or in a satellite moving in a circular orbit?
4. A block of wood floats in a pan of water in an elevator. When the elevator starts from rest and accelerates downward, does the 1 block floats higher above the water surface? What happens when the elevator accelerates upward?
5. The thrust on a human being due to atmospheric pressure is about 15 tons. How human being can withstand such an enormous thrust while it is impossible for him to carry a load of even one ton?
6. Why are sleepers used below the rails? Explain.
7. The passengers are advised to remove the ink from their f pens while going up in an airplane. Explain why?
8. Why a sinking ship often turns over as it becomes immersed in water?
9. Explain why a balloon filled with helium does not rise in the air indefinitely but halts after a certain height?
10. A light ball can remain suspended in a vertical jet of water flow?
11. In the case of an emergency, a vacuum brake is used to stop the train. How does this brake work?
12. Why dust generally settles down in a closed room?
13. How will the rise of a liquid be affected if the top of the capillary tube is closed?
14. What are buoyancy and the center of buoyancy?
15. Under what conditions:

(a) Centre of buoyancy coincides with the center of gravity?

(b) The center of buoyancy does not coincide with the center of gravity?

Long Questions:

1. A copper cube of mass 0.50 kg is weighed in water ($\rho = 10^3$ kg m^{-3}). The mass comes out to be 0.40 kg. Is the cube hollow or solid? Given density of copper = 8.96×10^3 kg m^{-3}.
2. A piece of pure gold ($\rho = 9.3$ g cm^{-3}) is suspected to be hollow. It weighs 38.250 g in air and 33.865 in water. Calculate the volume of the hollow portion in gold, if any.
3. A glass plate of length 20 cm, breadth 4 cm, and thickness 0.4 cm weights 40 g in air. If it is held vertically with the long side horizontal and the plate half breadth immersed in water, what will be its apparent weight, the surface tension of water = 70 dyne cm^{-1}.
4. What is the work done in blowing a soap bubble of diameter 0.07 m?
5. If 3.6960×10^3 J of work is done to blow it further, find the new radius. Surface tension of soap solution is 0.04 Nm1.

Assertion Reason Questions:

1. ***Directions:***

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false.

Assertion: It is easier to spray water in which some soap is dissolved.

Reason: Soap is easier to spread.

2. ***Directions:***

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false.

Assertion: The angle of contact of a liquid decrease with increase in temperature.

Reason: With increase in temperature, the surface tension of liquid increase.

Answer Key

Multiple Choice Answers-

1. **Answer:** (a) Capillarity
2. **Answer:** (b) 2M
3. **Answer:** (d) May decrease or increase depending upon size
4. **Answer:** (b) It expands
5. **Answer:** (b) Convex
6. **Answer:** (b) The liquid will wet the solid
7. **Answer:** (a) Surface tension of the liquid
8. **Answer:** (c) From a flat surface
9. **Answer:** (a) Surface tension
10. **Answer:** (c) Cohesive force = Adhesive force

Very Short Answers:

1. **Answer:** Law of floatation states that a body will float in a liquid, if weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.
2. **Answer:** The height of the blood column in the human body is more at the feet than at the brain as since pressure is directly dependent on height of the column, so pressure is more at feet than at the brain.
3. **Answer:** It is measured as the force acting on a unit length of a line imagined to be drawn tangentially anywhere on the free surface of the liquid at rest.
4. **Answer:** Archimedes's Principle will not hold in a vessel in free – fall as in this case, acceleration due to gravity is zero and hence buoyant force will not exist.
5. **Answer:** Since the surface tension of sea-water without oil is greater than the oily water, therefore the water without oil pulls the oily water against the direction of breeze, and sea waves calm down.
6. **Answer:** Since the cohesive forces between the oil molecules are less than the adhesive force between the oil molecules and the drop of oil spreads out and reverse holds for drop of water.
7. **Answer:** The terminal velocity is directly proportional to the square of radius of the ball, therefore the ratio of terminal velocities will be 1:4.
8. **Answer:** Viscosity is the property of a fluid by virtue of which an internal frictional force comes into play when the fluid is in motion and opposes the relative motion of its different layers.
9. **Answer:** Bernoulli's theorem is applied in atomizer and in lift of an aero plane wing.
10. **Answer:** According to Bernoulli's theorem, for an incompressible non – Viscous liquid (fluid) undergoing steady flow the total energy of liquid at all points is constant.

Short Questions Answers:

1. **Answer:** The upthrust on the bulb is larger than the upthrust on the iron weight. When the bell jar has evacuated the upthrust on both the bulb and the iron weight become zero. Clearly, the bulb is affected more than the iron weight. Thus the pan containing the bulb shall go down.
2. **Answer:** Due to the presence of salt, the density of seawater is more than that of river water. Hence seawater offers more upthrust as compared to river water. Therefore a lesser portion of our body is submerged in, seawater as compared to river water. Hence it is easier to swim in sea-water than in river water.
3. **Answer:** A vessel in free fall or in a satellite moving in a circular orbit is in the state of weightlessness. It means the value of 'g' is zero. Thus the weight of the vessel and upthrust will be zero. Hence Archimedes' Principle does not hold good.
4. **Answer:** When the elevator accelerates downward, the weight of the block of wood decreases. Hence it will float higher above the water's surface.
5. **Answer:** There is a large number of pores and openings on the skin of a body. Through these openings, air goes within the system and there is free communication between the inside and the outside. The presence of; the air inside the body counterbalances the pressure outside.
6. **Answer:** When sleepers are placed below the rails, the area of the cross- p section is increased. We know that P $= \frac{F}{A}$, so when the train runs on the rails, the pressure exerted on the ground due

to the weight of the train is small because of a large area of cross-section of the sleeper. Hence the ground will not yield under the weight of the train.

7. **Answer:** With the increase in height, the atmospheric pressure decreases. The ink in the pen is filled at the atmospheric pressure on the surface of the earth. So as the plane rises up, the pressure decreases \ and the ink will flow out of the pen from higher pressure to the low 'pressure region. This will spoil the clothes of passengers.

8. **Answer:** When the ship is floating, the metacenter of the ship is above the center of gravity. While sinking the ship takes in water and as a result, the center of gravity is raised above the metacenter. The ship turns over due to the couple formed by the weight and the buoyant force.

9. **Answer:** The balloon initially rises in the air because the weight of the displaced air i.e> upthrust is greater than the weight of the helium and the balloon. Since the density of air decreases with height, therefore, the balloon halts at a particular height where the density of air is such that the weight of air displaced is just equal to the weight of helium gas and the balloon. Hence the net force acting on the balloon is zero and the balloon stops rising.

10. **Answer:** The region where the ball and the vertical jet of water are in contact is a region of low pressure because of higher velocity. The pressure on the other side of the ball is larger. Due, to the pressure difference, the ball remains suspended.

11. **Answer:** Steam at high pressure is made to enter the cylinder of the vacuum brake. Due to high velocity, pressure decreases in accordance with Bernoulli's principle. Due to this decrease in pressure, the piston gets lifted. Hence the brake gets lifted.

12. **Answer:** Dust particles may be regarded as tiny spheres. They acquire terminal velocity after having fallen through some distance in the air. Since the terminal velocity varies directly as the square of the radius therefore the terminal velocity of dust particles is very small. So they settle down gradually.

13. **Answer:** The air trapped between the meniscus of the liquid and the closed end of the tube will be compressed. The compressed air shall oppose the rise of liquid in the tube.

14. **Answer:** 1. The upward thrust acting on the body immersed in a liquid is called buoyancy or buoyant force.

2. The center of buoyancy is the center of gravity of the displaced liquid by the body when immersed in a liquid.

15. **Answer:** (a) For a solid body of uniform density, the center of gravity coincides with the center of buoyancy.

(b) For a solid body having different densities over different parts, its center of gravity does not coincide with the center of buoyancy.

Long Questions Answers:

1. **Answer:** Let V be the volume of the cube, then according to Archimedes' principle,

Loss of weight in water = weight of water displaced .... (i)

Here, mass in air, ma = 0.5 kg

mass in water, mw = 0.4 kg .... (ii)

ρ of water = 10^3 kg m^3.

∴ From (i) and (ii), we get

$$(0.5 - 0.4)\, g = V \times 10^3 \times g$$

or $$V = \frac{0.1}{10^3} = 10^{-4} m^{-3}$$

Now density of cube $\frac{m_a}{V} = \frac{0.5}{10^{-4}} kg\, m^{-3}$

= 5 × 103 kg m−3

which is less than the density of copper (8.96 × 10^3 kg m^{-3}). So the cube must be hollow.

2. **Answer:** Density of pure gold, ρ = 9.3 g cm^3,

mass of gold piece, M = 3 8.250 g

∴ volume of the gold piece, V = $\frac{M}{P} = \frac{38.250}{9.3}$

= 4.113 cm^3

Also mass of gold piece in water

m' = 33.865 g

∴ apparent loss in mass of the gold piece in water = (M – m')

= (38.250 – 33.865)g

= 4.3.85 g

ρ_{water} = 1 g Cm^{-3}

∴ volume of displaced water = $\frac{m}{\rho} = \frac{4.385}{1} cm^{-3}$

= 4.385 cm^{-3}

∴ volume of the hollow portion in the gold piece

= 4.385 – 4.113

= 0.272 cm^{-3}.

3. **Answer:** Here, l = 20 m, b = 4 cm , t = 0.4 cm, T = 70 dyne cm-1

Following three forces are acting on the plate:

1. Weight of the plate, W = 40 grand actings vertically downward.
2. Force due to surface tension acting vertically downward.

If F be the force due to surface tension, then

F = T × length in contact with water

= 70 [2 (length + thickness)]

= 70 [2 (20 + 0.4)]

= 70 × (40.8) = 2856 dynes

$= \frac{2856}{980}$ gf = 2,9143 gf.

3. Upthrust, U = Vρg

Now volume of water displaced $= l \times \frac{b}{2} \times t$

$= 20 \times \frac{4}{2} \times 0.4$

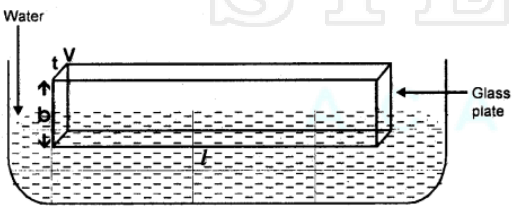
= 16 cm^3

r = 1 gm cm^{-3}

g = 980 cm s^{-2}

U = 16 × 1 × 980 dynes

$= \frac{16 \times 980}{980}$ gf = 16 gf



∴ Net weight = W + F – U

= 40 + 2.9143 – 16

= 26.9143 gf

4. **Answer:** Here, initial radius of soap bubble, r_1 = 0

Final radius of soap bubble, r_2 = 0.035 m (∵ D_2 = 0.07m)

Increase in surface area of soap bubble

$= 2(4\pi r_2^2 - 4\pi r_1^2)$

$= 2 \times 4\pi\left[(0.035)^2 - 0\right]$

$= 8\pi \times 0.1225 \times 10^{-2}$

$= 0.0308\ m^2$

surface tension of soap solution = T = 0.04 Nm^{-1}

∴ work done to blow soap bubble = increase in area × T

= 0.0308 × 0.04

= 1.232 × 10^{-3}

5. **Answer:** Let r be the new radius =?

∴ $3.6960 \times 10^{-3} = 2\left[4\pi\left(r^2 - r_2^2\right)\right] \times T$

$= 2 \times 4\pi\left[r^2 - (0.035)^2\right] \times 0.04$

or $r^2 = 1225 \times 10^{-6} + \frac{3.69 \times 10^{-3}}{8\pi \times 0.04}$

$= 1225 \times 10^{-3} + 3.67 \times 10^{-3}$

$= 4.875 \times 10^{-3} m$

∴ r = 0.07 m

Assertion Rason Answer:

1. (c) If assertion is true but reason is false.

Explanation:

When a liquid is sprayed, the surface area of the liquid increases. Therefore, work has to be done in spraying the liquid, which is directly proportional to the surface tension. Because on adding soap, surface tension of water decreases, the spraying of water becomes easy.

2. (c) If assertion is true but reason is false.

Explanation:

With increase in temperature surface tension of the liquid decreases and angle of contact also decreases.

Case Study Questions-

1. **Surface Tension:** The property due to which the free surface of liquid tends to have the minimum surface area and behaves like a stretched membrane is called surface tension. It is a force per unit length acting in the plane of interface between the liquid and the bounding surface i.e., S = F/L, where F = force acting on either side of an imaginary line on the surface and L = length of the imaginary line. Surface tension decreases with rise in temperature. Highly soluble impurities increase surface tension and sparingly soluble impurities decrease surface tension.

i. **The excess pressure inside a soap bubble is three times than excess pressure inside a second soap bubble, then the ratio of their surface area is**
   a. 9 : 1
   b. 1 : 3
   c. 1 : 9
   d. 3 : 1

ii. **Which of the following statements is not true about surface tension?**
   a. A small liquid drop takes spherical shape due to surface tension.
   b. Surface tension is a vector quantity.
   c. Surface tension of liquid is a molecular phenomenon.
   d. Surface tension of liquid depends on length but not on the area.

iii. **Which of the following statement is not true about angle of contact?**
   a. The value of angle of contact for pure water and glass is zero.
   b. Angle of contact increases with increase in temperature of liquid.
   c. If the angle of contact of a liquid and a solid surface is less than 90°, then the liquid spreads on the surface of solid.
   d. Angle of contact depend upon the inclination of the solid surface to the liquid surface.

iv. **Which of the following statements is correct?**
   a. Viscosity is a vector quantity.
   b. Surface tension is a vector quantity.
   c. Reynolds number is a dimensionless quantity.
   d. Angle of contact is a vector quantity

v. **A liquid does not wet the solid surface if the angle of contact is**
   a. 0°
   b. equal to 45°
   c. equal to 90°
   d. greater than 90°

2. A system is said to be isolated if no exchange or transfer of heat occurs between the system and its surroundings. When different parts of an isolated system are at different temperature a quantity of heat transfers from the part at higher temperature to the part at lower temperature. The heat lost by the part at higher temperature is equal to the heat gained by the part at lower temperature. Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings. A device in which heat measurement can be done is called a calorimeter. It consists of a metallic vessel and stirrer of the same material, like copper or aluminium. The vessel is kept inside a wooden jacket, which contains heat insulating material. Matter normally exists in three states: solid, liquid and gas. A transition from one of these states to another is called a change of state. Two common changes of states are solid to liquid and liquid to gas (and, vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings. The change of state from solid to liquid is called melting and from liquid to solid is called fusion. It is observed that the temperature remains constant until the entire amount of the solid substance melts. That is, both the solid and the liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid. The temperature at which the solid and the liquid states of the substance is in thermal equilibrium with each other is called its melting point. The change of state from liquid to vapour (or gas) is called vaporisation. It is observed that the temperature remains constant until the entire amount of the liquid is converted into vapour. That is, both the liquid and vapour states of the substance coexist in thermal equilibrium, during the change of state from liquid to vapour. The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point. The change from solid state to vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime. Dry ice (solid CO2) sublimes, so also iodine. During sublimation both the solid and vapour states of a substance coexist in thermal equilibrium.

i. **Device used for measurement of heat is**
   a. Calorimeter

b. Thermometer

c. Both a and b

d. No one of these

ii. **The change of state from solid to liquid is called**

a. Melting

b. Vaporization

c. Sublimation

d. None of these

iii. **Define melting point and boiling point**

iv. **What is sublimation?**

v. **Define fusion process**

Case Study Answer-

1. Answer

i. (c) 1 : 9

ii. (b) Surface tension is a vector quantity.

iii. (d) Angle of contact depend upon the inclination of the solid surface to the liquid surface.

iv. (c) Reynolds number is a dimensionless quantity.

v. (d) greater than 90°

2. Answer

i. (a) Calorimeter

ii. (a) Melting

iii. The change of state from solid to liquid is called melting process and temperature at which conversion of solid into liquid happens is called as melting point. The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point.

iv. The change from solid state directly into vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime.

The change of state from liquid state to solid state is called as fusion process.

Thermal Properties of Matter 11

Introduction

You might have noticed that you feel hotter on a sunny afternoon as compared to a windy night. This is because of the difference in temperatures. Temperature is very high in the afternoon as compared to night. This chapter basically gives us the

Examples: information about thermal properties of matter where we will study about the properties of different substances by virtue of heat / heat transfer.

In simple terms, we can say that when temperature is more heat is more and when temperature is less heat is less.

Hot Sunny day (Temperature is more) and ice-cold water (Temperature is less).

Thermal Properties of Matter

Thermal properties are those properties of a material which is related to its conductivity of heat. In other words, these are the properties which are exhibited by a material when heat is passed through it. Thermal properties come under the broader topic of physical properties of materials.

Thermal properties of a material decide how it reacts when it is subjected to heat fluctuation (excessive heat or very low heat, for example). The major components of thermal properties are:

- Heat capacity
- Thermal Expansion
- Thermal conductivity
- Thermal stress

Heat Capacity

Heat capacity of a material can be defined as the amount of heat required to change the temperature of the material by one degree. The amount of heat is generally expressed in joules or calories and the temperature in Celsius or Kelvin. In order to calculate the heat capacity of materials with a given dimension, Molar heat capacity or Specific heat capacity is used.

Heat capacity can be measured by the following formula:

$$Q = mc\Theta$$

is the amount of heat transferred, is the change in temperature.

Heat Transfer: Thermal Conductivity

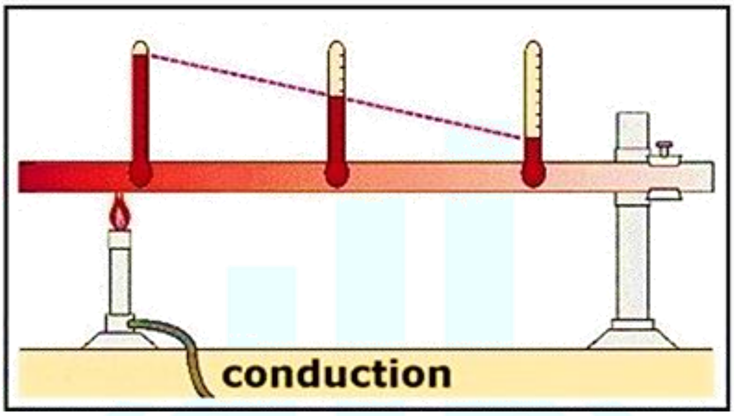
Heat Transfer

Heat is a very curious form of energy. It helps us stay warm, prepare hot and tasty food but its applications far exceed the domestic uses mentioned here. Understanding the properties of heat and heat transfer is the key to many fields of science. Thermodynamics is a massive field that deals only with the flow of heat through a system that is heat transfer through a system. Even nuclear energy uses the heat developed by the atom to create electricity. So it is clear that heat is quite important to us. That makes it imperative for us to take a closer look at heat.

Heat transfer can occur only through three means:

- Conduction
- Convection
- Radiation

Heat Transfer: Conduction



Heat Conduction refers to the transfer of heat between bodies due to physical contact between them. The transfer of heat by conduction actually occurs at a molecular level. Absorption of heat by a body causes the molecules of that body to gain excess energy. What do you do when you're too energetic? You get very jittery and shaky, don't you? You just want to move around to expend this energy. That is exactly what molecules do too.

In the process of gaining energy and vibrating excessively, they bump into their neighbours and transfer a little of its extra energy to them. This extra energy appears in the neighbouring molecules and heats them up too. This is how heat is transferred as long as heat is still being supplied.

Factors Affecting Thermal Conductivity

The rate of thermal conductivity depends on four basic factors:

Temperature Gradient: This is a physical quantity that illustrates to us in which direction and at what rate the temperature changes the most rapidly around a particular location. It basically tells us about the temperature difference between places and the direction of the transfer due to it. It is important to remember that heat always flows from the hottest to the coldest spot. This flow will continue till the temperature difference disappears and a state of thermal equilibrium is reached.

Cross-section and path length are dependent on the physical dimensions of the body. If the size of the body is large, then the heat required to heat it is also larger. With large bodies, we also have to consider the heat loss to the environment. Also, a greater surface area between the hot and the cold body implies a greater rate of heat transfer.

The physical properties of the body play an immense role in thermal conductivity through the body. Not all bodies are blessed with the same thermal behaviour. We measure the rate of transfer of heat through the material using a parameter called the Thermal Conductivity of the material (K). The more the value of K, the more easily and quickly it can conduct heat. The SI Unit of K is $JS^{-1}m^{-1}K^{-1}$. The thermal conductivity of a material is measured on a scale. This scale has two extremes; on the end of high thermal conductivity we have Silver with a perfect score of a 100 in heat conduction. On the other end of the scale, we have a vacuum, which is absent of molecules and hence is incapable of conducting heat. Everything else is ranked between this, for example, Copper (92), iron (11), water (0.1), Air (0.006), and Wood (0.03). Materials that are poor conductors of heat are called insulators.

Thermal Expansion

When heat is passed through a material, its shape changes. Generally, a material expands when heated. This property of a material is called Thermal Expansion. There can be a change in the area, volume and shape of the material. For example, railway tracks often expand and as a result, get misshapen due to extreme heat.

Thermal conductivity

It is the property of a material to conduct heat through itself. Materials with high thermal conductivity will conduct more heat than the ones with low conductivity. Some materials do not conduct heat at all because of the insulating properties of materials.

Thermal stress

The stress experienced by a body due to either thermal expansion or contraction is called thermal stress. It can be potentially destructive in nature as it can make the material explode.

Measurement of Temperature

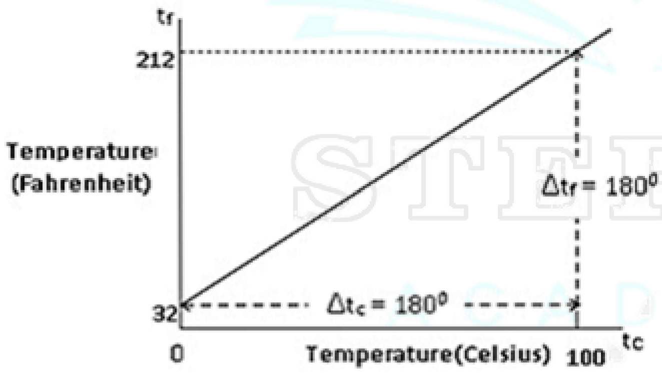
Temperature is measured with the help of thermometer. Mercury and Alcohol are commonly used liquids in the liquid-in-glass thermometers.

To construct a thermometer two fixed points are to be chosen as a reference points. These fixed points are known as freezing (ice point) and boiling point (steam point). The water freezes and boils at these two points under standard pressure.

The ice and steam point in Fahrenheit Temperature scale are 32°F and 212 °F resp. It has 180 equal intervals between two reference points.

On Celsius Scale values are 0°C and 100°C for ice and steam point resp. It has 100 equal intervals between two reference points.

Graphically the relation between the temperature in Celsius and in Fahrenheit is given by the following graph:



Mercury-in-Thermometer

And whose equation is:

$$\frac{t_f - 32}{180} = \frac{t_c}{100}$$

Where t_f = Fahrenheit temperature

t_c = Celsius temperature

Ideal-gas Equation and Absolute Temperature

A thermometer that uses any gas, however, gives the same readings regardless of which gas is used because all gases have same expansion at low temperature.

Variables that describe the behaviour of gas are:

Quantity (mass), Pressure, Volume

Temperature i.e. (P,V,T) where (T = t + 273.15; t is the temperature in °C)

Gases which have low density obey certain laws:

1.Boyle's Law– PV = constant (when temperature T is constant)

2.Charles' Law- V/T = constant (when pressure P is constant)

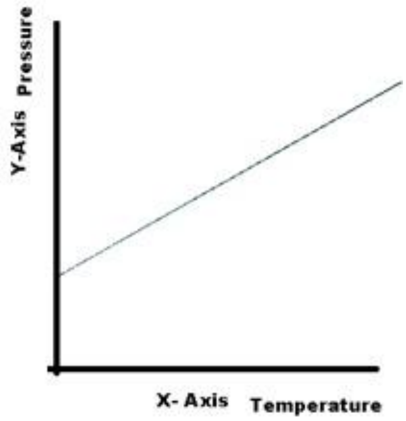
If combine both the above laws the equation becomes PV = RT where R is called universal gas constant and its value = 8.31 J mol^{-1} K^{-1}.

PV = RT is the ideal gas equation which is applicable only at low temperature.

For any quantity of dilute gas,

PV = μRT where μ, is the number of moles in the sample of gas.

In a constant volume gas thermometer temperature varies with respect to pressure. Temperature changes linearly with increase in pressure.



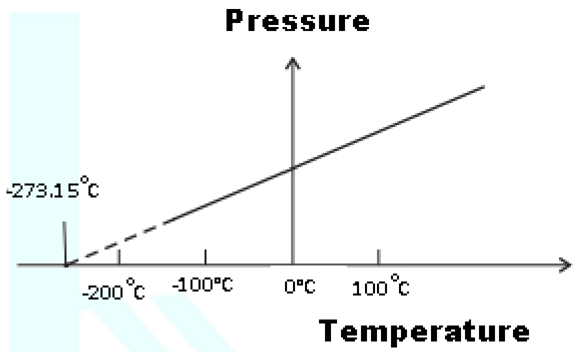
Absolute Zero

Absolute Zero is defined as minimum absolute temperature of an ideal gas.

If we plot pressure versus temperature, we get a straight line and if we extend the line backwards to the x-axis as shown in the graph below. The minimum temperature is found to be 273.15 °C (experimentally) and this value is known as absolute zero.

The relation between the temperature in kelvin and in Celsius scale is given by

$T = t_C + 273.15$



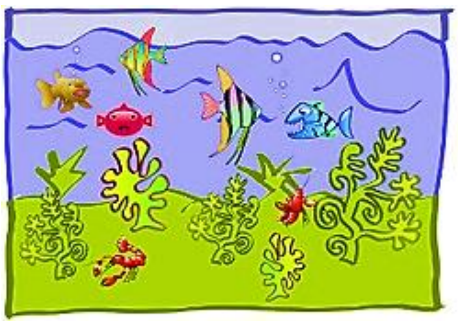
Thermal Expansion

Thermal expansion is the phenomenon of increase in dimensions of a body due to increase in its temperature.

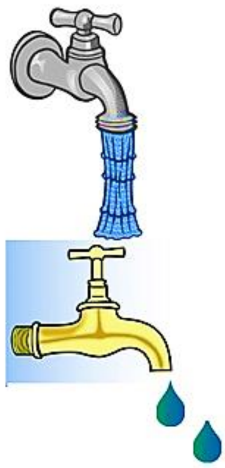
Examples of Thermal Expansion

The water is cold at the top of the lake because it expands and becomes less dense. So when this water freezes it insulates the water below it from the outside which means cold air is like a blanket. It is because of this property many fish can survive in the winter.

(1)



(2)

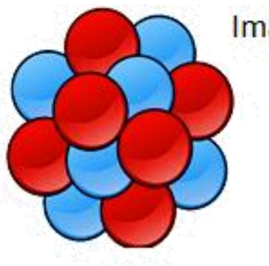


Even though the top layer of water is frozen as we can see in the image (1), the plant and animal life is not getting affected as shown in the image (2).

As soon as we turn on a hot water tap, the water comes very fast as water is still cold. But as soon as hot water starts coming , the flow of water becomes less and in some cases it stops. This is because the hot water heats the metal valve inside the tap which expands to block off any more flow of water.

Reason why Thermal Expansion happens is:

When any object is heated particles start moving in random motion and thus average distance between the molecules increases and a result the object appears to be expanded when heated. As we can see the picture below atoms is tightly packed but when we apply heat they will start moving in random motion.



As we can see in the Image (a) molecules are very tightly packed but when heated the molecules start moving apart in random motion, which can be seen in Image (b).

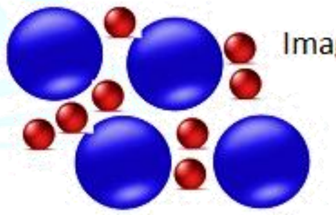
When an object is cooled it contracts which is referred as negative thermal expansion.

Types of Thermal Expansion

Linear Expansion: The expansion in length

Area Expansion: The expansion in area

Volume Expansion: The expansion in volume



Linear Expansion

Linear Expansion means expansion in length due to increase in temperature. Linear expansion means fractional change in length i.e. how the length is changing with respect to original length.

As we can see from above images the length has been increased from l to l + Δl.

Coefficient of Linear Expansion is a parameter which tells us how the size of the object changes with change in temperature. It is defined as degree of linear expansion divided by the change in temperature.

If the solid is in the form of long rod, then for small change in temperature, ΔT, the fractional change in length, Δl/l, is directly proportional to ΔT.

Mathematically can be written as:

$$\frac{\Delta l}{l} = \alpha_1 \Delta T$$

LinearExpansion $\frac{A}{A}\alpha_l T$

Where α_l = the coefficient of linear expansion

It is denoted by α_l

It is characteristic of the material of the rod. It varies for different substance.

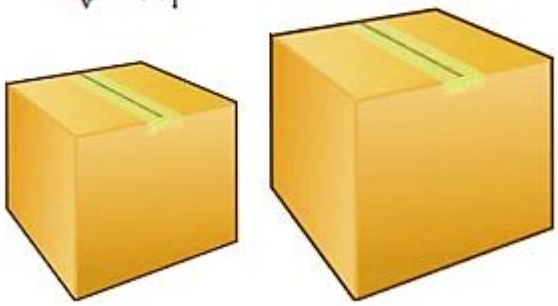
Example

Automatic hot water kettle switches off on its own when the water boils.

Metals expand more and have higher value of coefficient of linear expansion.

Relation between α_v and α_l

Relation between coefficient of linear expansion and coefficient of volume expansion =



To derive the above relation consider a block of cube initially its length is l ,suppose temperature is increased T + ΔT as a result length will also increase from (l+Δl)

Then $\boldsymbol{\alpha_l} = \frac{\left(\frac{\Delta l}{l}\right)}{\Delta T}$

Therefore, $\alpha_l l \Delta T = \Delta l$

Also as the temperature increases by ΔT the volume also increases V + ΔV

Where ΔV = change in volume which we can write as

$\Delta V = (l + \Delta l)^3 - l^3$

By solving we get $\Delta V = 3l^2\Delta l$ (we are neglecting $(\Delta l)^2$ and $(\Delta l)^3$ as they are very small as to compared to l.

Therefore, $\boldsymbol{\Delta V} = \frac{(3V\,\Delta l)}{l}$

$= 3V\alpha_l\Delta T$

Which gives $\alpha_v = 3\alpha_l$ the relation between coefficient of volume expansion and coefficient of linear expansion.

Heat Capacity

The change in temperature of a substance, when a given quantity of heat is absorbed or rejected by substance is characterised by a quantity called the heat capacity of that substance.

It is denoted by S.

It is given as $S = \Delta Q / \Delta T$

Where ΔQ = amount of heat supplied to the substance and T to T + ΔT change in its temperature.

Specific heat capacity:

Every substance has a unique value for the amount of heat absorbed or rejected to change the temperature of unit mass of it by one unit. This quantity is referred to as the specific heat capacity of the substance.

Mathematically can be written as:

$$S = \frac{\Delta Q}{\Delta T}$$

Where ΔQ = amount of heat absorbed or rejected by a substance

ΔT = temperature change

Specific Heat Capacity

Specific heat is defined as the amount of heat per unit mass absorbed or rejected by the substance to change its temperature by one.

Mathematically can be written as:

$$s = \frac{S}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Where

ΔQ = amount of heat absorbed or rejected by a substance

m = mass

ΔT = temperature change

It depends on the nature of the substance and its temperature.

The SI unit of specific heat capacity is $J\ kg^{-1}\ K^{-1}$.

Molar specific heat capacity:

Heat capacity per mole of the substance is the defined as the amount of heat (in moles) absorbed or rejected (instead of mass m in kg) by the substance to change its temperature by one unit.

Mathematically can be written as:

$$C = \frac{S}{\mu} = \frac{\Delta Q}{\mu} \Delta T$$

Where,

μ= amount of substance in moles

C = molar specific heat capacity of the substance.

ΔQ = amount of heat absorbed or rejected by a substance.

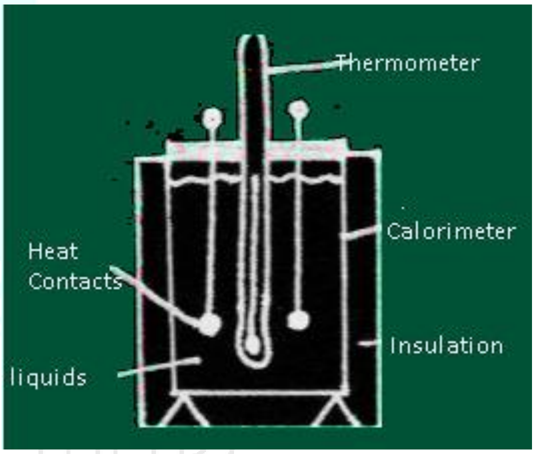
ΔT = temperature change

It depends on the nature of the substance and its temperature.

The SI unit of molar specific heat capacity is $Jmol^{-1}$ K^{-1}

Calorimetry

Calorimetry is made up of 2 words:



Calorie which means heat and metry means measurement. Therefore, Calorimetry means measurement of heat.

Calorimetry is defined as heat transfers from a body at a higher temperature to a body at a lower temperature provided there is no loss of heat to the atmosphere.

Principle of Calorimetry is heat lost by one body is equal to the heat gained by another body.

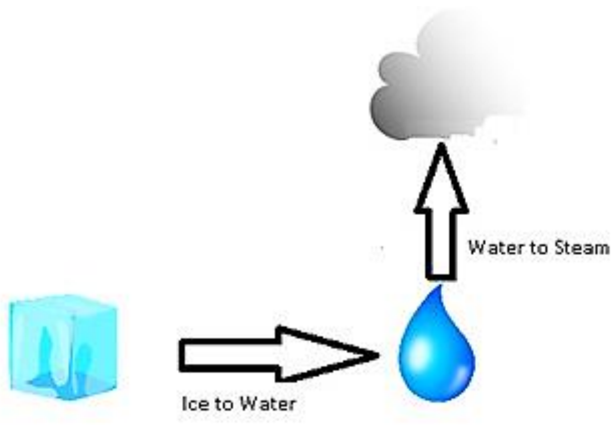
The Device which measures Calorimetry is known as Calorimeter.

Description of Calorimeter

A calorimeter consists of metallic vessel and a stirrer both are made of same material (copper or aluminum) and the vessel is kept in a wooden jacket so that there is no heat loss. A mercury thermometer can be inserted through a small opening in the outer jacket.

Change of State

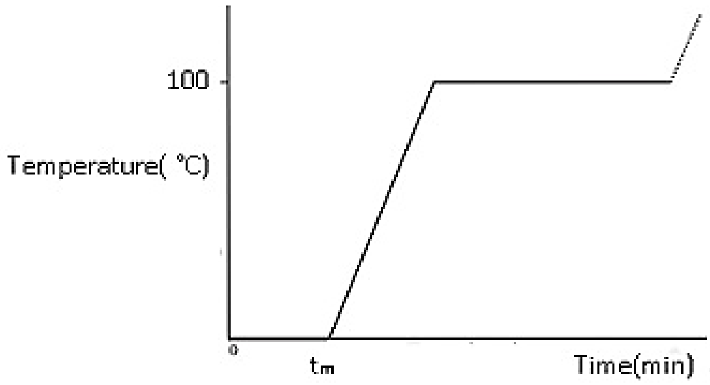
The transition from either solid to liquid or gas and gas to either liquid or solid is termed as change of state.



We can from the above image solid (ice) changes to liquid (water) and liquid changes to vapour (gas).

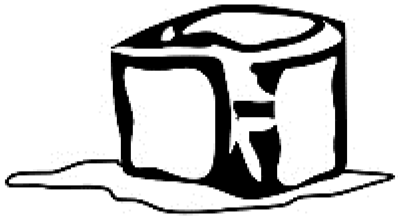
Change from solid (ice) to liquid (water) is known as Melting.

Change from liquid (water) to solid (ice) is known as Fusion.



Thermal Equilibrium: At this state there is no loss or gain of heat takes place.

The temperature at which the solid and the liquid states of the substance are in thermal equilibrium with each other is called its melting point.



It is depends on the

Substance

Pressure.

The melting point of a substance at standard atmospheric pressure is called its normal melting point.

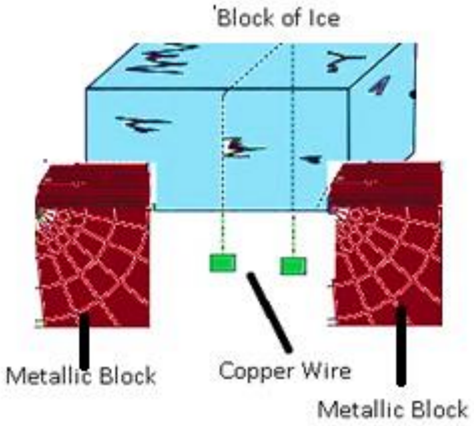
Regelation:

Regelation can be defined as phenomenon in which the freezing point of water is lowered by the application of pressure.

Example:

Cause of regelation:

If we have a block of ice and a copper wire pulled by two masses if we will observe that copper wire can pass through ice block this is because copper is good conductor of heat so as it passes through the ice it gets refreeze as the copper will generate heat and this heat will pass quickly to the ice below and it starts melting because there is increase in pressure which lowers temperature as a result the wire will move through the ice. This happens because of regelation.



The image above explains how a copper wire can pass through the block of ice.

Vaporisation: Transition from liquid to vapour.

The change of state from liquid to vapour (or gas) is called vaporisation.

The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point.

Boiling point at standard atmospheric pressure is known as normal boiling point.

It depends on nature of substance & pressure

It increases with increase in pressure and vice versa.

Example: As altitude increases, the density of the air becomes thinner, and thus exerts less pressure. At high altitudes, external pressure on water is therefore decreased and will hence take less energy to break the water. If less energy is required it means less heat and less temperature which means that water will boil at a lower temperature.

Sublimation: Transition from Solid to Vapour.

During the sublimation (solid changes to vapour without going through liquid state) process both the solid and vapour states of a substance coexist in thermal equilibrium.

Example:

Dry ice (solid CO_2) sublimes iodine.

Naphthalene balls sublimes to gaseous state.

Top Formulae

Ideal gas equation connecting pressure (P), volume (V) and absolute temperature (T)	$PV = \mu RT$, where μ is the number of moles and R is the universal gas constant.
Relation between temperatures on various scales	If T_C, T_F and T_K are temperature values of a body on the Celsius scale, Fahrenheit scale and Kelvin scale, then $\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$
Relation between critical temperature and pressure at triple point	If the triple point of water is chosen as the reference point, then $T_K = 273.16\left(\frac{P}{P_{tr}}\right)$, where P is the pressure at unknown temperature T and P_{tr} is the pressure at triple point.
Coefficients of expansion	(i) Coefficient of linear expansion $\alpha = \frac{\Delta L}{L(\Delta T)}$ (ii) Coefficient of area expansion $\beta = \frac{\Delta S}{S(\Delta T)}$ (iii) Coefficient of volume expansion $\gamma = \frac{\Delta V}{V(\Delta V)}$
Relation between coefficients of expansion	$\beta = 2\alpha$; $\gamma = 3\alpha$
Variation of density with temperature	$\rho = \rho_0 (1 - \gamma \Delta T)$
Specific heat capacity of a substance	$s = \frac{1}{m}\frac{\Delta Q}{\Delta T}$ where m is the mass of the substance and ΔQ is the heat required to change its temperature by ΔT.
Molar specific heat capacity	$C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$ where μ is the number of moles of the substance.

Change of heat	$\Delta Q = m\, s\, \Delta T$ where s is the specific heat of the substance.
Molar specific heat of substance	$C = m \times s$
Method of mixtures	Heat gained = Heat lost i.e. mass $\times$ specific heat $\times$ rise in temperature = mass $\times$ specific heat $\times$ fall in temperature
Change of state	$\Delta Q = mL$ where L is the latent heat of the substance.
Relation between specific heat capacity at constant volume and pressure	$C_P - C_v = \frac{R}{J}$, where $R = \frac{PV}{T}$ = gas constant for one gram mole of the gas.
Specific heat capacities of mono and polyatomic gases	For monatomic gases, $C_v = \frac{3}{2}R$; $C_p = \frac{5}{2}R$ For diatomic gases, $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$ For triatomic gases (non-linear molecule), $C_v = 3\,R$, $C_p = 4\,R$ For triatomic gases (linear molecule) $C_v = \frac{7}{2}R$, $C_p = \frac{9}{2}R$
Rate of conduction of heat	$\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{}$, where $\frac{\Delta T}{\Delta x}$ = temperature gradient = rate of fall of temperature with distance, A = area of the hot surface, K = coefficient of thermal conductivity.

Latent heat	If conducted heat is used in changing the state of m gram of the substance, then $\Delta Q = mL = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$, where L is the latent heat of the substance.
Specific heat: Change in temperature	If the conducted heat is used in increasing the temperature of the substance through range $\Delta\theta$, then $\Delta Q = sm\Delta\theta = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$

Class : 11th Physics

Chapter- 11 : Thermal Properties of Matter

Thermal Properties of Matter

Heat

A form of energy, transferred between two systems by virtue of temperature difference.

Mode of heat transfer

(i)Conduction : heat transfer through molecular collisions without any actual motion of matter.

(ii)Convection : heat transfer by actual motion of matter within the medium. Land breeze, sea breeze, trade winds based on natural convection are some examples.

(iii)Radiation : method of heat transfer requiring no material medium.

Specific Heat

Specific heat capacity $C = \frac{s}{m} = \frac{Q}{m\Delta T}$

Heat capacity $s = \frac{\Delta Q}{\Delta T}$

Molar specific heat capacity, $c = \frac{s}{n} = \frac{\Delta Q}{n\Delta T}$

Latent Heat

Heat required to change the state of unit mass substance without changing its temperature, $L = \frac{Q}{m}$

For water, latent heat of fusion, $L_f = 3.33 \times 10^5 \text{ J/kg}$

Latent heat of vapourisation, $L_v = 22.6 \times 10^5 \text{ J/kg}$

Thermal Laws

Wien displacement

$\lambda_m \propto \frac{1}{T}$ or $\lambda_m = \frac{b}{T}$

$b\text{(wine's constant)} = 2.9 \times 10^{-3} \text{ mK}$

Newton's law of cooling

For small temperature difference between a body and its surroundings, the loss of heat is given by $-\frac{dQ}{dt} = k(T_2 - T_1)$

Stefan's Boltzmann law

$E \propto T^4$; $E = \sigma T^4$

$\sigma = 5.672 \text{ Js}^{-1}\text{m}^{2}\text{k}^{4}$

For a perfectly black body

$E = \sigma(T^4 - T_0^4)$

Kirchhoff's Law

At any given temperature $\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant}$

Temperature

Degree of hotness or coldness of a body measuring device = Thermometer

Relation among different temperature scales

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_K - 273.15}{373.15 - 273.15} = \frac{R - 0}{80 - 0}$$

Thermal Conductivity

$(K) = \frac{Qx}{A(T_2 - T_1)t}$

Principle of calorimetry

Heat lost = Heat Gain

Thermal Expansion

Increase in dimensions due to increase in temperature

Thermal Stress

$\frac{\Delta F}{A} = Y\left(\frac{\Delta l}{l}\right)$, Y = Young's modulus

Types (In solids)

Linear expansion

Increase of length of a solid on heating. Coefficient of linear expansion

$$\alpha = \frac{\Delta l}{l_0 \Delta T} = \frac{L - l_0}{l_0(T - T_0)}$$

Superficial or area Expansion

Increase in area of a solid on heating. Coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A\Delta T} = \frac{dA}{A\,dT} = \frac{A - A_0}{A_0(T - T_0)}$$

Relation

$\gamma = 2\beta = 3a$

or

$a:b:\gamma = 1:2:3$

Cubical or volume expansion

Increase in volume of a solid on heating. Coefficient of cubical expansion.

$$\gamma = \frac{\Delta V}{V_0 \Delta T} = \frac{dV}{V_0 dT} = \frac{V - V_0}{V_0(T - T_0)}$$

Important Questions

Multiple Choice Questions-

1. Two stars A and B radiate maximum energy at 3600°A and 3600°A respectively. Then the ratio of absolute temperatures of A and B is
   (a) 256 : 81
   (b) 81 : 256
   (c) 3 : 4
   (d) 4 : 3
2. Which of the following will radiate heat to large extent?
   (a) Rough surface
   (b) Polished surface
   (c) Black rough surface
   (d) Black polished surface
3. Two spheres made of same material have radii in the ratio 2 : 1. if both the spheres are at same temperature, then what is the ratio of heat radiation energy emitted per second by them?
   (a) 1 : 4
   (b) 4 : 1
   (c) 3 : 4
   (d) 4 : 3
4. The earth intercepts approximately one billionth of the power radiated by the sun. if the surface temperature of the sun were to drop by a factor of 2, the average radiant energy incident on earth per second would reduce by factor of
   (a) 2
   (b) 4
   (c) 8
   (d) 16
5. A bucket full of hot water is kept in a room and it cools from 75°C to 70°C in t1 minutes from 70°C to 65°C in t2 minutes and from 65°C to 60°C in t3 minutes; then
   (a) t1 – t2 = t3
   (b) t1 < t2 < t3
   (c) t1 > t2 > t3
   (d) t1 < t2 > t3
6. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 3000°K, which of these will cool fastest?
   (a) Sphere
   (b) Cube
   (c) Plate
   (d) None
7. A perfectly black body emits radiation at temperature T^1K. if it sis to radiate 16 times this power, its temperature T^2. will be
   (a) $T^2 = 16\ T^1$
   (b) $T^2 = 8\ T^1$
   (c) $T^1 = 4\ T^1$
   (d) $T^2 = 2\ T^1$
8. Unit of Stefans constant is given by
   (a) $W/\ m\ K^2$
   (b) $W/\ m^2\ K^2$
   (c) $W^2/\ m^2\ K^4$
   (d) $W/\ mK$
9. The good absorber of heat are
   (a) Non-emitter
   (b) Poor-emitter
   (c) Good-emitter
   (d) Highly polished
10. A black body is at a temperature of 500K. it emits energy at a rate which is proportional to
    (a) 500
    (b) (500)2
    (c) (500)3
    (d) (500)4

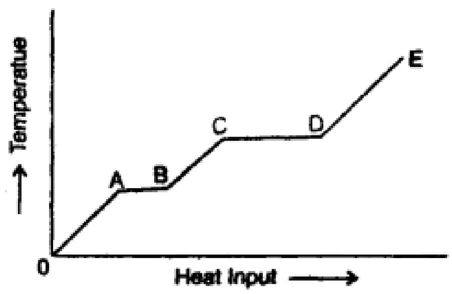
Very Short:

1. The fact that the triple point of a substance is unique is used in modern thermometry. How?
2. Is it possible for a body to have a negative temperature on the Kelvin scale? Why?
3. (a) Why telephone wires are often given snag?
   (b) The temperature of a gas is increased by 8°C. What is the corresponding change on the Kelvin scale?
4. There is a hole in a metal disc. What happens to the size of the hole if the metal disc is heated?

5. Milk is poured into a cup of tea and is mixed with a spoon. Is this an example of a reversible process? Why?
6. The top of a lake is frozen. Air ¡n contact with it is at -15°C. What do you expect the maximum temperature of water in contact with the lower surface ice? What do you expect the maximum temperature of water at the bottom of the lake?
7. How does the heat energy from the sun reaches Earth?
8. Why does not the Earth become as hot as the Sun although it has been receiving heat from the Sun for ages?
9. Why felt rather than air is employed for thermal insulation?
10. What are the three modes of transmission of heat energy from one point to another point?
11. Why a thick glass tumbler cracks when boiling liquid is poured into it?
12. What is the basic principle of a thermometer?
13. Out of mass, radius and volume of a metal ball, which one suffers maximum and minimum expansion on heating? Why?
14. The higher and lower fixed points on a thermometer are separated by 160 mm. If the length of the mercury thread above the lower point is 40 mm, then what is the temperature reading?
15. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which of the two will respond quickly to temperature changes.

Short Questions:

1. Why gas thermometers are more sensitive than mercury thermometers?
2. Why the brake drum of an automobile gets heated up when the automobile moves down a hill at constant speed?
3. A solid is heated at a constant rate. The variation of temperature with heat input is shown in the figure here:



(a) What is represented by AB and CD?
(b) What conclusion would you draw1 if CD = 2AB?
(c) What is represented by the slope of DE?
(d) What conclusion would you draw from the fact that the slope of OA is greater than the slope of BC?

4. Define:
(a) Thermal conduction.
(b) Coefficient of thermal conductivity of a material.
5. On what factors does the amount of heat flowing from the hot face to the cold face depend? How?
6. State Newton's law of cooling and define the cooling curve. What is its importance?
7. Explain why heat is generated continuously in an electric heater but its temperature becomes constant after some time?
8. A woollen blanket keeps our body warm. The same blanket if wrapped around ice would keep ice cold. How do you explain this apparent paradox?
9. A liquid is generally heated from below. Why?
10. If a drop of waterfalls on a very hot iron, it does not evaporate fora long time. Why?
11. On a hot day, a car is left in sunlight with all the windows closed. After some time, it is found that the inside of the car is considerably warmer than the air outside. Explain why?
12. It takes longer to boil water with a flame in a satellite in gravitational field-free space, why? How it will be heated?
13. Find γ for polyatomic gas and hence determine its value for a triatomic gas in which the molecules are linearly arranged.
14. Food in a hot case remains warm for a long time during winter, how?
15. You might have seen beggars sleeping on footpaths or in open in winter with their hands and knees pulled inside. Similarly dogs too curl while sleeping in winter. How does such action help anybody?

Long Questions:

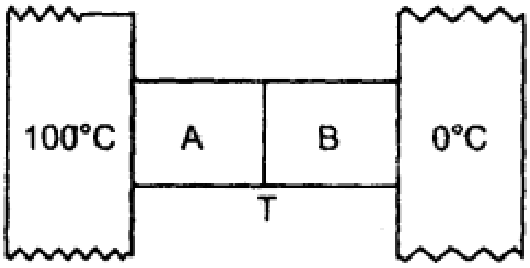
1. Calculate the increase in the temperature of the water which falls from a height of 100 m. Assume

that 90% of the energy due to fall is converted into heat and is retained by water. J = 4.2 J cal^{-1}.

2. A clock with a steel pendulum has a time period of 2s at 20°C. If the temperature of the clock rises to 30°C, what will be the gain or loss per day? The coefficient of linear expansion of steel is 1.2×10^{-5} C^{-1}

3. The thermal conductivity of brick is 1.7 W m^{-1} K^{-1} and that – of cement is 2.9 W m^{-1} K^{-1}. What thickness of cement will have the same insulation as the brick of thickness 20 cm.

4. Two metal cubes A and B of the same size are arranged as shown in the figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficient of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C respectively. After a steady-state is reached, what will be the temperature of the interface?



5. The heat of combustion of ethane gas is 373 Kcal per mole. Assuming that 50% of heat is lost, how many litres of ethane measured at STP must be burnt to convert 50 kg of water at 10°G to steam at 100°C? One mole of gas occupies 22.4 litres at S.T.P. Latent heat (L) of steam = 2.25 × 106 JK^{-1}.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
   (c) If assertion is true but reason is false.
   (d) If the assertion and reason both are false.

   Assertion: Specific heat capacity is the cause of formation of land and sea breeze.

   Reason: The specific heat of water is more than land.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
   (c) If assertion is true but reason is false.
   (d) If the assertion and reason both are false.

   Assertion: A brass disc is just fitted in a hole in a steel plate. The system must be cooled to loosen the disc from the hole.

   Reason: The coefficient of linear expansion for brass is greater than the coefficient of linear expansion for steel.

Answer Key

Multiple Choice Answers-

1. **Answer:** (d) 4 : 3
2. **Answer:** (c) Black rough surface
3. **Answer:** (b) 4 : 1
4. **Answer:** (d) 16
5. **Answer:** (b) t1 < t2 < t3
6. **Answer:** (c) Plate
7. **Answer:** (d) T^2 = 2 T1
8. **Answer:** (b) W/ m^2 K^2
9. **Answer:** (c) Good-emitter
10. **Answer:** (d) (500)4

Very Short Answers:

1. **Answer:** In modern thermometry, the triple point of water is a standard fixed point.
2. **Answer:** No. Because absolute zero of temperature is the minimum possible temperature on the Kelvin scale.
3. **Answer:**
   (a) It is done to allow for safe contraction in winter.
   (b) 8 K.
4. **Answer:** The size of the hole increases on heating the metal disc.
5. **Answer:** No. When milk is poured into tea, some work is done which is not recoverable. So the process is not reversible.
6. **Answer:** 0°C, 4°C.
7. **Answer:** It reaches by radiation.
8. **Answer:** Earth loses heat by convection and radiation.
9. **Answer:** In the air, loss of heat by convection is possible. But convection currents cannot be set up in felt.
10. **Answer:** Conduction, Convection and Radiation.
11. **Answer:** Its inner and outer surfaces undergo uneven expansion due to the poor conductivity of glass, hence it cracks.
12. **Answer:** The variation of some physical property of a substance with temperature constitutes the basic principle of the thermometer.
13. **Answer:** Volume and radius suffer maximum and minimum expansions respectively as $\gamma = 3\alpha$.
14. **Answer:** The temperature reading = $\frac{100\times40}{160} = 25$.
15. **Answer:** The thermometer with a cylindrical bulb will respond quickly as the area of the cylindrical bulb is greater than the area of the spherical bulb.

Short Questions Answers:

1. **Answer:** This is because the coefficient of expansion of a gas is very large as compared to the coefficient of expansion of mercury. For the same temperature change, the gas would undergo a much larger change in volume as compared to mercury.
2. **Answer:** Since the speed is constant so there is no change of kinetic energy. The loss in gravitational potential energy is partially the gain in the heat energy of the brake drum.
3. **Answer:** (a) The portions AB and CD represent a change of state. This is because the supplied heat is unable to change the temperature. While AB represents a change of state from solid to liquid, the CD represents a change of state from liquid to vapour state.
   (b) It indicates that the latent heat of vaporisation is twice the latent heat of fusion.
   (c) Slope of DE represents the reciprocal of the thermal or heat capacity of the substance in vapour state i.e. slope 0f DE $= \frac{dT}{dQ} = \frac{1}{mC}$ ($\because$ $dQ = mC\Delta T$).
   (d) Specific heat of the substance in the liquid state is greater than that in the solid-state as the slope of OA is more than that of BC i.e. $\frac{1}{mC_1} > \frac{1}{mC_2}$ where C_1, C_2 are specific heats mC_1 mC_2 of the material in solid and liquid state respectively.
4. **Answer:** (a) It h defined as the process of the transfer of heat energy from one part of a solid. to another part at a lower temperature without the actual motion of the molecules. It is also called the conduction of heat.
   (b) It is defined as the quantity of heat flowing per second across the opposite faces of a unit cube made of that material when the opposite faces are maintained at a temperature difference of 1K or 1°C.
5. **Answer:** If Q is the amount of heat flowing from hot to the cold face, then it is found to be:
   1. directly proportional to the cross-sectional area (A) of the face
      i.e. $Q \propto A$ ...(1)
   2. directly proportional to the temperature difference between the two faces, i.e. $Q \propto \Delta\theta$ ....(2)
   3. directly proportional to the time t for which the heat flows i.e. $Q \propto t$ .... (3)
   4. inversely proportional to the distance 'd' between the two faces.
      i.e. $Q \propto \frac{1}{\Delta x}$ ...(4)

   Combining factors (1) to (4), we get

$$Q \propto \frac{A\Delta\theta}{\Delta x} t$$

or

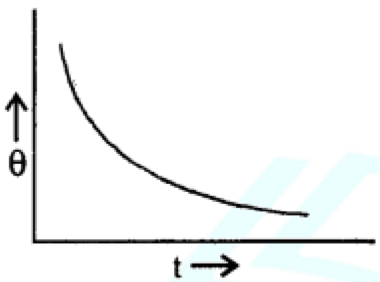
$$Q \propto K A \frac{\Delta\theta}{\Delta x} t$$

where K is the proportionality constant known as the coefficient – of thermal conductivity.

6. **Answer:** Newton's law of cooling: States that the rate of loss of heat per unit surface area of a body is directly proportional to the temperature difference between the body and the surroundings provided the difference is not too large.

Cooling Curve: It is defined as a graph between the temperature of a body and the time. It is as shown in the figure here.

The slope of the tangent to the curve at any point gives the rate of fall of temperature.



7. **Answer:** When the electric heater is switched on, a stage is quickly reached when the rate at which heat is generated by an electric current becomes equal to the rate at which heat is lost by conduction, convection and radiation and hence a thermal equilibrium is established. Thus temperature becomes constant.

8. **Answer:** Wool is a bad conductor of heat. Moreover wool encloses air in it which is a bad conductor. There can also be no loss of heat by convection. The woollen blanket keeps us warm by preventing the heat of the human body to flow outside and hence our body remains warm.

9. **Answer:** When a liquid is heated, it becomes rarer due to a decrease in density and it rises up. Liquid from the upper part of the vessel comes down to take its place and thus convection currents are formed. If the liquid is heated at the top, no such convection currents will be formed and only the liquid in the upper part of the vessel will become hot. However, the temperature in the lower part of the vessel will rise slightly due to a small amount of heat conducted by the hot liquid in the upper part of the vessel.

10. **Answer:** When a drop of waterfalls on a very hot iron, it gets insulated from the hot iron due to the formation of a thin layer of water vapour, which is a bad conductor in nature. It takes quite a long to evaporate as heat is conducted from hot iron to the drop through the layer of water vapour very slowly.

On the other hand, if a drop of waterfalls on an iron which is not very hot, then it comes in direct contact with the iron and evaporates immediately.

11. **Answer:** Glass possesses the property of selective absorption of heat radiation. It also transmits about 50% of heat radiation coming from a hot source like the sun and is more or less opaque to the radiation from moderately hot bodies (at about 100°C or so). Due to this, when a car is left in the sun, heat radiation from the sun gets into the car but as the temperature inside the car is moderate, it cannot escape through its windows. Thus glass windows of the car trap the sun rays and because of this, the inside of the car becomes considerably warmer.

12. **Answer:** Water boils with flame by the process of convection. Hot lighter particles raise up and heavier particles move down under gravity. In. a gravity-free space in the satellite, the particles cannot move down hence, water can't be heated by convection.

It will be heated by conduction.

13. **Answer:** The energy of a polyatomic gas having n degrees of freedom is given by

$$E = n \times \frac{1}{2} kT \times N = \frac{n}{2} RT$$

$$\therefore \; C_v = \frac{dE}{dT} = \frac{n}{2} R$$

$$\therefore \; C_p = C_v + R = \frac{n}{2} R + R$$

$$= \left(\frac{n}{2} + 1\right) R$$

$$\therefore \; \gamma = \frac{C_p}{C_v} = \frac{\frac{n}{2}+1}{\frac{n}{2}} = 1 + \frac{2}{n}$$

In case of a triatomic gas, n = 7

$$\therefore \gamma = 1 + \frac{2}{7} = \frac{9}{7}.$$

14. **Answer:** The hot case is a double-walled vessel. The space between the walls is evacuated in a good hot case. The food container placed inside the hot case is made of crowning steel, thus neither the outside low-temperature air can enter the container nor the heat from inside can escape through the hot case by conduction or convection. The highly polished shining surface of the food container helps in stopping loss of heat due to radiation. Thus, the heat of the food is preserved for a long time and food remains hot in winter.

15. **Answer:** The heat radiated or emitted from a body at a given temperature depends on

    1. the temperature difference between the body and the surrounding,
    2. area of the body in contact with the surroundings and
    3. the nature of the body.

    For man and animals in winter (1) and (2) factors remain what they are. So, in order to preserve, their body heat they curl up to reduce the surface area in contact with cold air.

Long Questions Answers:

1. **Answer:** Here, h = 100 m

Let m (kg) = mass of water

∴ Its P.E. at a height h = mgh

Energy of fall retained by water i.e. useful work done is given by,

W = 90% of mgh

$= \frac{900}{100}$ mgh

$= \frac{90}{100}$ m × 9.8 × 100

= 882 m J.

∴ Heat retained, $Q = WJ = \frac{m \times 882 J}{4.2 J\ cal^{-1}}$

= m × 210 cal ...(i)

Specific heat of water C = 10 cal kg^{-1} °C^{-1}

Let Δθ (°C) be the rise in the temperature of water.

∴ Heat gained, Q = mCΔθ

.= m × 10^3 × Δθ

= m × Δθ × 10^3 cal .... (ii)

∴ From (1) and (ii), we get

m × 210 = m × Δθ × 10^3

or

$\Delta\theta = \frac{210}{10^3} = 0.21°C.$

2. **Answer:** Here α = 1.2 × 10^{-1} °C^{-1}

Δt = 30 – 20= 10°C

T = 2s.

Using the relation, Δl = l α Δt, we get

$\frac{\Delta l}{l} = \alpha \Delta t$

= 1.2 × 10^{-5} × 10 = 1.2 × 10^{-4} ....(*i*)

$$\therefore T = 2\pi\sqrt{\frac{l}{g}} \ldots (ii)$$

If T' be the time period of the pendulum, when l increases by Δl, then

$$T' = 2\pi\sqrt{\frac{l+\Delta l}{g}}$$

$$= 2\pi\sqrt{\frac{l}{g}\left(1+\frac{\Delta l}{l}\right)} \quad \text{....}(iii)$$

$\frac{(iii)}{(ii)}$ gives $\frac{T'}{T} = \sqrt{1+\frac{\Delta l}{l}} = \sqrt{1+1.2\times10^{-4}}$

∴ loss in time in one oscillation T' – T

Hence loss in time in one day is given by

$$= \frac{T'-T}{T}\times24\times3600\,s$$

$$= \left(\frac{T'}{T}-1\right)\times24\times3600\,s$$

$$= \left[\sqrt{1+1.2\times10^{-4}}-1\right]\times24\times3600\,s$$

$$= \left[1+\frac{1}{2}\times1.2\times10^{-4}-1\right]\times24\times3600\,s$$

$$= \frac{1.2\times10^{-4}\times24\times3600}{2}s$$

= 5.18 s.

3. **Answer:** Here, KB = 1.7 W m^{-1} K^{-1}

KC = 2.9 W m^{-1} K^{-1}

dB = 20 cm

dc = ?

We know that the heat flow is given by

Q = KA $\frac{\Delta\theta}{d} t$

For the same insulation by the brick and the cement, Q, A, Δθ and t don't change

Thus $\frac{K}{d}$ should be a constant.

i.e. $$\frac{K_B}{d_B} = \frac{K_C}{K_B} \times d_B$$

or $$\frac{2.9}{1.7} \times 20 = 34.12\,\text{cm}$$

4. **Answer:** Let T (°C) be the temperature of the interface =?

Here, K_1 = 300 $Wm^{-1}\,°C^{-1}$ for A

K_2 = 200 $Wm^{-1}\,°C^{-1}$ for B

∴ $\Delta\theta_1$ = 100 – T for A

$\Delta\theta_2$ = T – 0 for B .

x = size of each cube A and B

∴ $x_1 = x_2 = x$

Let a = area of cross-section of the faces between which there is the flow of heat

If $\left(\frac{\Delta Q_1}{\Delta t}\right)_A$ and $\left(\frac{\Delta Q_2}{\Delta t}\right)_B$ be the rate of low of heat for A and B respectively, then in steady state,

$$\left(\frac{\Delta Q_1}{\Delta t}\right)_A = \left(\frac{\Delta Q_2}{\Delta t}\right)_B$$

or $$\frac{K_1 a \Delta\theta_1}{x} = \frac{K_1 a \Delta\theta_2}{x} \quad \left(\because \Delta Q = K\frac{\Delta\theta}{d}at\right)$$

or $$K_1\Delta\theta_1 = K_2\Delta\theta_2$$

or 300 (100 – T) = 200 (T – 0)

or (300 + 200) T = 30000

or $$T = \frac{300 \times 100}{500}$$

∴ $T = 60°C$

5. **Answer:** Here

L = 2.25 × 106 JK^{-1}

= $\frac{2.25}{4.2}$ × 106 cal°C^{-1}

Q = Heat of Combustion

= 373 × 103 Cal/mole

C = 103 J $Kg^{-1}\,°C^{-1}$

m = 50 kg

$\Delta\theta$ = 100- 10 = 90°C

V = 22.4 litres

If Q_1 = Total heat energy required to convert 50 kg of water at 10°C to steam at 100°C

$Q_1 = mC\Delta\theta + mL$

= 5.0 × 1000 × 90 + 50 × $\frac{2.25\times10^6}{4.2}$

= 4.5 × 106 + 26.79 × 106

= 31.29 × 106 cal

As 50% of heat is lost,

∴ total heat produced = $\frac{100}{50}$ × 3.129 × 106

Let n = no. of moles of ethane to be burnt, then

$$n = \frac{2 \times 31.29 \times 10^6}{373 \times 10^3}\text{mole}$$

∴ Volume of ethane = nV

$$= \frac{2 \times 31.29 \times 10^6}{373 \times 10^3} \times 22.4 \text{ litres}$$

= 3758.2 litres.

Assertion Reason Answer:

1. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
2. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

Case Study Questions-

1. We can say that heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference. The SI unit of heat energy transferred is expressed in joule (J) while SI unit of temperature is Kelvin (K), and degree Celsius (^{0}C) is a commonly used unit of temperature. When an object is heated, many changes may take place. Its temperature may rise; it may expand or change state. A measure of temperature is obtained using a thermometer. Many physical properties of materials change sufficiently with temperature. Some such properties are used as the basis for constructing thermometers. The two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale. The ice and steam point have values 32 °F and 212 °F, respectively, on the Fahrenheit scale and 0 °C and 100 °C on the Celsius scale. On the Fahrenheit scale, there are 180 equal intervals between two reference points, and on the Celsius scale, there are 100. A relationship for converting between the two scales may be obtained from a graph of Fahrenheit temperature (t_F) versus Celsius temperature (t_C) in a straight line. When temperature is held constant, the pressure and volume of a quantity of gas are related as PV = constant. This relationship is known as Boyle's law. When the pressure is held constant, the

volume of a quantity of the gas is related to the temperature as V/T = constant. This relationship is known as Charles' law. Low-density gases obey these laws, which may be combined into a single relationship. **$PV = \mu RT$** where, μ is the number of moles in the sample of gas and R is called universal gas constant: **$R = 8.31\ J\ mol^{-1}\ K^{-1}$** we have learnt that the pressure and volume are directly proportional to temperature: **$PV \alpha T$**. This relationship allows a gas to be used to measure temperature in a constant volume gas thermometer. The absolute minimum temperature for an ideal gas at which pressure becomes zero is found to be **– 273.15°C** and is designated as absolute zero. Absolute zero is the foundation of the Kelvin temperature scale or absolute scale temperature. The size of unit in Kelvin and Celsius temperature scales is the same. So, temperature on these scales are related by **$T = t_c + 273.15$**

i. **The SI unit of heat energy transferred is expressed in**
   a. Joule (J)
   b. Kelvin (K)
   c. Newton (N)
   d. None of these

ii. **Temperature is measured using**
   a. Thermometer
   b. Barometer
   c. Tachometer
   d. None of these

iii. **Relation between Kelvin (T) and Celsius temperature (t_c) scale is given by**
   a. $T = t_c + 273.15$
   b. $T = t_c - 273.15$
   c. $T = t_c$
   d. None of these

iv. **What is heat energy**

v. **What is absolute zero temperature**

2. A system is said to be isolated if no exchange or transfer of heat occurs between the system and its surroundings. When different parts of an isolated system are at different temperature a quantity of heat transfers from the part at higher temperature to the part at lower temperature. The heat lost by the part at higher temperature is equal to the heat gained by the part at lower temperature. Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings. A device in which heat measurement can be done is called a calorimeter. It consists of a metallic vessel and stirrer of the same material, like copper or aluminium. The vessel is kept inside a wooden jacket, which contains heat insulating material. Matter normally exists in three states: solid, liquid and gas. A transition from one of these states to another is called a change of state. Two common changes of states are solid to liquid and liquid to gas (and, vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings. The change of state from solid to liquid is called melting and from liquid to solid is called fusion. It is observed that the temperature remains constant until the entire amount of the solid substance melts. That is, both the solid and the liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid. The temperature at which the solid and the liquid states of the substance is in thermal equilibrium with each other is called its melting point. The change of state from liquid to vapour (or gas) is called vaporisation. It is observed that the temperature remains constant until the entire amount of the liquid is converted into vapour. That is, both the liquid and vapour states of the substance coexist in thermal equilibrium, during the change of state from liquid to vapour. The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point. The change from solid state to vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime. Dry ice (solid CO2) sublimes, so also iodine. During sublimation both the solid and vapour states of a substance coexist in thermal equilibrium.

i. **Device used for measurement of heat is**
   a. Calorimeter
   b. Thermometer
   c. Both a and b
   d. No one of these

ii. **The change of state from solid to liquid is called**

a. Melting

b. Vaporization

c. Sublimation

d. None of these

iii. **Define melting point and boiling point**

iv. **What is sublimation?**

v. **Define fusion process**

Case Study Answer-

1. Answer

i. (a) Joule (J)

ii. (a) Thermometer

iii. (a) $T = t_c + 273.15$

iv. Heat energy is the form of energy transferred between two or more systems or its surroundings due to temperature difference from higher temperature to lower temperature. The SI unit of heat energy transferred is expressed in joule (J).

v. The absolute minimum temperature for an ideal gas at which pressure becomes zero is found to be – 273.15 °C and is designated as absolute zero temperature. This is lowest temperature possible for ideal gas.

2. Answer

i. (a) Calorimeter

ii. (a) Melting

iii. The change of state from solid to liquid is called melting process and temperature at which conversion of solid into liquid happens is called as melting point. The temperature at which the liquid and the vapour states of the substance coexist is called its boiling point.

iv. The change from solid state directly into vapour state without passing through the liquid state is called sublimation, and the substance is said to sublime.

v. The change of state from liquid state to solid state is called as fusion process.

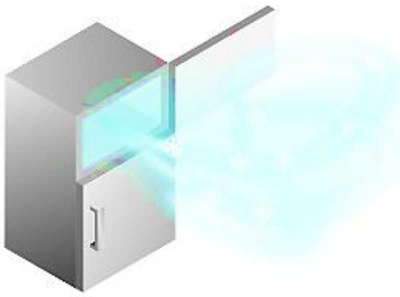
Thermodynamics 12

Introduction

Thermodynamics is that branch of physics which deals with concepts of heat and temperature and their relation to energy and work.

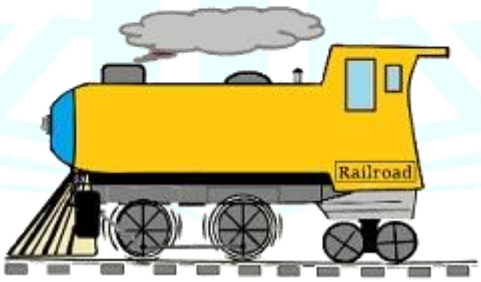
We can also consider it as a macroscopic science which deals with bulk systems and tells us about system as a whole.

In this chapter we will learn about the laws of thermodynamics which describes the system in terms of macroscopic variables, reversible and irreversible processes. Finally we will also learn on what principle heat engines, refrigerators and Carnot engine work.



Refrigerator

Examples: - Refrigerator, steam engine



Steam Engine

Thermal Equilibrium

Two systems are said to be in thermal equilibrium if the temperatures of the two systems are equal.

In mechanics if the net force on a system is zero, the system is in equilibrium.

In Thermodynamics equilibrium means all the macroscopic variables (pressure, temperature and volume) don't change with time. They are constant throughout.

For **Example:**

Consider two bodies at different temperatures one is at 30^0C and another at 60^0C then the heat will flow from body at higher temperature to the body at lower temperature.

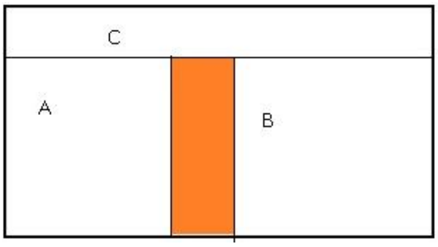
Heat will flow till both bodies acquire same temperature.

This state when there is no heat flow between two bodies when they acquire the same temperature is known as thermal equilibrium.

In the above case if consider a hot cup of coffee will become cold after sometime if it is kept on the table as there will heat flow between the hot coffee and surroundings. When the cup of coffee attains the same temperature as of room temperature then there will be no flow of heat.

Zeroth Law of Thermodynamics

Zeroth law of thermodynamics states that when two systems are in thermal equilibrium through a third system separately then they are in thermal equilibrium with each other also.



Foreg: Consider two systems A and B which are separated by an adiabatic wall. Heat flow happens between systems A and C, and between B and C, due to which all 3 systems attain thermal equilibrium.

Systems A and B are in thermal equilibrium with C. Then they will be in equilibrium with each other also.

Zeroth Law of Thermodynamics suggested that there should be some physical quantity which should have same value for the system to be in thermal equilibrium.

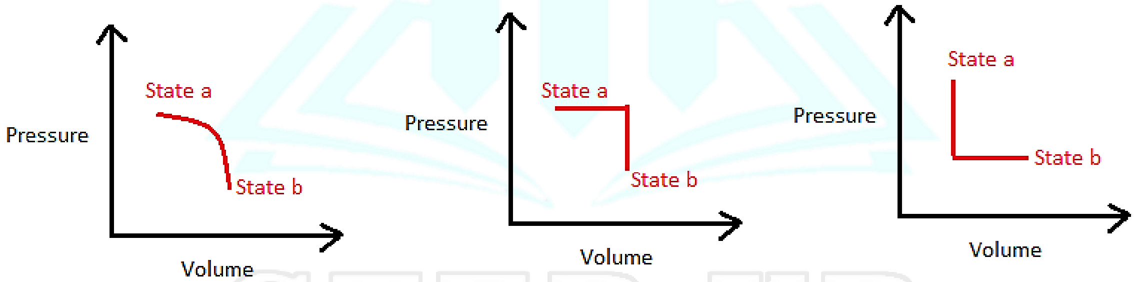
This physical quantity which determines whether system is in equilibrium or not is Temperature.

Temperature is the quantity which determines whether the system is in thermal equilibrium with the neighbouring system.

When the temperature becomes equal then the flow of heat stops.

Heat, Internal Energy and Work

We already know what heat is. It is a form of energy but it always comes into the picture when energy is being transferred from one system to another. Suppose we are at an initial state ‘a’ and want to go to a final state ‘b’. We can do that by various processes (as shown in the figure) and the heat energy released or absorbed in all the processes is different. So we can see that heat does not depend on the state of the system.



So there was a need to define another term that was dependent on the state of the system. This is termed as Internal Energy.

Internal Energy

Internal energy definition is given as:

The energy contained within the system associated with random motions of the particles along with the potential energies of the molecules due to their orientation.

The energy due to random motion includes translational, rotational, and vibrational energy. It is represented as U. So now we can say, since internal energy is a state function and in all the processes shown above the change in internal energy from state, ‘a’ to state ‘b’ will be the same.

Internal Energy Formula

The following is the formula for internal energy:

$\Delta U = Q + W$

Where,

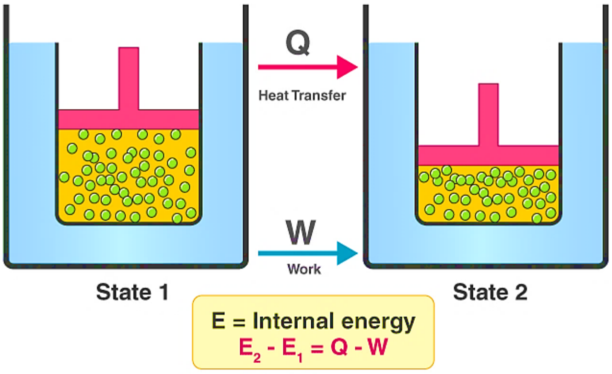
ΔU is the internal energy

Q is the heat added to the system

W is the work done by the system

First Law of Thermodynamics

A thermodynamic system in an equilibrium state possesses a state variable known as the internal energy(E). Between two systems the change in the internal energy is equal to the difference of the heat transfer into the system and the work done by the system.



The first law of thermodynamics states that the energy of the universe remains the same. Though it may be exchanged between the system and the surroundings, it can't be created or destroyed. The law basically relates to the changes in energy states due to work and heat transfer. It redefines the conservation of energy concept.

The First Law of Thermodynamics states that heat is a form of energy, and thermodynamic processes are therefore subject to the principle of conservation of energy. This means that heat energy cannot be created or destroyed. It can, however, be transferred from one location to another and converted to and from other forms of energy.

To help you understand the meaning of the First Law, we can take the common example of a heat engine. In a Heat engine, the thermal energy is converted into mechanical energy and the process also is vice versa. Heat engines are mostly categorized as an open system. The basic working principle of a heat engine is that it makes use of the different relationships between heat, pressure and volume of a working fluid which is usually a gas. Sometimes phase changes might also occur involving a gas to liquid and back to gas.

First Law of Thermodynamics Equation

The equation for the first law of thermodynamics is given as;

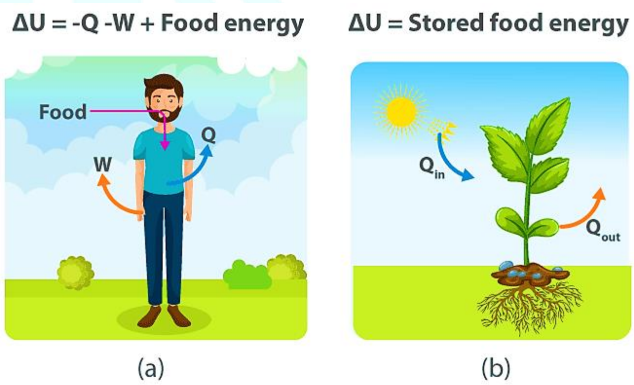
$\Delta U = q + W$

Where,

ΔU = change in internal energy of the system.

q = algebraic sum of heat transfer between system and surroundings.

W = work interaction of the system with its surroundings.



(a) (b)

For an isolated system, energy (E) always remains constant.

Internal Energy is a point function and property of the system. Internal energy is an extensive property (mass-dependent) while specific energy is an intensive property (independent of mass).

For an ideal gas, the internal energy is a function of temperature only.

Specific heat capacity

Specific heat is defined as the amount of heat required to raise the temperature of a body per unit mass.

It depends on:

Nature of substance

Temperature

Denoted by 's'

$$s = \left(\frac{\Delta Q}{m\Delta T}\right)$$

where m= mass of the body

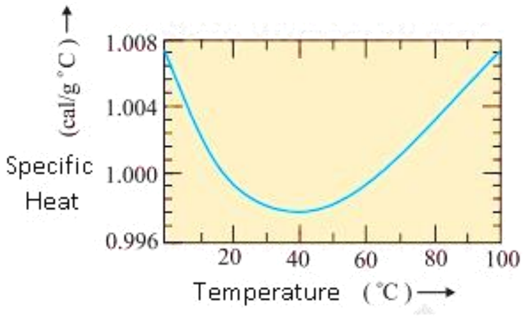
ΔQ = amount of heat absorbed or rejected by the substance

ΔT= temperature change

Unit - J kg^{-1} K^{-1}

If we are heating up oil in a pan, more heat is needed when heating up one cup of oil compared to just one tablespoon of oil. If the mass s is more the amount of heat required is more to increase the temperature by one degree.

Specific heat capacity of water



Calorie: - One calorie is defined to be the amount of heat required to raise the

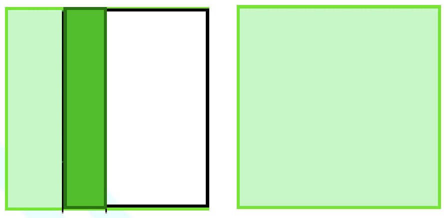
temperature of 1g of water from 14.5 °C to 15.5 °C.

In SI units, the specific heat capacity of water is 4186 J kg^{-1} K^{-1}e. 4.186 J g^{-1} K^{-1}.

The specific heat capacity depends on the process or the conditions under which heat capacity transfer takes place.

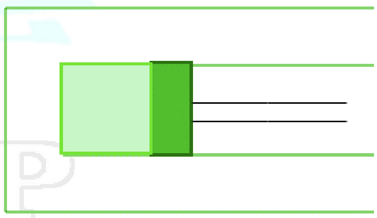
Thermodynamic State Variables and Equation of State

A system is said to be in a thermodynamics state of equilibrium if the macroscopic variables that change the state of the system do not change over time. These macroscopic variables include pressure, temperature, mass, and composition that does not change with time. For example, gas is stored inside a container that is completely insulated from its surroundings, with fixed values of pressure, volume, temperature, mass, and composition that do not change with time, is in a state of equilibrium.



(a)

The equilibrium of a gas can be described by its pressure, temperature, volume, and mass. It is not necessary that a thermodynamic system is always in equilibrium. For example, if gas at equilibrium is allowed to expand, it does not remain in thermodynamic equilibrium. The figure below shows the expansion of gases when they are left to expand:



(b)

Thermodynamics variables describe the state of the system at equilibrium. These various state variables are not necessarily independent.

These variables can be divided into two types:

- Extensive Variables
- Intensive Variables

Extensive Variables: These variables are the state variables that indicate the size of the system. For example, Volume can be considered an extensive variable because it gives us an idea about the size of the system.

Intensive Variables: These variables are the state variables that do not give us any information about the size of the system but indicate different information about the system. Examples of such variables are pressure, temperature, etc.

State Equation

State Equation describes the relationship between state variables of a thermodynamic system. The equation of state is completely defined in terms of pressure, temperature and volume. For example, in the case of ideal gases. The state equation becomes,

$$PV = RT$$

Or

PV = Constant

In the case of an isothermal process,

$$P_1V_1 = P_2V_2$$

These equations of state are for ideal gases.

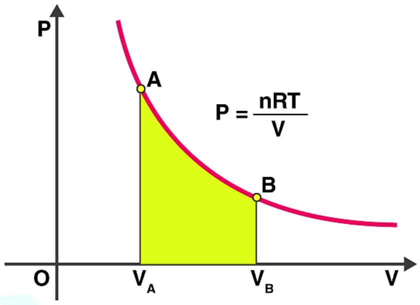
Thermodynamic Processes & Types

We know that if we have to take a thermodynamic system from the initial to the final state, we have several paths that can be taken. In this article, we will be discussing those thermodynamic processes. Before that, we will see what a quasi-static process is. It has been discussed that state variables are defined only when the thermodynamic system is in equilibrium with the surrounding. So a process in which the system is in thermodynamic equilibrium with the surrounding is known as a quasi-static process at each moment.

The Thermodynamic Processes

Isothermal Process:

It is a thermodynamic process in which temperature remains constant. We know,



$$W = \int PdV$$

According to Gas law,

$$PV = nRT$$

$$P = \frac{nRT}{v}$$

Using this value of P in work done, we get,

$$W = nRT\int_{V_A}^{V_B} \frac{dV}{V}$$

$$W = nRTln\frac{V_B}{V_A}$$

Adiabatic Process:

It is a thermodynamic process in which no heat is exchanged between the system and the surrounding. So, Q = 0. Mathematically this process is represented as:

$$PV^{\gamma} = K\,(\text{constant})$$

$$W = \int PdV$$

Substituting P, we get,

$$W = K\int_{V_i}^{V_f} \frac{dV}{V^{\gamma}}$$

$$W = K\frac{(V_f^{i-\gamma} - V_t^{i} - \gamma)}{1-\gamma}$$

For adiabatic process,

$$\Delta U = -W$$

Isochoric Process:

In isochoric process the change in volume of thermodynamic system is zero. A volume change is zero, so the work done is zero.

Volume of the system = Constant

Change in volume = 0

If, change in volume = 0, then work done is zero.

According to the 1st law of thermodynamic law

Q = W + dU

If W = 0

Q = dU

Isobaric Process:

The pressure remains constant during this process. So,

$$W = P(V_f - V_i)$$

So if volume increases, work done is positive, else negative.

Cyclic Process:

It is a process in which the final state of the system is equal to the initial state. As we know, change in internal energy is a state function, so, in this case, $\Delta U = 0$.

Quasi-static process:

In thermodynamics, a quasi-static process is referred to as a slow process.

It is a process that happens at an infinitesimally slow rate.

A quasi-static process has all of its states in equilibrium.

A quasi-static process is one in which the system is in thermodynamic equilibrium with its surroundings at all times.

Second Law of Thermodynamics

The second law of thermodynamics states that any spontaneously occurring process will always lead to an escalation in the entropy (S) of the universe. In simple words, the law explains that an isolated system's entropy will never decrease over time.

Nonetheless, in some cases where the system is in thermodynamic equilibrium or going through a reversible process, the total entropy of a system and its surroundings remains constant. The second law is also known as the Law of Increased Entropy.

The second law clearly explains that it is impossible to convert heat energy to mechanical energy with 100 per cent efficiency. For example, if we look at the piston in an engine, the gas is heated to increase its pressure and drive a piston. However, even as the piston moves, there is always some leftover heat in the gas that cannot be used for carrying out any other work. Heat is wasted and it has to de discarded. In this case, it is done by transferring it to a heat sink or in the case of a car engine, waste heat is discarded by exhausting the used fuel and air mixture to the atmosphere. Additionally, heat generated from friction that is generally unusable should also be removed from the system.

Second Law of Thermodynamics Equation

Mathematically, the second law of thermodynamics is represented as:

$\Delta S_{univ} > 0$

where ΔS_{univ} is the change in the entropy of the universe.

Entropy is a measure of the randomness of the system or it is the measure of energy or chaos within an isolated system. It can be considered as a quantitative index that describes the quality of energy.

Meanwhile, there are few factors that cause an increase in entropy of the closed system. Firstly, in a closed system, while the mass remains constant there is an exchange of heat with the surroundings. This change in the heat content creates a disturbance in the system thereby increasing the entropy of the system.

Secondly, internal changes may occur in the movements of the molecules of the system. This leads to disturbances which further causes irreversibilities inside the system resulting in the increment of its entropy.

Different Statements of The Law

There are two statements on the second law of thermodynamics which are;

Kelvin- Plank Statement

Clausius Statement

Kelvin-Planck Statement

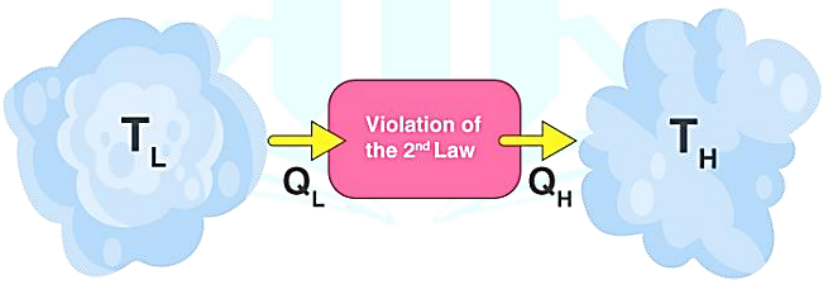
It is impossible for a heat engine to produce a network in a complete cycle if it exchanges heat only with bodies at a single fixed temperature.

Exceptions:

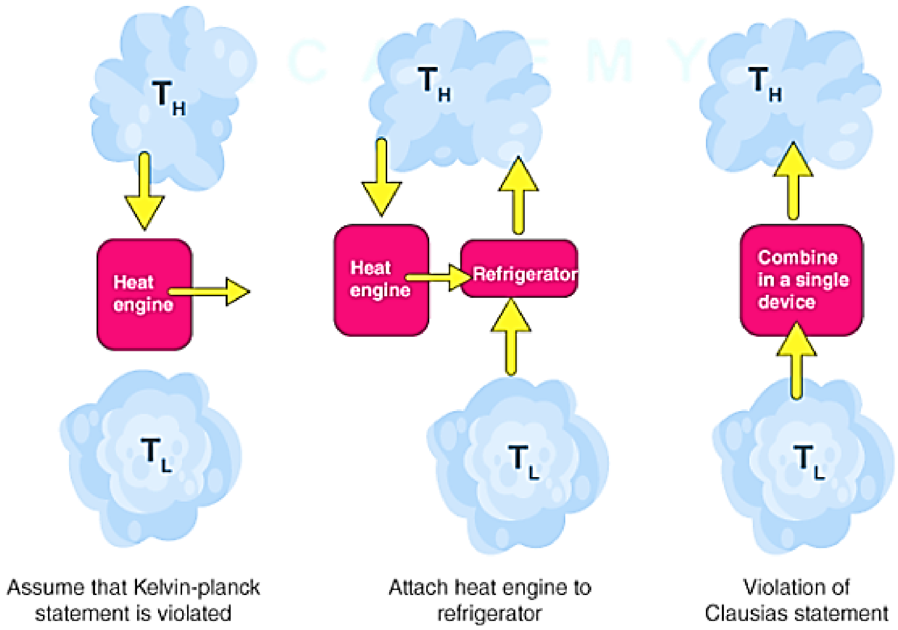
If Q_2 =0 (i.e., W_{net} = Q_1, or efficiency=1.00), the heat engine produces work in a complete cycle by exchanging heat with only one reservoir, thus violating the Kelvin-Planck statement.

Clausius's Statement

It is impossible to construct a device operating in a cycle that can transfer heat from a colder body to a warmer one without consuming any work. Also, energy will not flow spontaneously from a low-temperature object to a higher temperature object. It is important to note that we are referring to the net transfer of energy. Energy transfer can take place from a cold object to a hot object by transfer of energetic particles or electromagnetic radiation. However, the net transfer will occur from the hot object to the cold object in any spontaneous process. And some form of work is needed to transfer the net energy to the hot object. In other words, unless the compressor is driven by an external source, the refrigerator won't be able to operate. The heat pump and refrigerator work on Clausius's statement.

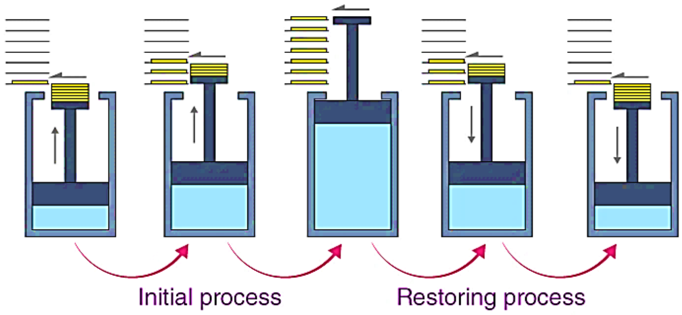


Both Clausius's and Kelvin's statements are equivalent i.e a device violating Clausius's statement will also violate Kelvin's statement and vice versa.



In addition to these statements, a French physicist named Nicolas Léonard Sadi Carnot also known as the "father of thermodynamics," basically introduced the Second Law of Thermodynamics. However, as per his statement, he emphasized the use of caloric theory for the description of the law. Caloric (self-repellent fluid) relates to heat and Carnot observed that some caloric was lost in the motion cycle.

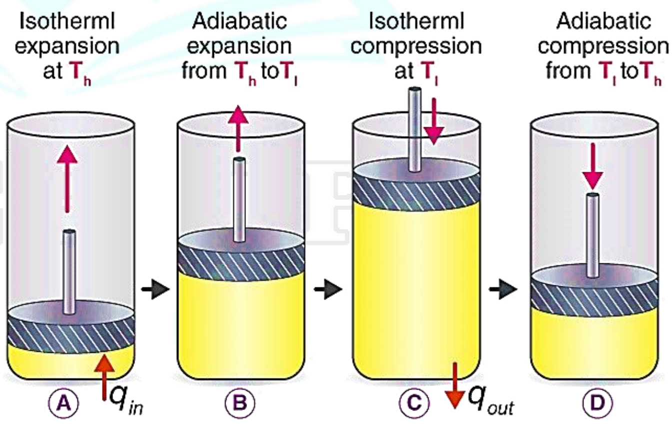
Reversible And Irreversible Processes



We see so many changes happening around us every day, such as boiling water, rusting of iron, melting ice, burning of paper, etc. In all these processes, we observe that the system in consideration goes from an initial state to a final state where some amount of heat is absorbed from the surroundings and some amount of work W is done by the system on the surrounding. Now, for how many such systems can the system and the surrounding be brought back to their initial state? With common examples such as rusting and fermentation, we can say that it is not possible in most cases. In this section, we shall learn about reversible and irreversible processes.

A thermodynamic process (state i → state f) is said to be reversible if the process can be turned back to such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe. As we know, in reality, no such processes as reversible processes can exist. Thus, the reversible processes can easily be defined as idealizations or models of real processes on which the limits of the system or device are to be defined. They help us in incurring the maximum efficiency a system can provide in ideal working conditions and thus the target design that can be set.

Carnot Cycle



A Carnot cycle is defined as an ideal reversible closed thermodynamic cycle. Four successive operations are involved: isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression. During these operations, the expansion and compression of the substance can be done up to the desired point and back to the initial state.

Following are the four processes of the Carnot cycle:

In (a), the process is reversible isothermal gas expansion. In this process, the amount of heat absorbed by the ideal gas is qin from the heat source at a temperature of T_h. The gas expands and does work on the surroundings.

In (b), the process is reversible adiabatic gas expansion. Here, the system is thermally insulated, and the gas continues to expand and work is done on the surroundings. Now the temperature is lower, T_l.

In (c), the process is a reversible isothermal gas compression process. Here, the heat loss q_{out} occurs when the surroundings do the work at temperature T_l.

In (d), the process is reversible adiabatic gas compression. Again, the system is thermally insulated. The temperature again rises back to Th as the surrounding continue to do their work on the gas.

Carnot Engine

The Carnot engine is a theoretical thermodynamic cycle proposed by Leonard Carnot. It estimates the maximum possible efficiency that a heat engine during the conversion process of heat into work and, conversely, working between two reservoirs can possess.

Carnot Theorem

According to Carnot Theorem:

Any system working between T_1 (hot reservoir) and T_2 (cold reservoir) can never have more efficiency than the Carnot engine operating between the same reservoirs.

Also, the efficiency of this type of engine is independent of the nature of the working substance and is only dependent on the temperature of the hot and cold reservoirs.

Top Formulae

- Equation of isothermal changes PV = constant or $P_2 V_2 = P_1V_1$
- Equation of adiabatic changes
    i. $P_2V_2^{\gamma} = PV_1$
    ii. $P^{1-\gamma}T_2^{\gamma} = P_1^{-\gamma}T^{\gamma}$
    iii. $T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$, where $\gamma = C_p / C_v$
- Work done by the gas in isothermal expansion

$$W = 2.3026\, RT \log_{10}\frac{V_2}{V_1}$$

$$W = 2.3026\, RT \log_{10}\frac{P_1}{P_2}$$

- Work done in adiabatic expansion

$$W = \frac{R}{(1-\gamma)}(T_2 - T)$$

- dQ = dU + dW
  Here, dW = P (dV), small amount of work done
  dQ = m L for change of state
  dQ = mc ΔT for rise in temperature
  dU = change in internal energy

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T}$$

- Where T_1 = temperature of source, T_2 = temperature of sink; Q_1 is the amount of heat absorbed/cycle from the source, Q_2 is the amount of heat rejected/cycle to the sink.
- Useful work done/cycle $W = Q_1 - Q_2$
- Efficiency of Carnot engine is also given by $\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$
- Coefficient of performance of a refrigerator

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T}; W = Q_1 - Q_2$$

where Q_2 is the amount of heat drawn/cycle from the sink (at T_2) and W is work done/cycle on the refrigerator. Q_1 is the amount of heat rejected/cycle to the source (air at room temperature T_1).

$$\beta = \frac{1-\eta}{\eta}$$

Class : 11th Physics
Chapter- 12 : Thermodynamics

Thermodynamics

Thermodynamics State variable
(i) Extensive – indicate the size of the system. e.g. U, volume, total mass.
(ii) Intensive – do not indicate size of the system. e.g. , pressure & temperature

Thermodynamics Equilibrium
Two systems are in thermal equilibrium with each other if they have the same temperature.

Laws

Zeroth Low of Thermodynamics
If two systems A&B are in thermal equilibrium with a third system C, then A&B are in thermal equilibrium with each other.

First Low of Thermodynamics
It is the statement of law of conservation of Energy;
$\Delta Q = \Delta U + \Delta W$
$= \Delta U + P\Delta U$ (Here, $\Delta U = P\Delta U$)

Second Low of Thermodynamics
It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work.

Reversible & Irreversible Process
Any process made to proceed in the reverse direction by changing its conditions is called Reversible Process.
Any process which cannot be retraced in the reverse direction exactly is called Irreversible Process.

Heat Engine
Converts continuously heat into mechanical energy in cyclic process.

Efficiency $\eta = \frac{W}{Q} = 1 - \frac{Q_2}{QT_1} = 1 - \frac{T_2}{T_1}$

Refrigerator and heat pump
Refrigerator is a heat engine working in the reverse direction.
Coefficient of performance,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Carnot Engine
An Ideal engine works on a reversible cycle of four operations in succession.
(i) isothermal expansion,
(ii) adiabatic expansion,
(iii) isothermal compression,
(iv)adiabatic compression.
Efficiency of Carnot's engine

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_2}$$

Specific heat capacity Relation
- $\frac{C_P}{C_V} = \gamma$
- Mayer's Equation
  $C_P - C_V = R$

Branch of Science which deals with concepts of heat & temperature and their interconversion by thermodynamic process

Cyclic Process
In this process, system returns to initial state for a cyclic process $\Delta U = 0$ (zero)

Quasi - static Process
Infinity slow process such that systems remain in thermal & mechanical equilibrium with the surroundings throughout.

Property
The pressure (P) & temperature (T) of the environment can differ from those of the system only infinitesimally.

Adiabatic Process
A thermally insulated system neither gains nor loses heat
$PV^{\gamma} =$ Constant
$TV^{\gamma-1} =$ Constant
$\frac{P^{n-\gamma}}{T^{\gamma}} =$ Constant; $\frac{C_P}{C_v} = \gamma$

Isochoric Process
Volume = Constant
$\frac{P}{T} =$ Constant
$\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Isobaric Process
Pressure = Constant
$\frac{V}{T} =$ Constant
$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Isothermal Process
Temperature = Constant
PV = Constant = n RT
$W = \int_{V_1}^{V_2} \frac{nRT}{V} dv$

Important Questions

Multiple Choice Questions

1. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O^2 to per N^2 molecule is
   (a) 1 : 1
   (b) 1 : 2
   (c) 2 : 1
   (d) depends on the moment of inertia of the two molecules
2. For a diatomic gas change in internal energy for a unit change in temperature for constant pressure and constant volume is U1 and U2 respectively. What is the ratio of U1 and U2?
   (a) 5 : 3
   (b) 3 : 5
   (c) 1 : 1
   (d) 5 : 7
3. An ideal gas heat engine operates in Carnot cycle between 227°C and 127°C. It absorbs 6 × 104 cal of heat at higher temperature. Amount of heat converted to work is:
   (a) 2.4 × 104 cal
   (b) 6 × 104 cal
   (c) 1.2 × 104 cal
   (d) 4.8 × 104 cal
4. Which of the following parameters dose not characterize the thermodynamic state of matter?
   (a) work
   (b) volume
   (c) pressure
   (d) temperature
5. A Carnot engine whose sink is at 300 K has an efficiency of 40%. By how much should the temperature of source be increased, so as to increase its efficiency by 50% of original efficiency?
   (a) 275 K
   (b) 325 K
   (c) 250 K
   (d) 380 K
6. The translational kinetic energy of gas molecules at temperature T for one mole of a gas is
   (a) (3/2) RT
   (b) (9/2) RT
   (c) (1/3) RT
   (d) (5/2) RT
7. The temperature of reservoir of Carnots engine operating with an efficiency of 70% is 1000 kelvin. The temperature of its sink is
   (a) 300 K
   (b) 400 K
   (c) 500 K
   (d) 700 K
8. A gas is taken through a number of thermodynamic states. What happens to its specific heat?
   (a) It is always constant.
   (b) It increases.
   (c) It decreases.
   (d) It can have any value depending upon the process of heat absorbed or evolved.
9. **Directions:** The following has four choices out of which ONLY ONE is correct. A refrigerator with its power on, is kept in a closed room with its door open, then the temperature of the room will ________.
   (a) rise
   (b) fall
   (c) remain the same
   (d) depend on the area of the room
10. **Directions:** The following has four choices out of which ONLY ONE is correct. Which of the following is incorrect regarding the first law of thermodynamics? A. It is not applicable to any cyclic process B. It is a restatement of the principle of conservation of energy C. It introduces the concept of the internal energy D. It introduces the concept of the entropy
   (a) A and D
   (b) B and C
   (c) C and A
   (d) A and B

Very Short:

1. What type of process is Carnot's cycle?
2. Can the Carnot engine be realized in actual practice?
3. A refrigerator transfers heat from a cold body to a hot body. Does this not violate the second law of thermodynamics?
4. What is a heat pump?
5. What forbids the complete conversion of work into heat?
6. Does the internal energy of an ideal gas change in:
   (a) an isothermal process?
   (b) an adiabatic process?
7. What is the specific heat of a gas in an isothermal process and in an adiabatic process? Why?
8. Can the temperature of an isolated system change?
9. Can we increase the coefficient of performance of a refrigerator by increasing the amount of working substance?
10. The door of an operating refrigerator is kept open in a closed room. Will it make the room warm or cool?

Short Questions:

1. Kelvin and Clausius's statements of the Second law of thermodynamics are equivalent. Explain?
2. Two identical samples of gas are expanded so that the volume is increased to twice the initial volume. However, sample number 1 is expanded isothermally while sample number 2 is expanded adiabatically. In which sample is the pressure greater? Why?
3. No real engine can have an efficiency greater than that of a Carnot engine working between the same two temperatures. Why?
4. Explain why two isothermal curves cannot intersect each other?
5. What is the source of energy when gas does work when expands adiabatically?
6. State and explain the zeroth law of thermodynamics?
7. State and explain the first law of thermodynamics. What are the sign conventions?
8. Why cannot a ship use the internal energy of seawater to operate the engine?
9. A certain amount of work is done by the system in a process in which no heat is transferred to or from the system. What happens to the internal energy and the temperature of the system?
10. If an electric fan is switched on in a closed room, will the air of the room be cooled? Why?

Long Questions:

1. Discuss the Carnot cycle and give essential features of a Carnot engine.
2. Derive the expression for the work done during:
   (a) Isothermal process
   (b) Adiabatic process
3. A gas is suddenly compressed to 1/3 of its original volume. Calculate the rise in temperature, the original temperature being 300K and $\gamma = 1.5$.
4. A perfect Qarjiotreiigifae utilizes an ideal gas. The source temperature is 500K and since the temperature is 375 K. If the engine takes 600 Kcal per cycle from the source, compute:
   (a) the efficiency of The engine.
   (b) work done per cycle,
   (c) heat rejected to the sink per cycle.
5. A refrigerator has, to transfer an average of 263 J of heat per second from temperature – 10°C to 25°C. Calculate the average power consumed assuming ideal reversible cycle and no other losses.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
   (c) If assertion is true but reason is false.
   (d) If the assertion and reason both are false.

   Assertion: When a bottle of cold carbonated drink is opened, a slight fog forms around the opening.

Reason: Adiabatic expansion of the gas causes lowering of temperature and condensation of water vapours.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
   (c) If assertion is true but reason is false.
   (d) If the assertion and reason both are false.

Assertion: In adiabatic compression, the internal energy and temperature of the system get decreased.

Reason: The adiabatic compression is a slow process

Answer Key

Multiple Choice Answers-

1. **Answer:** (a) 1 : 1
2. **Answer:** (c) 1 : 1
3. **Answer:** (c) 1.2 × 104 cal
4. **Answer:** (a) work
5. **Answer:** (c) 250 K
6. **Answer:** (a) (3/2) RT
7. **Answer:** (a) 300 K
8. **Answer:** (d) It can have any value depending upon the process of heat absorbed or evolved.
9. **Answer:** (a) rise
10. **Answer:** (a) A and D

Very Short Answers:

1. **Answer:** Cyclic process.
2. **Answer:** No. It is an ideal heat engine.
3. **Answer:** No. This is because external work is being performed.
4. **Answer:** A heat pump is a device that uses mechanical work to remove heat.
5. **Answer:** The second law of thermodynamics.
6. **Answer:** (a) No.
   (b) Yes.
7. **Answer:** It is infinite in isothermal process because $\Delta T = 0 \left(C = \frac{\Delta Q}{m\Delta T}\right)$ and zero in an adiabatic process as $\Delta Q = 0$.
8. Yes, in an adiabatic process the temperature of an isolated system changes. It increases when the gas is compressed adiabatically.
9. **Answer:** No.
10. **Answer:** The room will be slightly warmed.

Short Questions Answers:

1. **Answer:** Suppose we have an engine that gives a continuous supply of work when it is cooled below the temperature of its surroundings.

   This is a violation of Kelvin's statement. Now if the work done by the engine is used to drive a dynamo which produces current and this current produces heat in a coil immersed in hot water, then we have produced a machine which causes the flow of heat from a cold body to the hot body without the help of an external agent. This is a violation of Clausius's statement. Hence both statements are equivalent.

2. **Answer:** Pressure is greater in sample number 1 as can be explained: For isothermal expansion.

   $P_1V_1 = P_2V_2$ for no. 1 sample

   Now $V_2 = 2V_1$

   $\therefore P_1V_1 = P_2 2V_1$

   or

   $$P_2 = \frac{P_1}{2} \quad \text{...(i)}$$

   Now for adiabatic expansion (for sample 2)

   $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

   or

   $$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1\left(\frac{V_1}{2V_1}\right)^{\gamma}$$

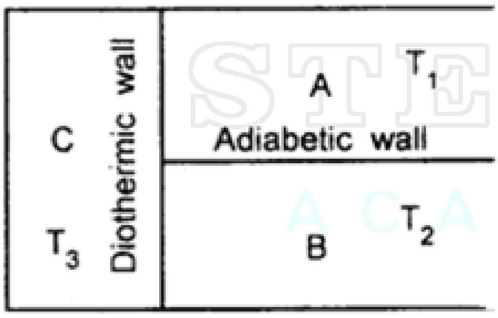
   $$= \frac{P_1}{2^{\gamma}} \quad \text{....}(ii)$$

   $\therefore$ From (i) and (ii) we find that pressure is greater in sample 1 as $\gamma > 1$.

3. **Answer:** A Carnot engine is an ideal engine from the following points of view:
   1. There is no friction between the walls of the cylinder and the piston.
   2. The working substance is an ideal gas *i.e.* the gas molecules do not have molecular attraction and they are points in size.

   However these conditions cannot be fulfilled in a real engine and hence no heat engine working between the same two temperatures can have an efficiency greater than that of a Carnot, engine.
4. **Answer:** If they intersect, then at the point of intersection, the volume and pressure of the gas will be the same at two different temperatures which is not possible.
5. **Answer:** During adiabatic expansion, the temperature and hence the internal energy of the gas decreases. Thus work is done by the gas at the cost of its internal energy.
6. **Answer:** It states that if two systems A and B are in thermal equilibrium with a third system C, then A and B must.be in thermal equilibrium with each other.

   Explanation: The three systems are shown in the figure. Let T1, T2, T3 be the temperatures of A, B, and C respectively.



   Systems A and C, B and C will exchange heat and after a certain time, they will attain thermal equilibrium separately.

   i.e. $T_1 = T_3$ ....(1)

   and $T_2 = T_3$ .... (2)

   Thus from (1) and (2),

   $T_1 = T_2$

   i.e. A and B are now in thermal equilibrium with each other.
7. **Answer:** It states that if an amount of heat dQ is added to a system then a part of it may increase its internal energy by an amount dU and the remaining part may be used up as the external work dW done by the system i.e. mathematically,

   $dQ = dU + dW$

   $= dU + PdV$

   Sign conventions:
   1. Work done by a system is taken as positive while the work done on the system is taken as -ve.
   2. The increase in the internal energy of the system is taken as positive while the decrease in the internal energy is taken as negative.
   3. Heat added (gained) by a system is taken as positive and the heat lost by the system is taken as negative.
8. **Answer:** The heat engine can convert the internal energy of seawater if there is a sink at a temperature lower than the temperature of seawater. Since there is no such sink and hence a ship can't use the internal energy of seawater to operate the engine.
9. **Answer:** The temperature of the system decreases as the system is doing work and no heat transfer is allowed to or from the system. As the temperature of the system decreases, the internal energy of the system also decreases.
10. **Answer:** No. It will not be cooled, rather it will get heated because the speed of the air molecules will increase due to the motion of the fan. We feel cooler because of the evaporation of the sweat when the fan is switched on.

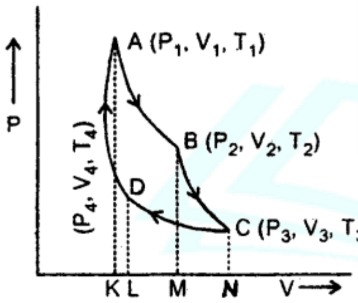
Long Questions Answers:

1. **Answer:** Carnot cycle: Heat engines essentially have
   1. a source of heat,
   2. a W0fkthg substance
   3. a sink (at a temperature lower than that source) and
   4. mechanical parts.

   Carnot designed an idea engine that operated in the reversible cycle. The cycle consisted of two isotherms and two adiabatic. The heat was taken in or rejected during isothermal expansion or contraction. The Carnot cycle thus consists of four steps (see fig.) Carnot took a perfect gas as the working substance enclosed in a cylinder with perfectly insulting walls fitted with an insulating piston but the bases of the cylinder were conducting

(1) In the first step of the cycle let P_1, V_1, by the pressure of the gas. It is placed, in contact with the. source of heat at temperature T_1 i.e the cylinder is out on the source. As the gas expands isothermally it absorbs some amount of heat to keep the temperature constant (curve AB)

The heat absorbed from the source Q_1 is equal to the work done W, in expanding the gas volume from V_1 to V_2 at temperature T_1 so that



$$Q_1 = W_1 = \int_{V_1}^{V_2} RT_1$$

In = Area ABMKA ....(1)

(2) The cylinder is put on insulating and gas is allowed to expand from V_2 to V_3 adiabatically. Its temperature falls from T_1 to T_2 and pressure becomes P_3 and P_2. The work done W is then.

$$W_1 = \int_{v_2}^{v_3} PdV = C_v(T_1 - T_2) = \text{Area BCNMB}$$

....(2)

(3) In this part of the cycle the cylinder is put with its conducting base in contact with a sink as temperature T_2 and gas is compressed isothermally. It rejects Q_2 heat at constant temperature T_2, the work done on the gas is [pressure volume change to (P_4, V_4) from (P_3, V_3)].

$$Q_2 = W_3 = \int_{v_3}^{v_4} PdV = RT\ In\frac{V_3}{V_2} = \text{Area CNLDC}$$

....(3)

(4) In the last step of the cycle, the cylinder's base is again put on the insulating stand, and the gas is compressed adiabatically so that the system returns back to its original state at A i.e. from (P4, V4) to (P1, V1) at temperature T1 via curve DA. Now the work done on the gas is.

$$W_4 = \int_{V_4}^{V_3} PdV = C_V(T_2 - T_1) = -C_V(T_1 - T_2)$$

= Area DLKAD .... (4)

From equation (2) and (4), it is clear that $W_4 = W_2$

If W = net work done by the engine in one cycle, then

$W = W_1 + W_2 + (-W_3) + (-W_4)$

$= W_1 - W_3 =$ Area ABCDA $= Q_1 - Q_2$ .....(5)

The efficiency of the Carnot engine (η): It is defined as the ratio of work done by the engine to the energy supplied to the engine in a cycle.

i.e. $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

Using equations (1) and (3)

$$\frac{Q_1}{Q_2} = \frac{RT_1 ln\frac{V_2}{V_1}}{RT_2 ln\frac{V_3}{V_4}} \quad ....(7)$$

Since B and C lie on the same adiabatic so

$$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$$

or $$\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad ....(8)$$

Also D and A lie on the same adiabatic so

$$T_1V_1^{\gamma-1} = T_2V_4^{\gamma-1}$$

or $$\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad ....(9)$$

∴ from (8) and (9), we get

$$\left(\frac{V_3}{V_2}\right)^{\gamma}=\left(\frac{V_4}{V_1}\right)^{\gamma}$$

$$\ln\frac{V_3}{V_4}=\ln\frac{V_2}{V_1} \quad \text{....(10)}$$

∴ from (7) and (10), we get

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}\frac{ln\left(\frac{V_2}{V_1}\right)}{ln\left(\frac{V_2}{V_1}\right)}=\frac{T_1}{T_2} \quad \text{....(11)}$$

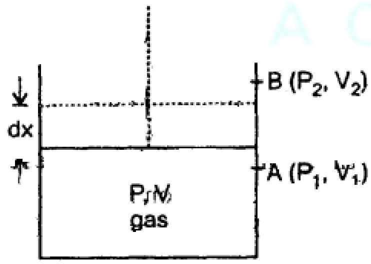
∴ from (6) and (11), we get

$$\eta=1-\frac{T_2}{T_1}$$

or $$\eta=\left(1-\frac{T_2}{T_1}\right)\times 100$$

1. The interesting aspect of η of Carnot engine is that it is independent of the nature of the working substance. But Carnot used an ideal gas operation which is not strictly followed by real gases or fuel
2. Theoretically, η can be 100%.
3. The efficiency of Carnot's ideal engine depends only on the temperature of the scarce and the sink.
4. The efficiency of any reversible engine working between the same two temperatures is the same.

2. **Answer:** Consider one mole of a perfect gas contained in a cylinder having conducting walls and fitted with a movable piston.



Let P, V be the pressure and volume of the gas corresponding to this state.

Let dx = distance by which piston moves outward at constant pressure P so that its volume increases by dV.

Let a = area of cross-section of the piston.

(a) If dW = work done in moving the piston by dx, then .

dW = force on piston × dx

= P a dx

= PdV ...(i)

Where dV = a dx = volume

Let the system goes from initial state A(P_1, V_1) to final state B(P_2, V_2)

If W = total work done from A to B, then

$$W=\int_A^B dW=\int_{V_1}^{V_2} PdV \quad \text{....}(ii)$$

Also we know that

PV = RT (n = 1 here)

$$P=\frac{RT}{V} \quad \text{....}(iii)$$

∴ from (*i*) and (*ii*), we get

$$W=RT\int_{V_1}^{V_2}\frac{1}{V}dV=RT\left[\log_e V\right]_{V_1}^{V_2}$$

$= RT (\log_e V_2 - \log_e V_1)$

$= RT \log_2 \frac{V_2}{V_1}$

$= 2.303\ RT \log_{10} \frac{V_2}{V_1}$

(b) From equation (ii) of case (a), we get

$$W=\int_{V_1}^{V_2} PdV \quad \text{....(ii)}$$

We know that an adiabatic process is represented mathematically by the equation:

PV^{γ} = constant = K

or

$$P=\frac{K}{V^{\gamma}} \quad \text{... (iii)}$$

∴ from (ii) and (iii), we get

$$W=\int_{V_1}^{V_2} KV^{\gamma}dV=K\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$=\frac{K}{1-\gamma}\left[V_2^{1-\gamma}-V_1^{1-\gamma}\right]$$

$$=\frac{1}{1-\gamma}\left[V_1^{1-\gamma}-V_2^{1-\gamma}\right]$$

$$\frac{1}{1-\gamma\left[KV_1^{1-\gamma}-KV_2^{1-\gamma}\right]}$$

$$= \frac{1}{1-\gamma}\left[P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}\right]$$

$$\frac{1}{1-\gamma}\left[P_1V_1 = P_2V_2\right]$$

$$= \frac{1}{1-\gamma}\left[RT_1 - RT_2\right] \qquad (\because PV = RT)$$

$$W = \frac{R}{\gamma-1}\left[T_1 - T_2\right]$$

3. **Answer:** Let V_1 = Initial volume

V_2 = Final volume = $\frac{V_1}{3}$

or

$\frac{V_1}{V_2} = 3$

T_1 = 300K

$T_2 - T_1$ = ?

γ = 1.5

We know that for an adiabatic change,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300\,(3)^{15-1} = 300\sqrt{3}$$

= 300 × 1.732 = 519.6 K

∴ Rise in temperature = $T_2 - T_1$

= 519.6 – 300 = 219.6 K

4. **Answer:** Here, T_1 = 50.0 K

T_2 = 375 k

Q_1 = Heat absorbed per cycle

= 600 K cal

∴ (a) Using tig relation,

$\eta = 1 - \frac{T_2}{T_1}$, we get

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{500-375}{500}$$

$$= \frac{125}{500} = 0.25$$

η% = 0.25 × 100 = 25%

(b) Let W = work done per cycle

∴ Using relation

$\eta = \frac{W}{Q_1}$, we get

W = ηQ1

= 0.25 × 600 K cal

= 150 K cal

= 150 × 10^3 × 4.2 J

= 6.3 × 10^5 J.

(c) Let Q_2 = heat rejected to the sink

∴ Using the relation

W = $Q_1 - Q_2$, we get

$Q_2 = Q_1 - W$ = 600 – 150 = 450 K cal

5. **Answer:** Here, T_1 = 25 + 273 = 298 K

T_2 = – 10 + 273 = 263 K

Q_2 = 263 Js^{-1}

we know that

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or $$Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{263} \times 263$$

= 298 Js^{-1}

∴ Average power consumed = $Q_1 - Q_2$

= (298 – 263) Js^{-1}

= 35W

Assertion Reason Answer:

1. If both assertion and reason are true and the reason is the correct explanation of the assertion.

Explanation:

When a bottle of cold carbonated drink is opened. A slight fog forms around the opening. This is because of adiabatic expansion of gas causes lowering of temperature and condensation of water vapours.

2. If the assertion and reason both are false.

Explanation:

Adiabatic compression is a rapid action and both the internal energy and the temperature increases.

Case Study Questions-

1. Zeroth Law of Thermodynamics states that two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other. The Zeroth Law clearly suggests that when two systems *A* and *B,* are in thermal equilibrium, there must be a physical quantity that has the same value for both. This

thermodynamic variable whose value is equal for two systems in thermal equilibrium is called temperature (T). Thus, if A and B are separately in equilibrium with C, $TA = TC$ and $TB = TC$. This implies that $TA = TB$ i.e. the systems A and B are also in thermal equilibrium. Zeroth Law of Thermodynamics leads to the concept of internal energy of a system. We know that every bulk system consists of a large number of molecules. Internal energy is simply the sum of the kinetic energies and potential energies of these molecules. A certain amount of heat is supplied to the system' or 'a certain amount of work was done by the system its energy changes.

i. **Three thermodynamic systems are at temperature of 50^0 c .what can we say about them?**
   a. Heat flows between them
   b. It obeys Zeroth Law of Thermodynamics
   c. Temperature of one system will increase and temperature of remaining two will decrease
   d. None of these
ii. **Zeroth law of thermodynamics helped in the creation of which scale?**
   a. Temperature
   b. Heat energy
   c. Pressure
   d. Internal energy
iii. **State Zeroth Law of Thermodynamics**
iv. **Define Internal energy of system**

2. **Kelvin-Planck statement:** No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work. **Clausius statement:** No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. It can be proved that the two statements above are completely equivalent. A thermodynamic process is reversible if the process can be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe. a reversible process is an idealized motion. A process is reversible only if it is quasi-static (system in equilibrium with the surroundings at every stage) and there are no dissipative effects. For example, a quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process. The free expansion of a gas is irreversible. The combustion reaction of a mixture of petrol and air ignited by a spark cannot be reversed. Cooking gas leaking from a gas cylinder in the kitchen diffuses to the entire room. The diffusion process will not spontaneously reverse and bring the gas back to the cylinder. The stirring of a liquid in thermal contact with a reservoir will convert the work done into heat, increasing the internal energy of the reservoir. The process cannot be reversed exactly; otherwise it would amount to conversion of heat entirely into work, violating the Second Law of Thermodynamics. Irreversibility is a rule rather an exception in nature.

i. **The diffusion process is**
   a. Reversible process
   b. Irreversible process
ii. **A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is**
   a. Reversible process
   b. Irreversible process
iii. **State Kelvin Planck statement.**
iv. **State Clausius statement.**
v. **Define reversible processes and irreversible processes of thermodynamics.**

Case Study Answer-

1. **Answer**

i. (b) It obeys Zeroth Law of Thermodynamics
ii. (a) Temperature
iii. Zeroth Law of Thermodynamics states that two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other. i.e. when two systems A and B, are in thermal equilibrium individually with system C then these two systems are also in thermal equilibrium with each other.

iv. Internal energy is the sum of the kinetic energies and potential energies of all the molecules possesses by system.

2. Answer

i. (b) Irreversible process

ii. (a) Reversible process

iii. Kelvin-Planck statement states that We cannot construct any device like the heat engine that operates on a cycle, absorbs the heat energy, and completely transforms this energy into an equal amount of work. Some of the heat gets released into the atmosphere. Practically no device bears 100% thermal efficiency.

iv. According to clausius It is nearly impossible for heat to move by itself from a temperature that is lower in temperature to a reservoir that is at a higher temperature. That is we can say that the transfer of heat can only occur spontaneously from high temperature to temperature. i.e No process is possible whose sole result is the transfer of heat from a colder object to a hotter object without any external work provided to do it in short we cannot construct a refrigerator that can operate without any input work.

A thermodynamic process is said to be reversible if both the system and the surroundings return to their original states, with no other change anywhere else in the universe. On the other hand an irreversible process can be defined as a process in which the system and surrounding will not return to their original condition once the process is initiated.

Kinetic Theory 13

Introduction

What is Kinetic Theory

Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

In solids the molecules are very tightly packed as inter molecular space is not present In liquids inter molecular spaces are more as compared to solids and in gases the molecules are very loosely packed as intermolecular spaces are very large.

The random movement of molecules in a gas is explained by kinetic theory of gases.

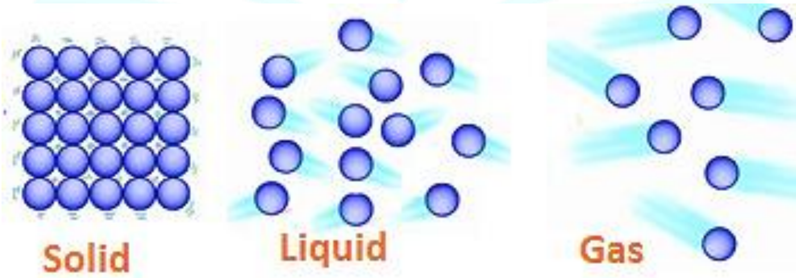
We will also see that why kinetic theory is accepted as a success theory.

Kinetic theory explains the following:

Molecular interpretation of pressure and temperature can be explained.

It is consistent with gas laws and Avogadro's hypothesis.

Correctly explains specific heat capacities of many gases.



Assumptions of Kinetic Theory of Gases

All gas molecules constantly move in random directions.

The size of molecules is very less than the separation between the molecules

The molecules of the sample do not exert any force on the walls of the container during the collision when the gas sample is contained.

It has a very small time interval of collision between two molecules, and between a molecule and the wall.

Collisions between molecules and wall and even between molecules are elastic in nature.

Newton's laws of motion can be seen in all the molecules in a certain gas sample.

With due course of time, a gas sample comes to a steady state. The molecule's distribution and the density of molecules do not depend on the position, distance and time.

Kinetic Theory of Gases

The kinetic theory of gases relates the macroscopic property of the gas, like – Temperature, Pressure, and Volume to the microscopic property of the gas, like – speed, momentum, and position. In this model, the atoms and molecules are continually in random motion, constantly colliding with one another and the walls of the container within which the gas is enclosed. It is this motion that results in physical properties such as heat and pressure. In this article, let us delve deeper into the kinetic theory of gases.

Molecular nature of matter

John Dalton

Atomic hypothesis was given by many scientists. According to which everything in this universe is made up of atoms.

Atoms are little particles that move around in a perpetual order attracting each other when they are little distance apart.

But if they are forced very close to each other then they rebel.

For **example:** Consider a block of gold. It consists of molecules which are constantly moving.

Dalton's atomic theory is also referred as the molecular theory of matter. This theory proves that matter is made up of molecules which in turn are made up of atoms.

According to Gay Lussac's law when gases combine chemically to yield another gas, their volumes are in ratios of small integers.

Avogadro's law states that the equal volumes of all gases at equal temperature and pressure have the same number of molecules.

Conclusion: All these laws proved the molecular nature of gases.

Dalton's molecular theory forms the basis of Kinetic theory.

Why was Dalton's theory a success?

Matter is made up of molecules, which in turn are made up of atoms.

Atomic structure can be viewed by an electron microscope.

Solids, Liquids, Gases in terms of molecular structure

Basis of Difference	Solids	Liquids	Gases
Inter Atomic Distance (distance between molecules).	Molecules are very tightly packed. Inter atomic distance is minimum.	Molecules are not so tightly packed. Inter atomic distance is more as compared to solids.	Molecules are loosely packed. Free to move. Inter atomic distance is maximum.
Mean Free Path is the average distance a molecule can travel without colliding.	No mean free path.	Less mean free path.	There is mean free path followed by the molecules.

Behaviour of Gas Molecules

The behaviour of gas molecules is dependent on the properties and laws obeyed by the molecules of the gas. The distribution of molecules in a gas is very different from the distribution of molecules in liquids and solids. There are five properties and five gas laws that govern the behaviour of gas molecules.

Gas is defined as a homogeneous fluid which has low density and low viscosity and the volume of the gas is assumed to have the volume equal to the volume of the vessel. The classification of gases are:

- Ideal gas
- Non-ideal gas or real gas

Following are the properties of gases:

Property	Symbol	Common units
Density	d	$g.l^{-1}$
Temperature	T	K
Pressure	P	mm Hg
Volume	V	cm
Amount of gas	n	mol

Kinetic Theory of Gases

The behaviour of gas molecules is explained with the help of the kinetic theory of gases. It is the study of gas molecules at the macroscopic level. Following are the five postulates of the kinetic theory of gases:

Gas is the composition of a large number of molecules that are constantly in a random movement.

The volume of the molecules is negligible as the distance between the gas molecules is greater than the size of the molecules.

The intermolecular interactions are also negligible.

The collision of molecules with each other and with the walls of the container is always elastic.

The average kinetic energy of all the molecules is dependent on the temperature.

Boyle's Law

According to Boyle's law, the volume of the gas is inversely related to pressure when the amount of gas is fixed at a constant temperature.

$$P \propto \frac{1}{V}$$

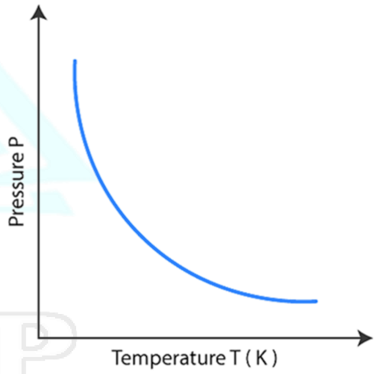
PV = constant

$P_1V_1 = P_2V_2 = P_3V_3$ = constant

Where,

P is the pressure of a gas.

V is the volume of gas.



BOYLE'S LAW

Charles's Law

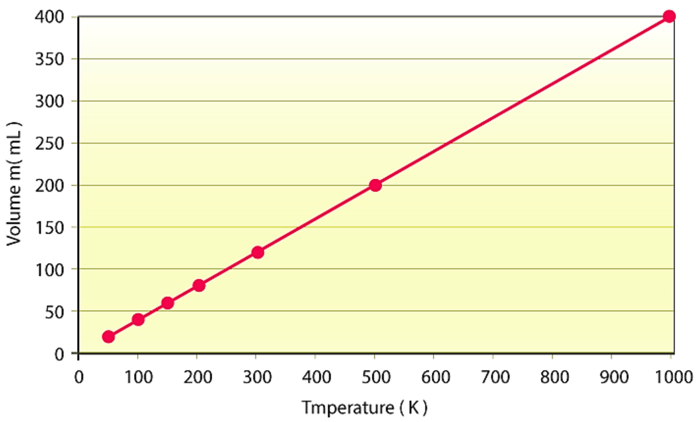
According to Charles's law, the volume of the gas with a fixed mass is directly proportional to the temperature.

$$V \propto T$$

Where,

T is the temperature of a gas.

V is the volume of gas.



CHARLES'S LAW

Gay-Lussac's Law

According to Gay-Lussac's law, when the volume of the gas is constant, the pressure of a given mass of gas varies directly with the absolute temperature of the gas.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

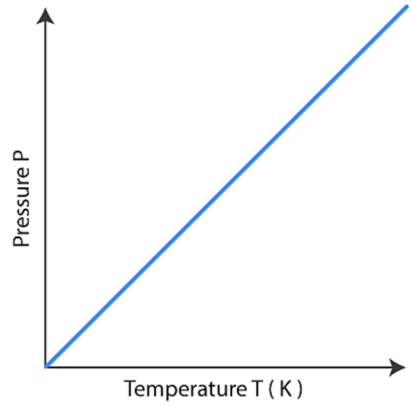
Where,

T_1 is the initial temperature.

P_1 is the initial pressure.

T_2 is the final temperature.

P_2 is the final pressure.



GAY-LUSSAC'S LAW

Avogadro's Law

According to Avogadro's law, when the pressure and temperature of the given gas are constant, then the number of moles and the volume of the gas are in a direct relationship.

$$V \propto n$$

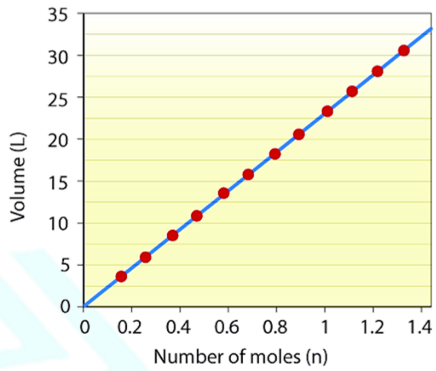
or $$\frac{V}{n} = k$$

Where,

V is the volume of the gas.

n is the number of moles.

k is the proportionality constant.



AVOGADRO'S LAW

Ideal Gas Law

According to ideal gas law, the product of pressure and volume of one gram molecule of an ideal gas is equal to the product of a number of moles of the gas, universal gas constant and the absolute temperature.

$$PV = nRT = NkT$$

Where,

P is the pressure of the gas.

V is the volume of the gas.

n is the number of moles.

R is the universal gas constant = 8.3145 J. $mol^{-1}.K^{-1}$

T is the temperature of the gas

N is Avogadro's number, $N_A = 6.0221 \times 10^{23}$

Specific Heat

Specific heat, Csp, is the amount of heat required to change the heat content of exactly 1 gram of a material by exactly 1°C.

Specific heat values can be determined in the following way: When two materials, each initially at a different temperature, are placed in contact with one another, heat always flows from the warmer material into the colder material until both the materials attain the same temperature. From the law of conservation of energy, the heat gained by the initially colder material must equal the heat lost by the initially warmer material.

We know that when heat energy is absorbed by a substance, its temperature increases. If the same quantity of heat is given to equal masses of different substances, it is observed that the rise in temperature for each substance is different. This is due to the fact that different substances have different heat capacities. So heat capacity of a substance is the quantity of the heat required to raise the temperature of the whole substance by one degree. If the mass of the substance is unity then the heat capacity is called Specific heat capacity or the specific heat.

Specific Heat Capacity Formula

$$Q = C\, m\, \Delta t$$

Where,

Q = quantity of heat absorbed by a body

m = mass of the body

Δt = Rise in temperature

C = Specific heat capacity of a substance depends on the nature of the material of the substance.

S.I unit of specific heat is J kg^{-1} K^{-1}.

Specific Heat Capacity Unit

Heat capacity = Specific heat x mass

Its S.I unit is J K^{-1}.

Monatomic Gases

"Monatomic" is a combination of two words "mono", and "atomic" means a single atom. This term is used in both Physics and Chemistry and is applied to the gases as monatomic gases. In the gaseous phase at sufficiently high temperatures, all the chemical elements are monatomic gases.

Noble gases are monatomic gases as they are unreactive, which is a property of these gases. They do find applications in daily life like

Helium is used in filling balloons as their density is lower than the air's.

Neon is used for creating advertising signs as they glow when electricity flows through them.

Argon is used in a light bulb to prevent the burning of the filament as it is unreactive

Diatomic Molecules

Diatomic molecules are those molecules that are composed of only two atoms. If a diatomic molecule is composed of the same element, it is known as a homonuclear, and if it is composed of two different elements, it is known as heteronuclear.

Polyatomic Ion

A polyatomic ion is also known as a molecular ion that is composed of two or more covalently bonded atoms. It is also referred to as a radical.

Top Formulae

Boyle's law	PV = constant
Charles' law	V/T = constant
Gay-Lussac's law	P/T = constant
Gas equation	PV = μRT, where μ is the number of moles of the given gas.
Pressure exerted by gas	$P = \frac{1}{3}n m\overline{v^2}$
Mean KE of translation per molecule of a gas	$= \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$
Mean KE of translation per mole of a gas	$= \frac{1}{2}M\overline{v^2} = \frac{3}{2}RT = \frac{3}{2}NkT$
Total KE per mole of gas	$= \frac{n}{2}RT$, where n is the number of degrees of freedom of each molecule.

Boyle's law	PV = constant
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Total KE per mole of gas	$= \frac{n}{2} RT$, where n is the number of degrees of freedom of each molecule.
rms speed	$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}}$
Effect of temperature	$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$
Mean free path	$\bar{l} = \frac{k_B T}{\sqrt{2}\, d\, p} = \frac{1}{\sqrt{2}\pi d^2 n}$ where n is the number of molecules per unit volume of the gas.
Collision frequency	$f = v / \lambda$

Class : 11th Physics
Chapter- 13 : Kinetic Theory

Kinetic Theory

Degree of freedom
For monoatomic gas: f = 3 For diatomic gas:
(i) at room temperature, f = 5
(ii) at high temperature, f = 7
For polyatomic gas:
(i)at room temperature, f = 6
(ii)at high temperature, f = 8

Assumption of kinetic Theory of Gases
- All the molecules of a gas are identical.
- The molecules of different gases are different.
- The molecules of gases are in a state of random motion.
- The collisions of gases molecules are perfectly elastic.

Average Speed
$$v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$$
$$v_{av} = \sqrt{\frac{8RT}{\pi M}} = \left(\sqrt{\frac{8}{3\pi}}\right) v_{rms} = 0.92\ v_{rms}$$

Pressure exerted by gas
$$P = \frac{1}{3}\rho v_{rms}^2$$

Root mean square speed
$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

Mean Free Path
$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

Most probable speed
$$v_{mp} = \sqrt{\frac{2RT}{M}} = \left(\sqrt{\frac{2}{3}}\right) v_{rms} = 0.816\ v_{rms}$$

Relation between pressure (p) and Kinetic Energy (E)
$$E = \frac{3}{2}PV$$

Behaviour of Gases

An ideal gas satisfies equation $PV = nRT$ all pressure and temperature n = no. of moles, $R = N_A k_B$ universal gas constant

Specific Heat Capacity
Specific Heat Capacity for an ideal gas, $C_p - C_v = R$

for monoatomic $\frac{C_p}{C_v} = \gamma = \frac{5}{3}$

for diatomic $\frac{C_p}{C_v} = \gamma = \frac{7}{5}$

for polyatomic gases, $\frac{C_p}{C_v} = \gamma = \frac{(4+f)}{(3+f)}$

Hence, f is the degree of freedom

Law of Equipartition Energy
Energy associated with each degree of freedom per molecule $= \frac{1}{2}kT$

Charles's s Law
$V \propto T$ (If P = constant)

$\frac{V}{T}$ = constant

or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Avogadro's law
Under the same condition of temperature and pressure equal volumes of all gases contain equal no. of molecules.

i.e. $N_1 = N_2$

Boyles's Law
$P \propto \frac{1}{V}$ (If T = constant)

PV = constant

or $P_1V_1 = P_2V_2$

Guy Lussac's Law
$P \propto T$ (If V = constant)

$\frac{P}{T}$ = constant

or $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Dalton's Law of partial pressure
Total pressure of a mixture of non - reacting gases,

$P = P_1 + P_2 + ... + P_n$

Important Questions

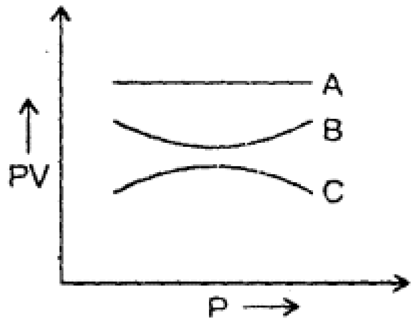
Multiple Choice Questions-

1. A room temperature the r.m.s. velocity of the molecules of a certain diatomic gas is found to be 1930 m/sec. the gas is
   (a) H^2
   (b) F^2
   (c) O^2
   (d) Cl^2
2. Energy supplied to convert unit mass of substance from solid to liquid state at its melting point is called
   (a) Latent heat of fusion
   (b) Evaporation
   (c) Solidification
   (d) Latent heat of fission
3. One any planet, the presence of atmosphere implies [nrms = root mean square velocity of molecules and ne = escape velocity]
   (a) nrms << ne
   (b) nrms > ne
   (c) nrms = ne
   (d) nrms = 0
4. Calculate the RMS velocity of molecules of a gas of which the ratio of two specific heats is 1.42 and velocity of sound in the gas is 500 m/s
   (a) 727 m/s
   (b) 527 m/s
   (c) 927 m/s
   (d) 750 m/s
5. The r.m.s. speed of the molecules of a gas in a vessel is 200 m/s. if 25% of the gas leaks out of the vessel, at constant temperature, then the r.m.s. speed of the remaining molecules will be
   (a) 400 m/s
   (b) 150 m/s
   (c) 100 m/s
   (d) 200 m/s
6. A gas is taken in a sealed container at 300 K. it is heated at constant volume to a temperature 600 K. the mean K.E. of its molecules is
   (a) Halved
   (b) Doubled
   (c) Tripled
   (d) Quadrupled
7. Moon has no atmosphere because
   (a) It is far away form the surface of the earth
   (b) Its surface temperature is 10°C
   (c) The r.m.s. velocity of all the gas molecules is more then the escape velocity of the moons surface
   (d) The escape velocity of the moons surface is more than the r.m.s velocity of all molecules
8. A unit mass of solid converted to liquid at its melting point. Heat is required for this process is:
   (a) Specific heat
   (b) Latent heat of vaporization
   (c) Latent heat of fusion
   (d) External latent heat
9. One mole of ideal gas required 207 J heat to rise the temperature by 10°K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10°K the heat required is (R = 8/3 J/mole °K)
   (a) 1987 J
   (b) 29 J
   (c) 215.3 J
   (d) 124 J
10. The r.m.s velocity of the molecules of an ideal gas is C at a temperature of 100K. at what temperature is r.m.s. velocity will be doubted?
    (a) 200 K
    (b) 400 K
    (c) 300 K
    (d) 50 K

Very Short:

1. What does gas constant R signify? What is its value?
2. What is the nature of the curve obtained when:
   (a) Pressure versus reciprocal volume is plotted for an ideal gas at a constant temperature.
   (b) Volume of an ideal gas is plotted against its absolute temperature at constant pressure.
3. The graph shows the variation of the product of PV with the pressure of the constant mass of

three gases A, B and C. If all the changes are at a constant temperature, then which of the three gases is an ideal gas? Why?



4. On the basis of Charle's law, what is the minimum possible temperature?
5. What would be the ratio of initial and final pressures if the masses of all the molecules of a gas are halved and their speeds are doubled?
6. Water solidifies into ice at 273 K. What happens to the K.E. of water molecules?
7. Name three gas laws that can be obtained from the gas equation.
8. What is the average velocity of the molecules of a gas in equilibrium?
9. A vessel is filled with a mixture of two different gases. Will the mean kinetic energies per molecule of both gases be equal? Why?
10. The density of a gas is doubled, keeping all other factors unchanged. What will be the effect on the pressure of the gas?

Short Questions:

1. Why cooling is caused by evaporation?
2. On reducing the volume of the gas at a constant temperature, the pressure of the gas increases. Explain on the basis of the kinetic theory of gases.
3. Why temperature less than absolute zero is not possible?
4. There are n molecules of a gas in a container. If the number of molecules is increased to 2n, what will be:

   (a) the pressure of the gas.

   (b) the total energy of the gas.

   (c) r.m.s. speed of the gas molecules.
5. Equal masses of O_2 and He gases are supplied equal amounts of heat. Which gas will undergo a greater temperature rise and why?
6. Two bodies of specific heats S_1 and S_2 having the same heat capacities are combined to form a single composite body. What is the specific heat of the composite body?
7. Tell the degree of freedom of:

   (a) Monoatomic gas moles.

   (b) Diatomic gas moles.

   (c) Polyatomic gas moles.
8. State law of equipartition of energy.
9. Explain why it is not possible to increase the temperature of gas while keeping its volume and pressure constant?
10. A glass of water is stirred and then allowed to stand until the water stops moving. What has happened to the K.E. of the moving water?

Long Questions:

1. Calculate r.m.s. the velocity of hydrogen at N.T.P. Given the density of hydrogen = 0.09 kg m^4.
2. Calculate the temperature at which r.m.s. the velocity of the gas molecule is double its value at 27°C, the pressure of the gas remaining the same.
3. Calculate the K.E./mole of a gas at N.T.P. Density of gas at N.T.P. = 0.178 g dm^{-3} and molecular weight = 4.
4. Calculate the diameter of a molecule if n = 2.79 × 10^{25} molecules per m3 and mean free path = 2.2 × 10^{-8} m.
5. Calculate the number of molecules in 1 cm^3 of a perfect gas at 27°C and at a pressure of 10 mm of Hg. Mean K.E. of a molecule at 27°C = 4 × 1025 J. ρ_{Hg} = 13.6 × 103 kg m^{-3}.

Assertion Reason Questions:

1. **Directions:** Choose the correct option from the following:

   (a) Both A and R are true, and R is the correct explanation of A

   (b) Both A and R are true, but R is NOT the correct explanation of A

   (c) A is true but R is false

   (d) A is false and R is also false

   Assertion (A): The number of degrees of freedom of a linear triatomic molecules is 7.

   Reason (R): The number of degrees of freedom depends on number of particles in the system.

2. **Directions:** Choose the correct option from the following:
   (a) Both A and R are true, and R is the correct explanation of A
   (b) Both A and R are true, but R is NOT the correct explanation of A
   (c) A is true but R is false
   (d) A is false and R is also false

   Assertion (A): Absolute zero is not the temperature corresponding to zero energy.

   Reason (R): The temperature at which no molecular motion ceases is called absolute zero temperature.

Answer Key

Multiple Choice Answers-

1. **Answer:** (a) H^2
2. **Answer:** (a) Latent heat of fusion
3. Answer (a) nrms << ne
4. **Answer:** (a) 727 m/s
5. **Answer:** (d) 200 m/s
6. **Answer:** (b) Doubled
7. **Answer:** (c) The r.m.s. velocity of all the gas molecules is more then the escape velocity of the moons surface
8. **Answer:** (c) Latent heat of fusion
9. **Answer:** (d) 124 J
10. **Answer:** (b) 400 K

Very Short Answers:

1. **Answer:** The universal gas constant (R) signifies the work done by (or on) a gas per mole per kelvin. Its value is 8.31 J mol^{-1} K
2. **Answer:** (a)It is a straight line.
   (b) It is a straight line.
3. **Answer:** A is an ideal gas because PV is constant at constant temperature for an ideal gas.
4. **Answer:** – 273.15°C.
5. **Answer:** 1: 2 ($\because$ P $= \frac{13mn}{V} C^2$)
6. **Answer:** It is partly converted into the binding energy of ice.
7. **Answer:**
   1. Boyle's law
   2. Charle's law
   3. Gay Lussac's law.
8. **Answer:** Zero.
9. Yes. This is because the mean K.E. per molecule i.e. $\frac{3}{2}$kT depends only upon the temperature.
10. It will be doubled. ($\because$ P $\propto \rho$ if other factors are constant).

Short Questions Answers:

1. **Answer:** Evaporation occurs on account of faster molecules escaping from the surface of the liquid. The liquid is therefore left with molecules having lower speeds. The decrease in the average speed of molecules results in lowering the temperature and hence cooling is caused.
2. **Answer:** On reducing the volume, the space for the given number of molecules of the gas decreases i.e. no. of molecules per unit volume increases. As a result of which more molecules collide with the walls of the vessel per second and hence a larger momentum is transferred to the walls per second. Due to which the pressure of gas increases.
3. **Answer:** According to the kinetic interpretation of temperature, absolute temperature means the kinetic energy of molecules.

   As heat is taken out, the temperature falls and hence velocity decreases. At absolute zero, the velocity of the molecules becomes zero i.e. kinetic energy becomes zero. So no more decrease in K.E. is possible, hence temperature cannot fall further.
4. **Answer:** (a) We know that

   $P = \frac{1}{3} mnC^2$.

   where n = no. of molecules per unit volume.

   Thus when no. of molecules is increased from n to 2n, no. of molecules per unit volume (n) will increase from n 2n

   $\frac{n}{V}$ to $\frac{2n}{V}$, hence pressure will become double.

   (b) The K.E. of a gas molecule is,

$\frac{1}{2}mC^2 = \frac{3}{2}kT$

If the no. of molecules is increased from n to 2n. There is no effect on the average K.E. of a gas molecule, but the total energy is doubled.

r.m.s speed of gas is $C_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3P}{mn}}$

When n is increased from n to 2n. both n and P become double and the ratio $\frac{P}{n}$ remains unchanged. So there will be no effect of increasing the number of molecule from n to 2n on r.m.s. speed of gas molecule.

5. **Answer:** Helium is monoatomic while O_2 is diatomic. In the case of helium, the supplied heat has to increase only the translational K.E. of the gas molecules.

   On the other hand, in the case of oxygen, the supplied heat has to increase the translations, vibrational and rotational K.E. of gas molecules. Thus helium would undergo a greater temperature rise.

6. **Answer:** Let m_1 and m_2 be the masses of two bodies having heat capacities S1 and S1 respectively.

   $\therefore (m_1 + m_2)S = m_1S_1 + m_2S_2 = m_1S_1 + m_1S_1 = 2m_1S_1$

   $$S = \frac{2m_1S_1}{m_1 + m_2}$$

   Also, $m_2S_2 = m_1S_1$

   ($\because$ Heat capacities of two bodies are same.)

   or $$m_2 = \frac{m_1S_1}{S_2}$$

   $\therefore$ $$S = \frac{2m_1S_1}{m_1 + \frac{m_1S_1}{S_2}} = \frac{2S_1S_2}{S_1 + S_2}$$

7. **Answer:** (a) A monoatomic gas possesses 3 translational degrees of freedom for each molecule.

   (b) A diatomic gas molecule has 5 degrees of freedom including 3 translational and 2 rotational degrees of freedom.

   (c) The polyatomic gas molecule has 6 degrees of freedom (3 translational and 3 rotational).

8. **Answer:** It states that in equilibrium, the total energy of the system is divided equally in all possible energy modes with each mode i.e. degree of freedom having an average energy equal to $\frac{1}{2}$ K_BT.

9. **Answer:** It is not possible to increase the temperature of a gas keeping volume and pressure constant can be explained as follows:

   According to the Kinetic Theory of gases,

   $$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2$$

   $$= \frac{1}{3}\frac{M}{V}kT$$

   ($\because C^2 = kT$, when k is a constant)

   $$T \propto PV$$

   Now as T is directly proportional to the product of P and V. If P and V are constant, then T is also constant.

10. **Answer:** The K.E. of moving water is dissipated into internal energy. The temperature of water thus increases.

Long Questions Answers:

1. **Answer:**

   Here, $\rho = 0.09 \text{ kg m}^{-3}$

   P = 76 cm of Hg

   $= 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$

   $= 1.01 \times 105 \text{ Nm}^{-2}$

   C = ?

   Using the relation,

   $P = \frac{1}{3}\rho C^2$, we get

   $$C = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$$

   $$= \sqrt{3.37 \times 10^6}$$

   $$C = 1.836 \times 10^3 \text{ ms}^{-1}$$

   $= 1836 \text{ ms}^{-1}$

2. **Answer:** Let t be the required temperature = ? and Ct, C27 be the r.m.s. velocities of the gas molecules at t°C and 27°C respectively.

   $\frac{C_t}{C_{27}} = 2$ (given)

   Also let M = molecular weight of the gas

   Now T = t + 273

   and T27 = 27 + 273 = 300 K

   $\therefore$ Using the relation

$$C = \sqrt{\frac{3RT}{M}}, \text{ we get}$$

$$C_t = \sqrt{\frac{3RT}{M}}$$

and $$C_{27} = \sqrt{\frac{3RT_{27}}{M}}$$

$\therefore$ $$\frac{C_t}{C_{27}} = \sqrt{\frac{T}{T_{27}}} = \sqrt{\frac{t+273}{300}}$$

or $$2 = \sqrt{\frac{t+273}{300}}$$

or $$4 = \frac{t+273}{300}$$

or $t = 1200 - 273 = 9274°C$

3. **Answer:** Here, $\rho = 0.178$ g dm^{-3}

$= 0.178 \times 10^{-3}$ g cm^{-3} ($\because$ 1 $dm^3 = 10^{-3}$ cm^3)

$= 178 \times 10^{-6}$ g cm^{-3}

Volume of 1 mole of gas i.e. 4 g of gas = $\frac{\text{Mass}}{\text{Density}}$

or $$V = \frac{M}{\rho} = \frac{4}{178 \times 10^{-6}} cm^3$$

T = 273 K at NTP

$\therefore$ $$R = \frac{PV}{T} = \frac{76 \times 13.6 \times 980 \times 4}{178 \times 10^{-6} \times 273}$$

K.E/mole = ?

We know that

$$\text{K.E/mole} = \frac{3}{2}\text{RT}$$

$$= \frac{3}{2} \times \frac{76 \times 13.6 \times 980 \times 4}{178 \times 10^{-6} \times 273} \times 273$$

$= 3.42 \times 10^{10}$ erg

$$= \frac{3.42 \times 10^{10}}{10^7} J = 3.42 \times 10^3 J$$

4. **Answer:** Here, $n = 2.79 \times 10^{25}$ molecules m^{-3}

$\lambda = 2.2 \times 10^{-8}$ m

d = ?

Using the relation.

$$\lambda = \frac{1}{\sqrt{2}} \frac{1}{\pi n d^2}, \text{ we get}$$

$$d^2 = \frac{1}{\sqrt{2}} \frac{1}{\pi n \lambda}$$

$$= \frac{1}{1.414 \times 3.142 \times 2.79 \times 10^{35} \times 2.2 \times 10^{8}}$$

$= 0.03666 \times 10^{-7}$ m^2

$= 0.367 \times 10^{-18}$ m^2

$\therefore$ $$d = \sqrt{0.367 \times 10^{-18} m^2}$$

$= 0.606 \times 10^{-9}$ m = 606 nm

5. **Answer:** Here, K..E. per molecule at 27°C = 4×10^{-11} J

Let μ = number of molecules in 1 cm^3 or 10-6 m^3

$\therefore$ Mean K.E. per cm3 = $\mu \times 4 \times 1011$ J ....(i)

Now K.E. per gram molecule = $\frac{3}{2}$ RT

for a perfect gas, PV = RT

$\therefore$ K.E, per gram molecule = $\frac{3}{2}$ PV

or

K.E. per cm3 of gas = $\frac{3}{2}$ PV

P = 10 mm of Hg = 10^{-2} m of Hg

$= 10^{-2} \times 13.6 \times 10^3 \times 9.8$

$= 136 \times 9.8$ Nm^{-2} V

= 1 cm^3

= 10^{-6} m^3

$\therefore$ K.E per cm^3 of gas = $\frac{3}{2} \times 136 \times 9.8 \times 106$

$= 1.969 \times 10^{-3}$ J ....(ii)

$\therefore$ from (i) and (ii) we get

$\mu \times 4 \times 10^{-11} = 1.969 \times 10^{-3}$

or

$$\mu = \frac{1.969 \times 10^{-3}}{4 \times 10^{11}}$$

= 4.92 × 107 molecules.

Assertion Reason Answer:

1. (b) Both A and R are true, but R is NOT the correct explanation of A
2. (a) Both A and R are true, and R is the correct explanation of A

Case Study Questions-

1. Boyle's law is a gas law which states that the pressure exerted by a gas (of a given mass, kept at a constant temperature) is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept constant. For a gas, the relationship between volume and pressure (at constant mass and

temperature) can be expressed mathematically as follows. P ∝ (1/V) Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant, k. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure. The law also states that the Kelvin temperature and the volume will be in direct proportion when the pressure exerted on a sample of a dry gas is held constant. Charles law and Boyle's law applied to low density gas only. The total pressure of a mixture of ideal gases is the sum of partial pressures. This is Dalton's law of partial pressures.

i. **Boyle's law is obeyed by high as well as low density gases. True or False?**
   a. True
   b. False
ii. **Charles law is states that volume of an ideal gas is directly proportional to temperature at constant**
   a. Temperature
   b. Pressure
   c. Volume
   d. None of these
iii. **State Daltons law of partial pressures**
iv. **State Boyle's law**
v. **State Charles law**

2. Pressure of an Ideal Gas: according to kinetic theory of gases pressure is given by P = 1/3 nmv2 Where, n is number of molecules per unit volume, m is mass and v2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial. The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature, a macroscopic measurable parameter of a gas (a thermodynamic variable as it is called) to a molecular quantity, namely the average kinetic energy of a molecule. The two domains are connected by the Boltzmann constant and given by E = kbT. Where kb is Boltzmann constant having value of 1.38*10-23 joule per Kelvin. We have seen that in thermal equilibrium at absolute temperature T, for each translational mode of motion, the average energy is ½ Kb T. The most elegant principle of classical statistical mechanics (first proved by Maxwell) states that this is so for each mode of energy: translational, rotational and vibrational. That is, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to ½ kB T. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes ½ kB T to the energy, while each vibrational frequency contributes 2 × ½ kB T = kB T, since a vibrational mode has both kinetic and potential energy modes.

i. **Boltzmann constant has value of**
   a. **$1.38*10^{-23}$** joule per Kelvin.
   b. **$1.38*10^{-28}$** joule per Kelvin.
   c. **$1.38*10^{-30}$** joule per Kelvin.
   d. None of these
ii. **SI unit of Boltzmann constant is given by**
   a. Joules per meter
   b. Joules per Kelvin
   c. Joules per Newton
   d. None of these
iii. **According to kinetic theory give formula for pressure of idea gas.**
iv. **According to kinetic theory what is average kinetic energy of molecules of ideal gas?**
v. **What is law of equipartition of energy?**

Case Study Answer-

1. Answer

i. (a) True
ii. (b) Pressure
iii. The total pressure of a mixture of ideal gases is the sum of partial pressures exerted by all the molecules of gas. This is Dalton's law of partial pressures.
iv. Boyle's law is a gas law which states that at constant temperature the pressure exerted by a gas is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept

constant. For a gas, the **P (1/V)** Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant k.

v. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure.

2. Answer

i. (a) **$1.38*10^{-23}$** joule per Kelvin.

ii. (b) Joules per Kelvin

iii. According to kinetic theory of gases pressure is given by P = **$1/3\ nmv^2$** Where, n is number of molecules per unit volume, m is mass and v^2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial.

iv. **The average kinetic energy** of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas and given by **$E = 3/2\ k_bT$.**

v. Where k_b is Boltzmann constant having value of **$1.38*10^{-23}$** joule per Kelvin.

We know that for each translational mode of motion, the average energy is **½ Kb T.** classical statistical mechanics states that in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to **½ k_BT.** This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes **½ k_BT** to the energy, while each vibrational frequency contributes **$2 \times ½\ k_B T = k_BT$**, since a vibrational mode has both kinetic and potential energy modes.

Oscillations 14

Introduction

In this chapter we will learn about oscillatory motion or oscillations. Any motion which repeats itself at regular intervals of time is known as periodic motion. If a body moves back and forth repeatedly about its mean position, then it is said to be in oscillatory motion.

For example: The to and fro movement of pendulum, jumping on a trampoline, a child swinging on a swing.

Oscillations can be defined as Periodic to and fro motion which repeat itself at regular intervals of time.

To and from motion of pendulum

Child on a swing

Kids jumping on the trampoline

Oscillatory Motion and Periodic Motion

Periodic motion is defined as the motion that repeats itself after fixed intervals of time. This fixed interval of time is known as time period of the periodic motion. Examples of periodic motion are motion of hands of the clock, motion of planets around the sun etc.

Oscillatory motion is defined as the to and from motion of the body about its fixed position. Oscillatory motion is a type of periodic motion. Examples of oscillatory motion are vibrating strings, swinging of the swing etc.

Oscillatory Motion

Oscillatory motion is defined as the to and from motion of an object from its mean position. The ideal condition is that the object can be in oscillatory motion forever in the absence of friction but in the real world, this is not possible and the object has to settle into equilibrium.

To describe mechanical oscillation, the term vibration is used which is found in a swinging pendulum. Likewise, the beating of the human heart is an example of oscillation in dynamic systems.

Examples of Oscillatory Motion

Following are the examples of oscillatory motion:

- Oscillation of simple pendulum
- Vibrating strings of musical instruments is a mechanical example of oscillatory motion
- Movement of spring
- Alternating current is an electrical example of oscillatory motion
- Series of oscillations are seen in cosmological model

Simple Harmonic Motion

Simple harmonic motion (SHM) is a type of oscillatory motion which is defined for the particle moving along a straight line with an acceleration which is moving towards a fixed point on the line such that the magnitude is proportional to the distance from the fixed point.

For any simple mechanical harmonic system (system of the weight hung by the spring to the wall) that is displaced from its equilibrium position, a restoring force which obeys the Hooke's law is required to restore the system back to equilibrium. Following is the mathematical representation of restoring force:

$$F = -kx$$

Where,

F is the restoring elastic force exerted by the spring (N)

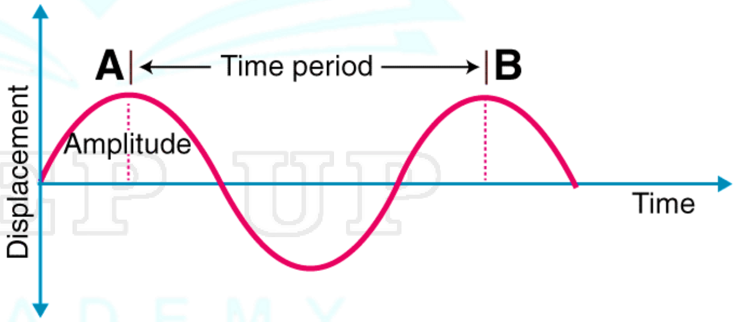
k is the spring constant (Nm^{-1})

x is the displacement from equilibrium position (m)

Periodic Motion

We can classify the motion of various bodies on the basis of the way they move. For example, a car moving on a straight road is said to have linear motion. Similarly, the motion of the earth around the sun is circular motion. In this session, we shall be discussing periodic motion along with its formula.

A motion that repeats itself after equal intervals of time is known as periodic motion.



Examples of periodic motion: a tuning fork or motion of a pendulum if you analyze the motion you will find that the pendulum passes through the mean position only after a definite interval of time. We can also classify the above motion to be oscillatory. An oscillatory is a motion in which the body moves to and from about a fixed position. So an oscillatory motion can be periodic but it is not necessary.

So taking an example of a wave motion we will see some parameters related to periodic motion. Let's take the following figure:

Periodic Motion Formula

Time Period (T): It is the time taken by the motion to repeat itself. So the unit of a time period is seconds.

Frequency (f): It is defined as a number of times the motion is repeated in one second. The unit of frequency is Hz (Hertz). Frequency is related to Time period as:

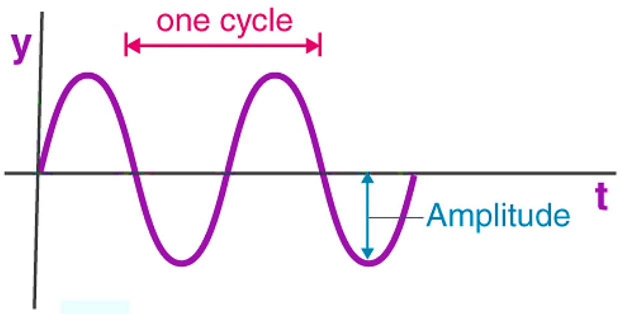
$$f = \frac{1}{T}$$

Frequency, Time Period and Angular Frequency

As we know, many forms of energy like light and sound travel in waves. A wave is defined through various characteristics like frequency, amplitude and speed. In wave mechanics, any given wave enfolds parameters like – frequency, time period, wavelength, amplitude etc. This article lets us understand and learn in detail about frequency, time period, and angular frequency.

Parameters of a Wave

Frequency definition states that it is the number of complete cycles of waves passing a point in unit time. The time period is the time taken by a complete cycle of the wave to pass a point. Angular frequency is angular displacement of any element of the wave per unit of time.



Consider the graph shown below. It represents the displacement y of any element for a harmonic wave along a string moving in the positive x-direction with respect to time. Here, the string element moves up and down in simple harmonic motion.

The relation describing the displacement of the element with respect to time is given as:

$y(0,t) = a \sin(-\omega t)$, here we have considered the inception of wavefrom x=0

$y(0,t) = -a \sin(\omega t)$

As we know, sinusoidal or harmonic motion is periodic in nature, i.e. the nature of the graph of an element of the wave repeats itself at a fixed duration. To mark the duration of periodicity following terms are introduced for sinusoidal waves.

Time Period

As shown above, the particles move about the mean equilibrium or mean position with time in a sinusoidal wave motion. The particles rise until they reach the highest point, the crest, and then continue to fall until they reach the lowest point, the trough. The cycle repeats itself in a uniform pattern. The time period of oscillation of a wave is defined as the time taken by any string element to complete one such oscillation. For a sine wave represented by the equation:

$y(0, t) = -a \sin(\omega t)$

The time period formula is given as:

$$T = \frac{2\pi}{\omega}$$

Frequency

We define the frequency of a sinusoidal wave as the number of complete oscillations made by any wave element per unit of time. By the definition of frequency, we can understand that if a body is in periodic motion, it has undergone one cycle after passing through a series of events or positions and returning to its original state. Thus, frequency is a parameter that describes the rate of oscillation and vibration.

The equation gives the relation between the frequency and the period:

The relation between the frequency and the period is given by the equation:

$f=1/T$

For a sinusoidal wave represented by the equation:

$y(0,t) = -a \sin(\omega t)$

The formula of the frequency with the SI unit is given as:

Formula

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

SI unit

Hertz

Angular Frequency

For a sinusoidal wave, the angular frequency refers to the angular displacement of any element of the wave per unit of time or the rate of change of the phase of the waveform. It is represented by ω. Angular frequency formula and SI unit are given as:

Formula

$$\omega = \frac{2\pi}{T} = 2\pi f$$

SI unit

rads^{-1}

Where,

ω = angular frequency of the wave.

T = time period of the wave.

f = ordinary frequency of the wave.

Displacement as a function of time and Periodic function

To understand this idea of displacement as a function of time, we will have to derive an expression for displacement, assume a body traveling at an initial velocity of v_1 at the time t_1 and then the body accelerates at a constant acceleration of 'a' for some time and a final velocity of v_2 at the time t_2, keeping these things in assumption let's derive the following.

Let's write displacement as

$$d = V_{average} * \Delta t$$

Where Δt is the change in time, assuming that the object is under constant acceleration.

$$d = \left(\frac{V_1 + V_2}{2}\right) * \Delta t$$

Where V_2 and V_1 are final and initial velocities respectively, let's rewrite final velocity in terms of initial velocity for the sake of simplicity.

$$d = \left(\frac{V_1 + (V_1 + a * \Delta t)}{2}\right) * \Delta t$$

Where a is the constant acceleration the body is moving at, now if we rewrite the above as,

$$d = \left(\frac{2 * V_1}{2} + \frac{a * \Delta t}{2}\right) * \Delta t$$

The above expression is one of the most fundamental expressions in kinematics, it is also sometimes given as

$$d = V_i t + \frac{1}{2} a t^2$$

Where Vi is the initial velocity, and t is actually the change in time, all the quantities in this derivation, like Velocity, displacement and acceleration, are vector quantities.

Velocity of a particle executing Simple Harmonic Motion

Velocity in SHM is given by $v = dx/dt$,

$$x = A\sin(\omega t + \phi)$$

$$v = \frac{d}{dt}A\sin(\omega t + \phi) = \omega A\cos(\omega t + \phi)$$

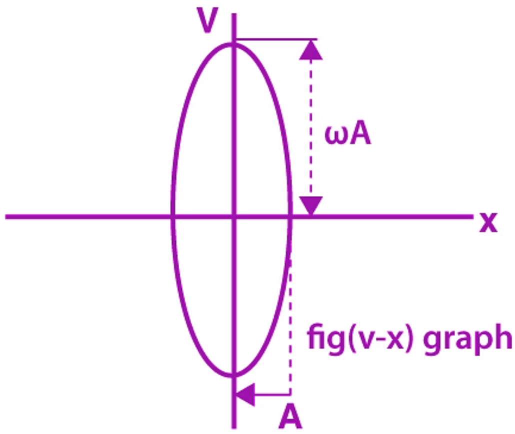
$$v = A\omega\sqrt{1 - \sin^2 \omega t}$$

since, $x = A\sin \omega t$

$$\frac{x^2}{A^2} = \sin^2 \omega t$$

$$\Rightarrow \quad v = A\omega\sqrt{1 - \frac{x^2}{A^2}}$$

$$\Rightarrow \quad v = \omega\sqrt{A^2 - x^2}$$



fig(v-x) graph

On squaring both sides

$$\Rightarrow \quad v^2 = \omega^2(A^2 - x^2)$$

$$\Rightarrow \quad \frac{v^2}{\omega^2} = (A^2 - x^2)$$

$$\Rightarrow \quad \frac{v^2}{\omega^2 A^2} = \left(1 - \frac{x^2}{A^2}\right)$$

$$\Rightarrow \quad \frac{v^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

this is an equation of an ellipse.

The curve between displacement and velocity of a particle executing the simple harmonic motion is an ellipse.

When ω = 1, then the curve between v and x will be circular.

Acceleration in SHM

$$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t + \phi)$$

$$\Rightarrow \quad \vec{a} = -\omega^2 A\sin(\omega t + \phi)$$

$$\Rightarrow \quad |a| = -\omega^2 x$$

Hence the expression for displacement, velocity and acceleration in linear simple harmonic motion are:

$$x = A\sin(\omega t + \phi)$$

$$v = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$$

and $\quad a = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$

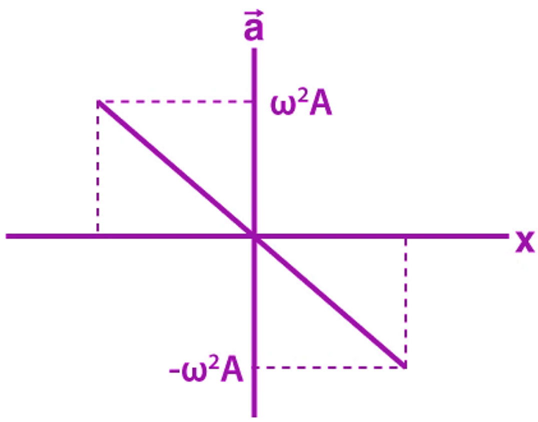


fig. $\vec{a}$ - $\vec{x}$ graph

Energy in Simple Harmonic Motion (SHM)

The system that executes SHM is called the harmonic oscillator. Consider a particle of mass m, executing linear simple harmonic motion of angular frequency (ω) and amplitude (A), the displacement $(\vec{x})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ at any time t are given by:

$$x = A\sin(\omega t + \phi)$$

$$v = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$$

$$a = -\omega^2 A\sin(\omega t + \phi) = -\omega^2 x$$

The restoring force $(\vec{F})$ acting on the particle is given by:

$$F = -kx, \text{ where } k = m\omega^2$$

Kinetic Energy of a Particle in SHM

$$= \frac{1}{2}mv^2 \left[\text{Since}, v^2 = A^2\omega^2\cos^2(\omega t + \phi)\right]$$

$$= \frac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi)$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$

Therefore, the Kinetic Energy

$$= \frac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi) = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Potential Energy of SHM

The total work done by the restoring force in displacing the particle from (x = 0) (mean position) to $x = x$:

When the particle has been displaced from x to $x + dx$, the work done by restoring force is $dw = F\,dx = -kx\,dx$

$$w = \int dw = \int_0^x -kx\,dx = \frac{-kx^2}{2}$$

$$= -\frac{m\omega^2 x^2}{2}$$

$$\left[k = m\omega^2\right]$$

$$= -\frac{m\omega^2 x^2}{2}A^2\sin^2(\omega t + \phi)$$

Potential Energy = –(work done by restoring force)

$$= \frac{m\omega^2 x^2}{2} = \frac{m\omega^2 A^2}{2}\sin^2(\omega t + \phi)$$

Total Mechanical Energy of the Particle Executing SHM

E = KE + PE

$$E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$E = \frac{1}{2}m\omega^2 A^2$$

Hence, the particle's total energy in SHM is constant, independent of the instantaneous displacement.

⇒ Relationship between Kinetic Energy, Potential Energy and time in Simple Harmonic Motion at t = 0, when x = ±A.

⇒ Variation of Kinetic Energy and Potential Energy in Simple Harmonic Motion with displacement:

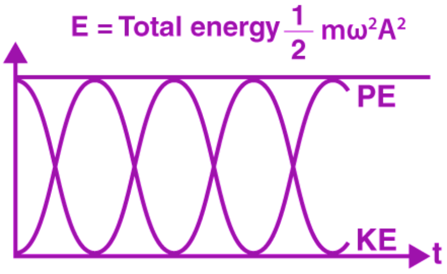
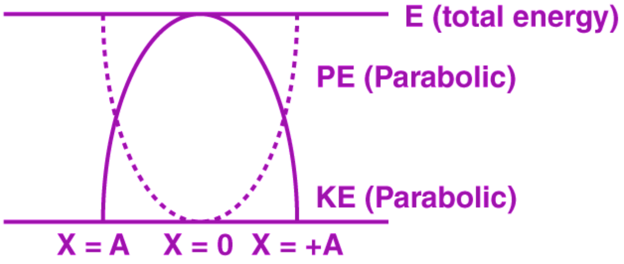


fig. (PE-KE-t) graph



Simple Pendulum Definition

A simple pendulum is a mechanical arrangement that demonstrates periodic motion. The simple pendulum comprises a small bob of mass 'm' suspended by a thin string secured to a platform at its upper end of length L.

The simple pendulum is a mechanical system that sways or moves in an oscillatory motion. This motion occurs in a vertical plane and is mainly driven by the gravitational force. Interestingly, the bob that is suspended at the end of a thread very light somewhat we can say it is even massless. The period of a simple pendulum can be made extended by increasing the length string while taking the measurements from the point of suspension to the middle of the bob. However, it should be noted that if the mass of the bob is changed it will the period remains unchanged. Period is influenced mainly by the position of the pendulum in relation to Earth as the strength of gravitational field is not uniform everywhere.

Meanwhile, pendulums are a common system whose usage is seen in various instances. Some are used in clocks to keep track of the time while some are just used for fun in case of a child's swing. In some cases, it is used in an unconventional manner such as a sinker on a fishing line. In any case, we will explore and learn more about the simple pendulum on this page. We will discover the conditions under which it performs simple harmonic motion as well as derive an interesting expression for its period.

Important Terms

The oscillatory motion of a simple pendulum: Oscillatory motion is defined as the to and fro motion of the pendulum in a periodic fashion and the centre point of oscillation known as equilibrium position.

The time period of a simple pendulum: It is defined as the time taken by the pendulum to finish one full oscillation and is denoted by "T".

The amplitude of simple pendulum: It is defined as the distance travelled by the pendulum from the equilibrium position to one side.

Length of a simple pendulum: It is defined as the distance between the point of suspension to the centre of the bob and is denoted by "l".

Spring Mass System Arrangements

Spring mass systems can be arranged in two ways. These include;

The parallel combination of springs

Series combination of springs

We will discuss them below;

Parallel Combination of Springs

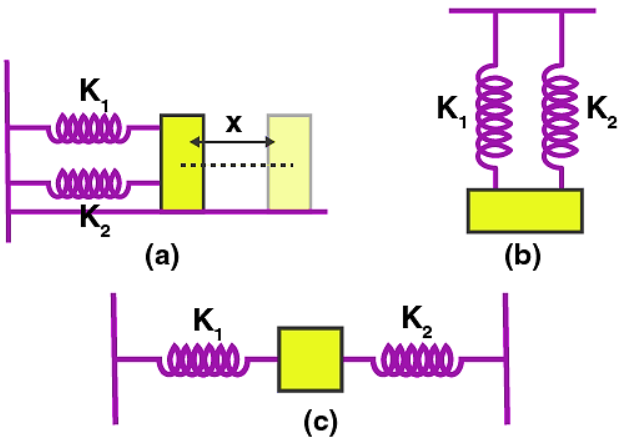


Fig (a), (b) and (c) – are the parallel combination of springs.

Displacement on each spring is the same.

But restoring force is different;

$$F = F_1 + F_2$$

Since, F = -kx, the above equation can be written as

$$\Rightarrow \qquad -k_p x = -k_1 x = k_2 x$$

$$\Rightarrow \quad -xk_p = -x(k_1 + k_2)$$

$$\Rightarrow \quad k_p = k_1 + k_2$$

Time Period of Simple Pendulum Derivation

Using the equation of motion, T – mg cosθ = mv²L

The torque tending to bring the mass to its equilibrium position,

τ = mgL × sinθ = mgsinθ × L = I × α

For small angles of oscillations sin θ ≈ θ,

Therefore, Iα = -mgLθ

$$\alpha = -(mgL\theta)/I$$

$$-\omega_0^2\theta = -(mgL\theta)/I$$

$$\omega_0^2 = (mgL)/I$$

$$\omega_0 = \sqrt{(mgL/I)}$$

Using I = ML², [where I denote the moment of inertia of bob]

we get, $\omega_0 = \sqrt{(g/L)}$

Therefore, the time period of a simple pendulum is given by,

$$T = 2\Pi/\omega_0 = 2\Pi \times \sqrt{(L/g)}$$

Top Formulae

Displacement in SHM	$y = a \sin(\omega t \pm \phi_0)$
Velocity in SHM	$v = \omega\sqrt{a^2 - y^2}$
Acceleration in SHM	$a = -\omega^2 y$ and $\omega = 2\pi\nu = 2\pi/T$
Potential energy in SHM	$U = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}k y^2$
Kinetic energy in SHM	$K = \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}k(a^2 - y^2)$
Total energy	$E = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}ka^2$
Spring constant	$k = F/y$
Spring constant of parallel combination of springs	$k = k_1 + k_2$
Spring constant of series combination of springs	$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k}$
Time period	$T = 2\pi\sqrt{\frac{m}{K}}$
Equation of displacement in damped oscillation	If the damping force is given by $F_d = -b v$, where v is the velocity of the oscillator and b is its damping constant, then the displacement of the oscillator is given by $x(t) = A e^{-bt/2m} \cos(\omega' t + \Phi)$, where ω' is the angular frequency of the damped oscillator and is given by $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
Mechanical energy E of damped oscillator	$E(t) = \frac{1}{2}kA^2e^{-bt/m}$

Class : 11th Physics
Chapter- 14 : Oscillations

Velocity in SHM

$v=\frac{dx}{d(t)}=-\omega A\sin(\omega t+\varphi)$

$v_{max}=\omega A$

Acceleration in SHM

$a=\frac{dv}{dt}=-\omega^2 A\cos(\omega t+\varphi)$

$a_{max}=\omega^2 A$

Energy in SHM

Kinetic Energy

$K.E=\frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi)$

Potential Energy

$P.E=\frac{1}{2}m\omega^2A^2\cos^2(\omega t+\varphi)$

Total energy, T.E.=K.E.+ P.E.

$=\frac{1}{2}kA^2[\cos^2(\omega t+\varphi)+\sin^2(\omega t+\varphi)]$

Displacement in SHM

x(t)= A(cosωt+φ)

A= Amplitude i.e.,maximum displacement of particles.

Force law equation for SHM

$F=-k(x), k=m\omega^2$

$\therefore \omega=\sqrt{\frac{k}{m}}$

Phase

Time varying quantity(ωt + φ)

Simple Harmonic Motion is the simplest form of oscillatory motion.

Phase constant or Phase angle (φ)

- It depends upon velocity (v) and displacement of particle at t = 0

Time period(T).

The smallest interval of time after which the motion is repeated

SHM

Frequency

The number of oscillations per second.

i.e., $f=\frac{1}{T}=\frac{\omega}{2\pi}$

Oscillations

- Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.
- To and from motion repeatedly about a fixed point in a definite interval of time.

Forced Oscillations

Driving force, $F(t)=F_o\cos\omega_d t$

Displacement, $x(t)=A\cos(\omega_d t+\varphi)$

Amplitude, $A=\frac{F_o}{\left\{m^2(\omega^2-\omega_d^2)^2+\omega_d^2b^2\right\}^{\frac{1}{2}}}$

$\tan\varphi=\frac{-v_o}{\omega_d x_o}$

Oscillations due to a spring

Time period $T=2\pi\sqrt{\frac{m}{k}}$

Here, $k=\frac{F}{x}$ = spring constant

Simple pendulum

Time period $T=2\pi\sqrt{\frac{I}{mgl}}=2\pi\sqrt{\frac{l}{g}}$

Damped Oscillations

Oscillation of a body whose amplitude goes on decreasing with time.

Damping force $F_d=-bv$, b = damping constant

Displacement of damped oscillator

$x=x_m e^{-bt/2m}\sin(\omega t+\varphi)$

Amplitude decreases with time as

$x=x_m e^{-(b/2m)t}$

Resonance

When frequency of external force is equal to natural frequency of oscillator.

Important Questions

Multiple Choice Questions-

1. If an simple pendulum oscillates with an amplitude of 50 mm and time period of 2s, then its maximum velocity is
   (a) 0.10 m/s
   (b) 0.16 m/s
   (c) 0.25 m/s
   (d) 0.5 m/s
2. If the frequency of the particle executing S.H.M. is n, the frequency of its kinetic energy becoming maximum is
   (a) n/2
   (b) n
   (c) 2n
   (d) 4n
3. Spring is pulled down by 2 cm. What is amplitude of motion?
   (a) 0 cm
   (b) 6 cm
   (c) 2 cm
   (d) cm
4. The period of thin magnet is 4 sec. if it is divided into two equal halves then the time period of each part will be
   (a) 4 sec
   (b) 1 sec
   (c) 2 sec
   (d) 8 sec
5. The acceleration of particle executing S.H.M. when it is at mean position is
   (a) Infinite
   (b) Varies
   (c) Maximum
   (d) Zero
6. A spring of force constant k is cut into two pieces such that on piece is double the length of the other. Then the long piece will have a force constant of
   (a) 2 k/3
   (b) 3 k/2
   (c) 3 k
   (d) 6 k
7. Particle moves from extreme position to mean position, its
   (a) Kinetic energy increases, potential increases decreases
   (b) Kinetic energy decreases, potential increases
   (c) Both remains constant
   (d) Potential energy becomes zero and kinetic energy remains constant
8. Grap of potential energy vs. displacement of a S.H. Oscillator is
   (a) parabolic
   (b) hyperbolic
   (c) elliptical
   (d) linear
9. The time-period of S.H.O. is 16 sec. Starting from mean position, its velocity is 0.4 m/s after 2 sec. Its amplitude is
   (a) 0.36 m
   (b) 0.72 m
   (c) 1.44 m
   (d) 2.88 m
10. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will
    (a) Remain unchanged
    (b) Increase
    (c) Decrease
    (d) Become erratic

Assertion Reason Questions:

1. ***Directions:***
   (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
   (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
   (c) If assertion is true but reason is false.
   (d) If the assertion and reason both are false

Assertion: Sine and cosine functions are periodic functions.

Reason: Sinusoidal functions repeats it values after a definite interval of time.

2. ***Directions:***

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false

Assertion: Simple harmonic motion is a uniform motion.

Reason: Simple harmonic motion is not the projection of uniform circular motion.

Answer Key

MCQ Answers-

1. **Answer:** (b) 0.16 m/s
2. **Answer:** (c) 2n
3. **Answer:** (c) 2 cm
4. **Answer:** (c) 2 sec
5. **Answer:** (d) Zero
6. **Answer:** (b) 3 k/2
7. **Answer:** (a) Kinetic energy increases, potential increases decreases
8. **Answer:** (a) parabolic
9. **Answer:** (c) 1.44 m
10. **Answer:** (b) Increase

Very Short Questions-

1. How is the time period effected, if the amplitude of a simple pendulum is in Creased?
2. Define force constant of a spring.
3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?
4. How is the frequency of oscillation related with the frequency of change in the of K. E and PE of the body in S.H.M.?
5. What is the frequency of total energy of a particle in S.H.M.?
6. How is the length of seconds pendulum related with acceleration due gravity of any planet?
7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.
8. How is the time period of the pendulum effected when pendulum is taken to hills Or in mines?
9. Define angular frequency. Give its S.I. unit.
10. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Very Short Answers-

1. **Ans.** No effect on time period when amplitude of pendulum is increased or decreased.
2. **Ans.** The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.
3. **Ans.** Not at the mid-point, between mean and extreme position. it will be at $x = a\sqrt{2}$.
4. **Ans.** P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of S.H.M
5. **Ans.** The frequency of total energy of particle is S.H.M is zero because it retain constant.
6. **Ans.** Length of the seconds pendulum proportional to acceleration due to gravity)
7. **Ans.** Increased

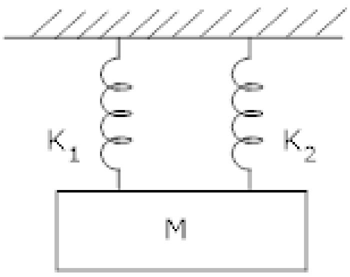
$$T\alpha \frac{1}{\sqrt{g}}$$

8. **Ans.** As T will increase.
9. **Ans.** It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π. ω = 2π v, S.I. unit is rads s^{-1}

10. **Ans.** No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

Short Questions-

1. A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period 't' of the combined system?



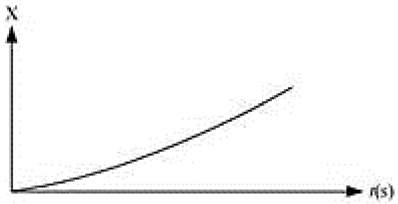
2. Show that the total energy of a body executing SHN is independent of time?
3. A particles moves such that its acceleration 'a' is given by a = -b x where x = displacement from equilibrium position and b is a constant. Find the period of oscillation? 2
4. A particle is S.H.N. is described by the displacement function:

$$x = A\cos(wt + \Phi);\, w = \frac{2\pi}{T}$$

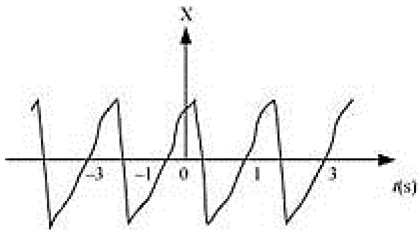
If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm | s, What are its amplitude and phase angle?

5. Determine the time period of a simple pendulum of length = l when mass of bob = m Kg? 3
6. Which of the following examples represent periodic motion?
   (a) A swimmer completing one (return) trip from one bank of a river to the other and back.
   (b) A freely suspended bar magnet displaced from its N-S direction and released.
   (c) A hydrogen molecule rotating about its center of mass.
   (d) An arrow released from a bow.
7. Figure 14.27 depicts four *x-t* plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

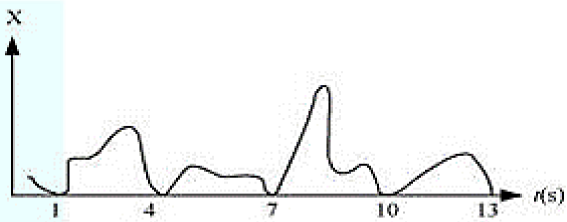
(a)



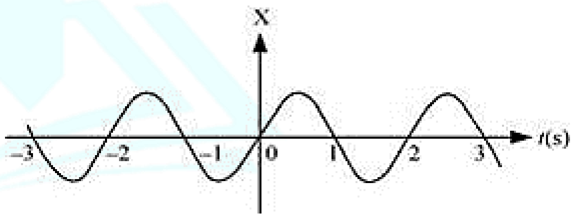
(b)



(c)



(d)



8. Which of the following relationships between the acceleration *a* and the displacement *x* of a particle involve simple harmonic motion?
   (a) $a = 0.7x$
   (b) $a = -200\,x^2$
   (c) $a = -10x$
   (d) $a = 100\,x^3$
9. The acceleration due to gravity on the surface of moon is 1.7ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (*g* on the surface of earth is 9.8 ms^{-2})
10. A simple pendulum of length *l* and having a bob of mass *M* is suspended in a car. The car is moving on a circular track of radius *R* with a uniform speed *v*. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Short Answers-

1. **Ans.** If T = Time Period of simple pendulum

m = Mass

k = Spring constant

$$T = 2\pi\sqrt{\frac{m}{k}}$$

then,

or $$k = \frac{4\pi^2 m}{T^2}$$

$$\rightarrow k_1 = \frac{4\pi^2 m}{t_1^2} \text{let } T = t_1$$

For first spring:

$$\rightarrow k_2 = \frac{4\pi^2 m}{t_2^2} \text{let } T = t_2$$

For second spring:

When springs is connected in parallel, effective spring constant, k = k = $k_1 + k_2$

or $$k = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

If t = total time period

$$\frac{4\pi^2 m}{t^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

$$\frac{1}{t^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$$

or $$t^{-2} = t_1^{-2} + t_2^{-2}$$

2. **Ans.** Let y = displacement at any time't'

a = amplitude

w = Angular frequency

v = velocity,

y = a Sin wt

$$v = \frac{dy}{dt} = \frac{d}{dt}(a \sin wt)$$

So, $$v = a\, w \cos wt$$

Now, kinetic energy = K. E. $= \frac{1}{2}mv^2$

$$K.E. = \frac{1}{2}mw^2a^2\cos^2 wt \rightarrow 1)$$

Potential energy $= \frac{1}{2}ky^2$

$$P.E. = \frac{1}{2}ka^2\sin^2 wt \rightarrow 2)$$

Adding equation 1) & 2)

Total energy = K.E. + P.E.

$$= \frac{1}{2}mw^2a^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$$

Since $$w = \sqrt{\frac{k}{m}} \Rightarrow w^2 m = k^2$$

Total energy $$= \frac{1}{2}mw^2a^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$$

$$= \frac{1}{2}ka^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$$

$$= \frac{1}{2}ka^2(\cos^2 wt + \sin^2 wt)$$

Total energy $= \frac{1}{2}ka$

Thus total mechanical energy is always constant is equal to $\frac{1}{2}ka^2$. The total energy is independent to time. The potential energy oscillates with time and has a maximum value of $\frac{ka^2}{2}$. Similarly, K. E. oscillates with time and has a maximum value of $\frac{ka^2}{2}$. At any instant = constant = $\frac{ka^2}{2}$. The K. E or P.E. oscillates at double the frequency of S.H.M.

3. **Ans.** Given that $a = -b\,x$, Since $a \propto x$ and is directed apposite to x, the particle do moves in S. H. M.

a = b x (in magnitude)

or $$\frac{x}{a} = \frac{1}{b}$$

or $$\frac{\text{Displacement}}{\text{Accleration}} = \frac{1}{b} \rightarrow 1)$$

Time period = $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Accleration}}}$

Using equation 1)

$$T = 2\pi\sqrt{\frac{1}{b}}$$

4. **Ans.** Att = 0; x = 1 cm; $w = \pi/s$

t = Time

x = Position

w = Argular frequency

$\therefore$ $$x = A\cos(Wt + \phi)$$

$$1 = A\cos(\pi \times 0 + \phi)$$

$$1 = A\cos\phi$$

Now, $v = \frac{dx}{dt} = \frac{d}{dt}(A\cos(wt+\phi))$

Att = 0; $v = \pi$ cm/s; $w = \pi/s$

$$\cancel{\pi} = -A\cancel{\pi}\sin(\pi\times 0+\phi)$$

$\Rightarrow$ $$-1 = A\sin\phi \rightarrow 2)$$

Squaring and adding 1) & 2)

$$A^2\cos^2\phi + A^2\sin^2\phi = 1+1$$

$$A^2(\cos^2\phi+\sin^2\phi) = 2$$

$$A^2 = 2$$

$$A = \sqrt{2}\,cm$$

Dividing 2) by 1), we have:

$$\frac{\cancel{A}\sin\phi}{\cancel{A}\cos\phi} = -1$$

$$\tan\phi = -1$$

or $$\phi \equiv \tan^{-1}(-1)$$

$$\phi = \frac{3\pi}{4}$$

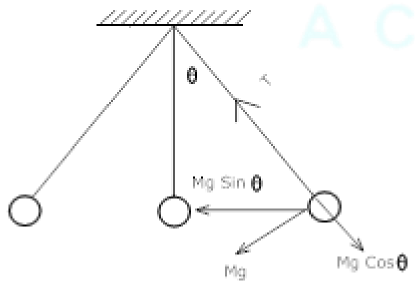
5. **Ans.** It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of B ob

x = Displacement = OB

l = length of simple pendulum



Let the bob is displaced through a small angle θ the forces acting on it:-

1) weight = Mg acting vertically downwards.

2) Tension = T acting upwards.

Divide Mg into its components → Mg Cos θ & Mg Sin θ

T = Mg Cos θ

F = Mg Sin θ

–ve sign shows force is divested towards the ocean positions. If θ = Small,

$$\cong \theta = \frac{Arc\ OB}{l} = \frac{x}{l}$$

Sin θ

$$F = -Mg\frac{x}{l}$$

In S.H.M., vestoring fore,

$$F = -mg\,\theta\, F = -mg\frac{x}{l} \rightarrow 1)$$

Also, if k = spring constant

F = –k x

$$\cancel{-}mg\frac{\cancel{x}}{l} = \cancel{-}k\,\cancel{x}\left(\text{equating } F = -mg\frac{x}{l}\right)$$

$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{\cancel{m}\times 1}{\cancel{m}g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

i.e. 1) Time period depends on length of pendulum and 'g' of place where experiment is done.

2) T is independent of amplitude of vibration provided and it is small and also of the mass of bob.

6. **Ans. (b)** and **(c)**

(a) The swimmer"s motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

7. **Ans. (b)** and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

8. **Ans.** (c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$$\therefore \quad a = -\frac{k}{m}x$$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation $a = -10\,x$ is written in the above form with $\frac{k}{m} = 10$

Hence, this relation represents SHM.

9. **Ans.** Acceleration due to gravity on the surface of moon, $g' = 1.7\ ms^{-2}$

Acceleration due to gravity on the surface of earth, $g = 9.8\ ms^{-2}$

Time period of a simple pendulum on earth, T = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

l is the length of the pendulum

$$\therefore \quad l = \frac{T^2}{(2\pi)^2} \times g$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\,m$$

The length of the pendulum remains constant.

On moon"s surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4\times(3.14)^2}\times 9.8}{1.7}} = 8.4\,s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

10. **Ans.** The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration $= \frac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

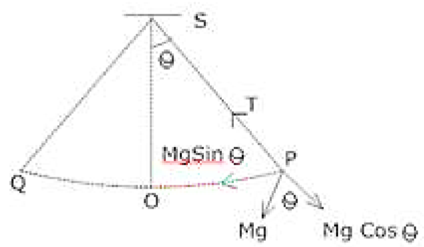
Time period, $T = 2\pi\sqrt{\frac{1}{a_{eff}}}$

Where, l is the length of the pendulum

$\therefore$Time period, $T = 2\pi\sqrt{\frac{1}{g^2 + \frac{v^4}{R^2}}}$

Long questions-

1. What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?



2. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

(b) at the end B,

(c) at the mid-point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from B going towards A.

3. The motion of a particle executing simple harmonic motion is described by the displacement function,

$x(t) = A\cos(\omega t + \omega)$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s-1. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ω t + α), what are the amplitude and initial phase of the particle with the above initial conditions.

4. In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

5. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

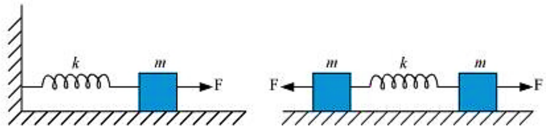
(a) x = -2 sin (3t + π/3)

(b) x = cos (π/6 - t)

(c) x = 3 sin (2πt + π/4)

(d) x = 2 cos πt

6. Figure 14.30 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force F.

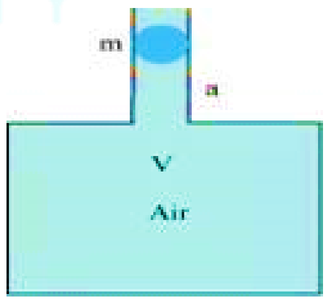


(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

8. An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal seeFig.14.33.



Long Answers-

1. **Ans.** A simple pendulum is the most common example of the body executing S.H.M, it consist of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support, which is free to oscillate.

Let m = mass of bob

l = length of pendulum

Let O is the equilibrium position, OP = X

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

1) The weight = M g acting vertically downwards.

2) The tension = T in string acting along Ps.

Resolving Mg into 2 components as Mg Cos θ and Mg Sin θ,

Now, T = Mg Cos θ

Restoring force F = - Mg Sin θ

-ve sign shows force is directed towards mean position.

Let θ = Small, so Sin θ ≈ θ = $\frac{Arc(op)}{1} = \frac{x}{1}$

Hence F = - mg θ

$F = -mg\frac{x}{l} \to 3)$

Now, In S.H.M, F = k x →4) k = Spring constant

Equating equation3) & 4) for F

$-k\,x = -mg\ \frac{x}{l}$

Spring factor $= k = \frac{mg}{l}$

Inertia factor = Mass of bob = m

Now, Time period = T

$= 2\pi\sqrt{\frac{Inertia\ factor}{Spring\ factor}}$

$= 2\pi\sqrt{\frac{m}{mg/l}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

2. **Ans. (a)** Zero, Positive, Positive

(b) Zero, Negative, Negative

(c) Negative, Zero, Zero

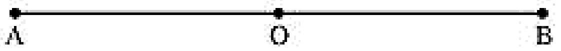
(d) Negative, Negative, Negative

(e) Zero, Positive, Positive

(f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.



A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along AO.

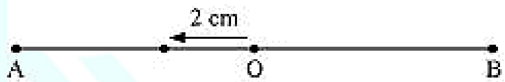
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along B.

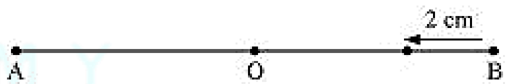
Force is also negative in this case as the particle is directed leftward.

(c)



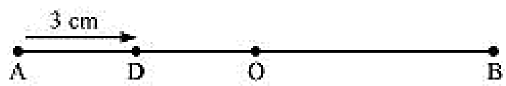
The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)



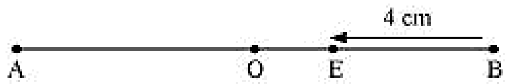
The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle's velocity and acceleration, and the force on it are all negative.

(e)



The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in **(d)**.

3. **Ans.** Initially, at $t = 0$:

Displacement, $x = 1$ cm

Initial velocity, $v = \omega$ cm/sec.

Angular frequency, $\omega = \pi$ rad/s^{-1}

It is given that:

$$x(x) = A\cos(\omega t + \phi)$$

$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$

$$A\cos\phi = 1 \qquad ....(i)$$

$$\text{Velocity, } v = \frac{dx}{dt}$$

$$\omega = -A\omega\sin(\omega t + \phi)$$

$$1 = A\sin(\omega \times 0 + \phi) = A\sin\phi$$

$$A\sin\phi = 1 \qquad ....(ii)$$

Squaring and adding equations (*i*) and (*ii*), we get:

$$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$$

$$A^2 = 2$$

$$\therefore \qquad A = \sqrt{2}\text{ cm}$$

Dividing equation (*ii*) by equation (*i*), we get:

$$\tan\phi = -1$$

$$\therefore \qquad \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, ...$$

SHM is given as:

$$x = B\sin(\omega t + a)$$

Putting the given values in this equation, we get:

$$1 = B\sin\left[\omega \times 0 + a\right]$$

$$B\sin a = 1 \qquad ....(iii)$$

$$\text{Velocity, } \quad v = \omega B\cos(\omega t + a)$$

Substituting the given values, we get:

$$\pi = \pi B\sin a$$

$$B\sin a = 1 \qquad ....(iv)$$

Squaring and adding equations (*iii*) and (*iv*), we get:

$$B^2\left[\sin^2 a + \cos^2 a\right] = 1 + 1$$

$$B^2 = 2$$

$$\therefore \qquad B = \sqrt{2}\text{ cm}$$

Dividing equation (*iii*) by equation (*iv*), we get:

$$\frac{B\sin a}{B\cos a} = \frac{1}{1}$$

$$\tan a = 1 = \tan\frac{\pi}{4}$$

$$a\frac{\pi}{4}, \frac{5\pi}{4}, ...$$

4. **Ans. (a)** $x = 2\sin 20t$

(b) $x = 2\cos 20t$

(c) $x = -2\cos 20t$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200$ Nm^{-1}

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\, rads^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin\omega t$

$= 2\sin 20t$

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

$$\text{Displacement, } \quad x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= 2\sin\left(20t + \frac{\pi}{2}\right)$$

$$= 2\cos 20t$$

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

$$\text{Displacement, } \quad x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$$

$$= 2\sin\left(20t + \frac{\pi}{2}\right)$$

$= -2\cos 20t$

The functions have the same frequency $\left(\frac{20}{2\pi}Hz\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

5. Ans.(a)

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$= 2\cos\left(3t + \frac{5\pi}{6}\right)$$

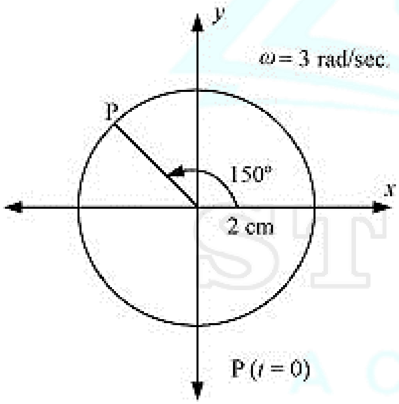
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3 rad/\text{sec}.$

The motion of the particle can be plotted as shown in the following figure.



(b) $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$

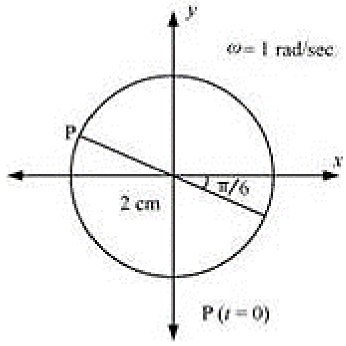
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A=2

Phase angle, $\phi = \frac{\pi}{6} = 30°$

Angular velocity, $\omega = \frac{2\pi}{T} = 1\, rad/\text{sec}.$

The motion of the particle can be plotted as shown in the following figure.



(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

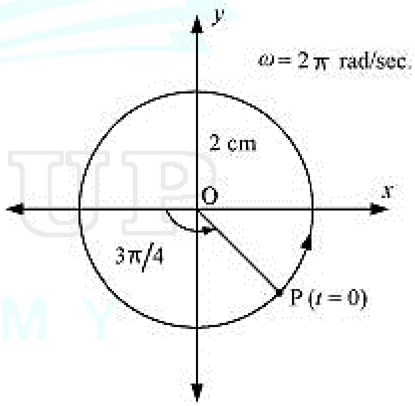
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 3 cm

Phase angle, $\phi = \frac{3\pi}{4} = 135°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3 rad/\text{sec}.$

The motion of the particle can be plotted as shown in the following figure.



(d) $x = 2\cos \pi t$

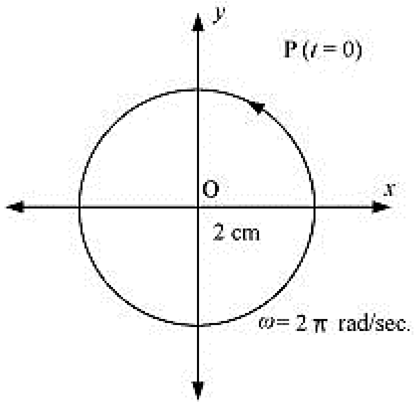
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = 0$

Angular velocity, ω = π rad/s

The motion of the particle can be plotted as shown in the following figure.



6. **Ans.(a)** For the one block system:

When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:

$$F = kl$$

Where, k is the spring constant

Hence, the maximum extension produced in the spring, $l = \frac{F}{k}$

For the two block system:

The displacement (x) produced in this case is:

$$x = \frac{1}{2}$$

Net force, $F = +2\ kx\ \ 2k\frac{1}{2}$

$$\therefore \qquad l = \frac{F}{k}$$

(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\frac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore \qquad m\frac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

Where, $\omega^2 = \frac{k}{m}$

ω is angular frequency of the oscillation

$\therefore$ Time period of the oscillation, $T = \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

For the two block system:

$$F = m\frac{d^2x}{dr^2}$$

$$m\frac{d^2x}{dr^2} = -2kr$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dr^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where,

Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

7. **Ans.** Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

$F = -(\text{Volume} \times \text{Density} \times g)$

$F = -(A \times 2h \times \rho \times g) = -2\rho gh$

$= -k \times$ Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, m = Volume of mercury $\times$ Density of mercury

$= Al\rho$

$$T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{1}{2g}}$$

∴ Hence, the mercury column executes simple harmonic motion with time period

$2\pi\sqrt{\frac{l}{2g}}$.

8. **Ans.** Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

Volumetric strain $\frac{\text{Change in volume}}{\text{Original volume}}$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{ax}{V}$$

Bulk Modulus of air $B = \frac{Stress}{Strain} = \frac{-p}{\frac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball, $F = p \times a$

$$\frac{-Bax}{V} \cdot a$$

$$= \frac{-Ba^2x}{V}$$

In simple harmonic motion, the equation for restoring force is:

$$F = -kx \quad \dots (ii)$$

Where, k is the spring constant

Comparing equations (i) and (ii), we get:

$$= \frac{Ba^2}{V}$$

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

Assertion Reason Answer:

1. (a) If both assertion and reason are true but reason is not the correct explanation of the assertion.
2. (d) If the assertion and reason both are false.

Case Study Questions-

1. A motion that repeats itself at regular intervals of time is called periodic motion. Very often, the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory. The smallest interval of time after which the motion is repeated is called its period. Let us denote the period by the symbol T. Its SI unit is second. The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz). Answer the following. a)

   i. Every oscillatory motion is periodic motion true or false?
      a. True
      b. False

   ii. Circular motion is
      a. Oscillatory motion
      b. Periodic motion
      c. Rotational motion
      d. None of these

   iii. Define period. Give its SI unit and dimensions

   iv. Define frequency of periodic motion. How it is related to time period

   v. What is oscillatory motion

2. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations

are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations. We consider the case when the external force is itself fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. The maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

i. When a system oscillates with its natural frequency ω, and the oscillations are called
   a. Free oscillations
   b. Forced oscillations
ii. All free oscillations eventually die out because of
   a. Damping force
   b. electromagnetic force
   c. None of these
iii. What is free oscillation?
iv. What is forced oscillations?
v. What is resonance?

Case Study Answer-

1. Answer

i. (a) True
ii. (b) Periodic motion
iii. The smallest interval of time after which the motion is repeated is called its period. Its SI unit is second and dimensions are [T1].
iv. Reciprocal of Time period (T) gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T i.e. they are inversely proportional to each other. The unit of n is thus s-1 or hertz.
v. Oscillatory motion is type of periodic motion in which body performs periodic to and fro motion about some mean position. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

2. Answer

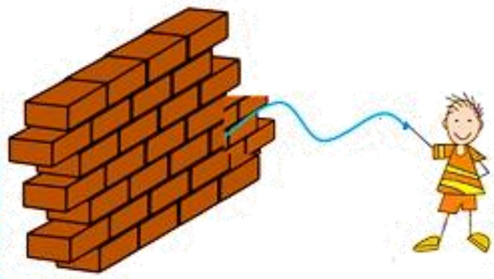
i. (a) Free oscillations
ii. (b) Damping force
iii. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations.
iv. Forced oscillations are oscillations where external force drives the oscillations with frequency given by external force.

   The phenomenon of increase in amplitude when the driving force is close to encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

Waves 15

Introduction

In this chapter we will see the importance of waves in our life.



We will also study about the different properties of waves, some terms related to waves and also about different types of waves.We will also learn how waves propagate.

For example: -

1. Medium required by the waves to travel from one point to another:-
   - Consider a boy holding a thread and one end of thread is tied to the wall.
   - When a boy moves the thread, the thread moves in the form of a wave.
   - Similarly a boat sailing over the sea,the boat is able to move because of waves.
   - The ripples formed in a lake when we drop a stone in the lake.They are also waves.
   - Earthquakes are caused due to the waves under the surface of the earth.
   - The strings of the guitar when we play them are also waves again.
   - Music system which we use to hear songs.This is due to sound waves.
   - When 2 people talk they are able to hear each other because of the sound waves.

   In the below Picture we can see waves need a medium to propagate.
2. Medium not required by some type of waves to move from one point to another:-
   - TV remote waves play important part.
   - Satelliteshelp ustouseTV, mobile phones, music system, the sun, the traffic lights, microwave,x-rays.

     Some type of waves can propagate from one point to another without any medium.

3. Waves which are related to matter:-
    - There are some set of waves which are inside the matter.
    - For example: - whole of universe.

    Waves propagating inside the matter

What is a wave?

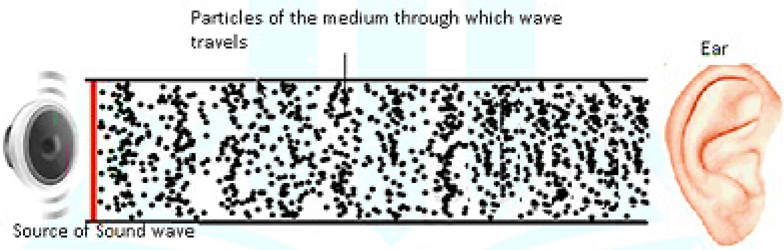
A wave is s disturbance that propagates through spaceand time,usually with transference of energy.

For example: -

- Consider the sound of the horn; this sound reaches our ear because of sound waves.
- There is transfer of energy from one point to another with the help of particles in the medium.
- These particles don't move they just move around their mean position,but the energy is getting transferred from one particle to another and it keeps on transferring till it reaches the destination.
- The movement of a particle is initiated by the disturbance.And this disturbance is transferred from one point to another through space and time.

  Note:- Energy and not the matter is transferred from one point to another.

1. When a source of energy causes vibration to travel through the medium a wave is created.



Types of Waves

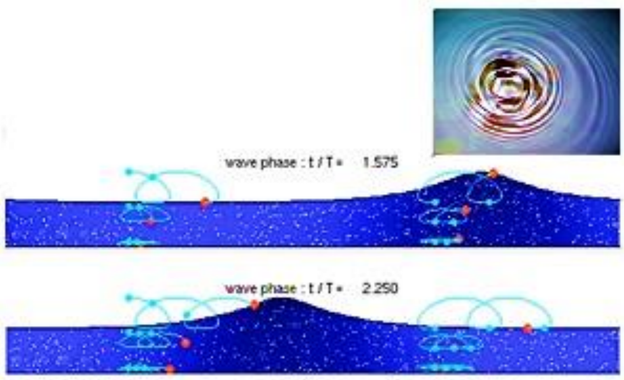
1. Mechanical waves
2. Electromagnetic waves
3. Matter waves

Mechanical Waves:

1. The mechanical waves are governed by all the Newton's laws of motion.
2. Medium is needed for propagation of the wave.

   For Example: Water Waves, Sound Waves

   Water waves: They are mechanical waves for which a medium is required to propagate.



Sound waves: A guitar or music system. Sound waves need a medium to propagate.They cannot travel in vacuum.

Electromagnetic Waves:

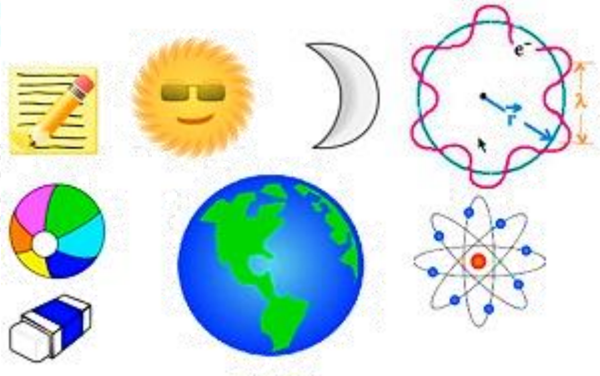
- Electromagnetic waves are related to electric and magnetic fields.
- An electromagnetic wave, does not need a medium to propagate, it carries no mass,does carry energy.

  Examples: Satellite system, mobile phones,radio, music player, x-rays and microwave.

Matter waves:-

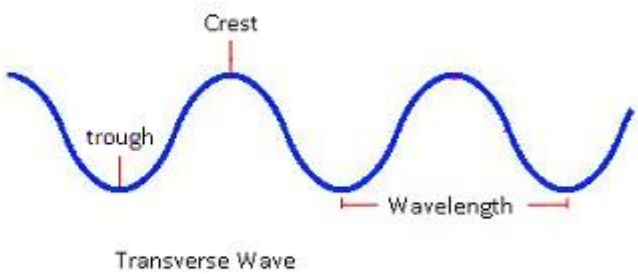
- Waves related to matter. Matter consists of small particles.
- Matter waves are associated with moving electrons,protons,neutrons & other fundamental particlesetc.
- It is an abstract concept.

  Examples:- pencil, sun, moon, earth, ball, atoms.



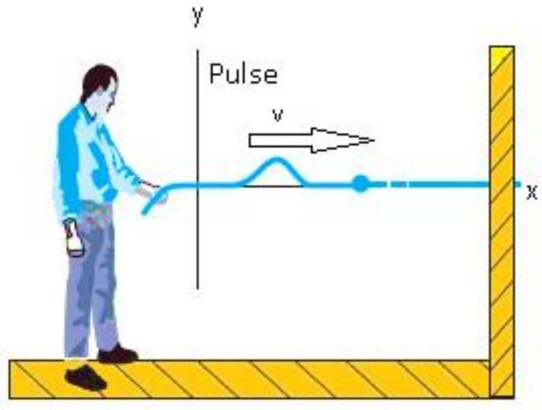
Transverse Waves

- The transverse waves are those in which direction of disturbance or displacement in the medium is perpendicular to that of the propagation of wave.
- The direction in which a wave propagates is perpendicular to the direction of disturbance.



For example:-

- Consider a manholding one end of a thread and other end of the threadis fixed to wall.
- When a little jerk is given to the thread in the upward direction.The entire thread moves in a wavy manner.
- The jerk propagated along the entire length of the thread.
- The small disturbance which came from the source at one end, that disturbance getting propagatedand that is known as direction of propagation.
- Disturbance is vertically upward and wave is horizontal.They are perpendicular to each other.
- This type of wave is known as transverse wave.

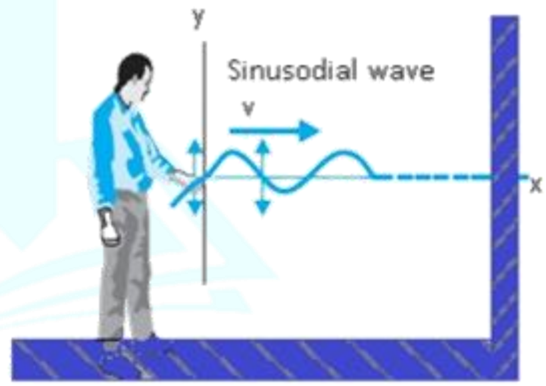


A single pulse is sent along a stretchedstring. A typical element of the string (suchas that marked with a dot) moves up andthen down as the pulse passes through.

The element's motion is perpendicular to thedirection in which the wave travels.

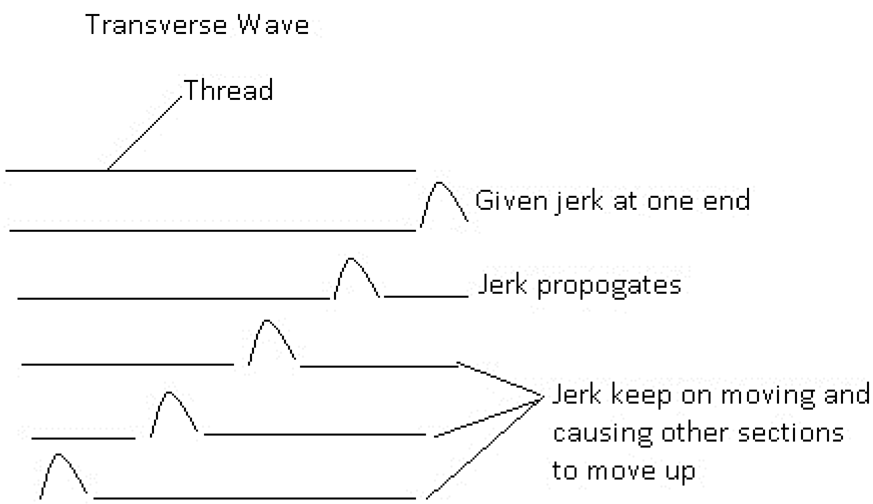
How are transverse waves caused?

- When we pull a thread in upward direction the formation and propagation of the waves are possible because entire thread is under tension.
- This tension is the small disturbance which is given at one end and it gets transferred to its neighbouring molecules.
- This will keep on continuing.So this small pulse will get propagated along the length of the thread.
- The movement of the particles is perpendicular to the propagation of the wave and the wave will propagate horizontally.



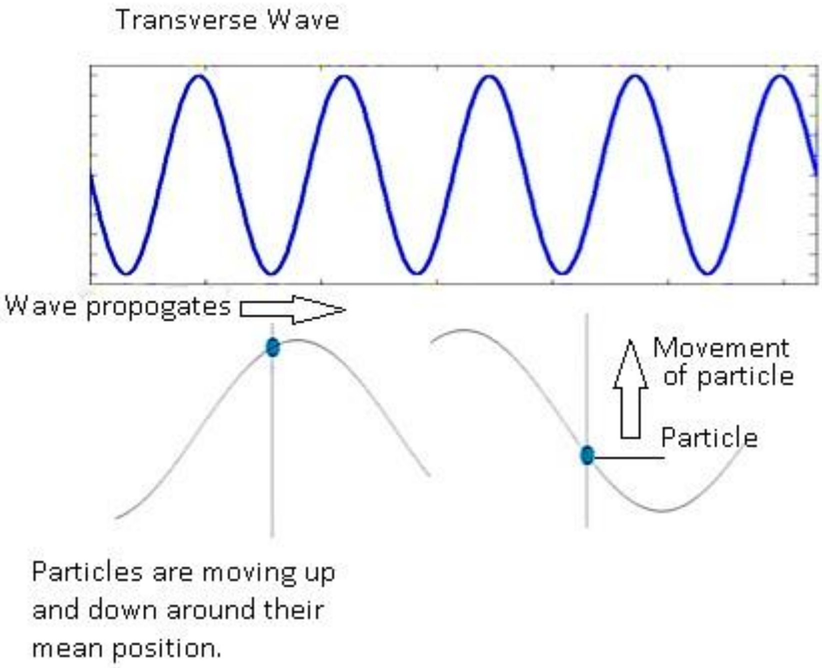
It is a transverse wave

A sinusoidal wave is sent along the string.A typical element of the string moves up and down continuously as the wave passes.

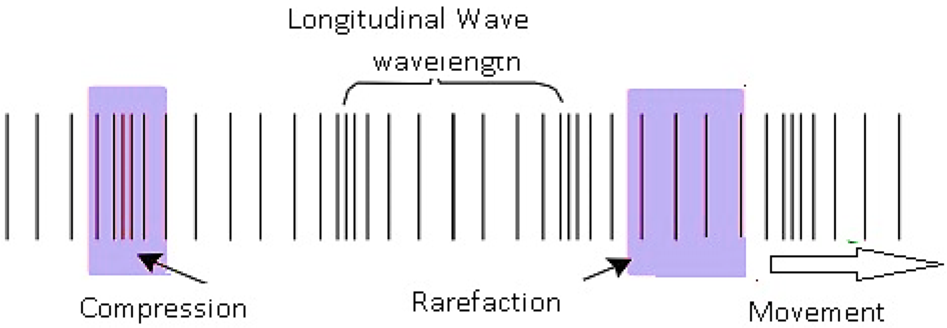


Conclusion:-

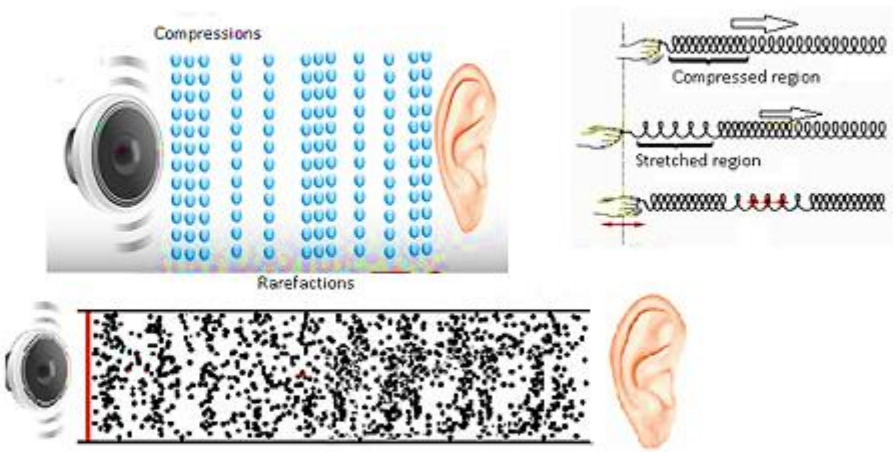
1. Transverse waves are those waves which propagates perpendicular to the direction of the disturbance.
2. Direction of disturbance is the direction of motion of particles of the medium.



Longitudinal Waves



- Longitudinal means something related to length.
- In longitudinal waves direction of disturbance or displacement in the medium is along the propagation of the wave.



- For example: - Sound waves. Particles and wave moving along the horizontal direction. So both are in the same direction.
- In a Longitudinal wave there are regions where particles are very close to each other.These regions are known as compressions.
- In some regions the particles are far apart.Those regions are known as rarefactions.

Differentiate between transverse and longitudinal waves

Transverse	Longitudinal
Constituents of the medium oscillate perpendicular to the direction of the wave propagation.	Constituents of the medium oscillate parallel to the direction of wave propagation.
	

Displacement in a progressive wave

- Amplitude and phase together describe the complete displacement of the wave.
- Displacement function is a periodic in space and time.
- Displacement of the particles in a medium takes place along the y-axis.
- Generally displacement is denoted as a function of X and T, but here it is denoted by y.
- In case of transverse wave displacement is given as:
- y(x,t) where x=propagation of the wave along x-axis, and particles oscillates along y-axis.
- Therefore $y(x,t)= A \sin(kx - \omega t + \varphi)$.This is the expression for displacement.
- This expression is same as displacement equation which is used in oscillatory motion.
- As cosine function;$y(x,t)= B \cos(kx - \omega t + \varphi)$,As both sine and cosine function)$y (x, t) = A \sin (kx - \omega t + \varphi) + B \cos(kx - \omega t + \varphi)$

Mathematically:

- Wave travelling along +X-axis: $y(x, t) = a \sin (kx - \omega t + \varphi)$.
- Consider $y=a\sin(kx - \omega t + \varphi) \Rightarrow y/a=\sin(kx - \omega t + \varphi)$
- $\sin^{-1}(y/a) = kx-\omega t \Rightarrow kx=\sin^{-1}(y/a) +\omega t$
- $x=(1/k)\sin^{-1}(y/a)+ (\omega t/k)$
- Wave travelling along –X-axis: $x=(1/k)\sin^{-1}(y/a)-(\omega t/k)$(only change in the sign of ωt)

Conclusion:-

- As time t increases the value of x increases. This implies the x moves along x-axis.
- As time t decreases the value of x decrease. This implies the x moves along (-)ive x-axis.

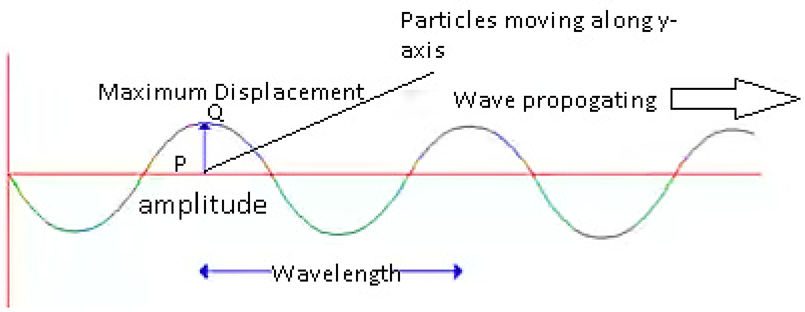
Amplitude and Phase of a wave

- Amplitude and phase together describes the position of the particle.

- Amplitude is the maximum displacement of the elements of the medium from their equilibrium positions as wave passes through them.
- It is denoted by A.

In case of transverse wave,

- The distance between the point P and Q (in the Figure) is maximum displacement. This maximum displacement of the particles is known as amplitude.



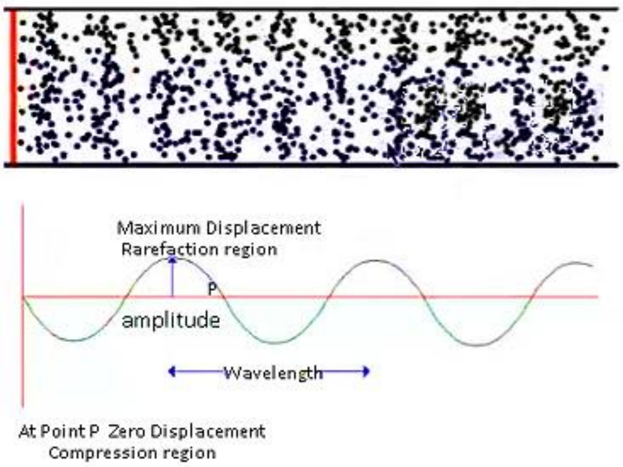
In case of longitudinal wave,

- There are regions of compressions (particles are closely packed) and rarefactions (particles are far apart).
- In compressions density of the wave medium is highest and in rarefactions density of the wave is lowest.
- Consider when the particle is at rarefaction, in that region as particle gets more space as a result the particles oscillates to the maximum displacement.
- Whereas in compressed region the particles oscillates very less as the space is not very much.
- The peak or the maximum amplitude is the centre of two compressed regions. Because at the center of the two compressed region the particle is most free to displace to maximum displaced position.

Conclusion:-

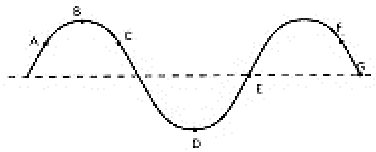
- In case of longitudinal wave the particles will not oscillate to a very large distance. This displacement won't represent the amplitude as it is not maximum possible displacement.

  Amplitude is represented basically by the centre of the rarefaction region where the particle is most free to oscillate to its maximum displacement.



Phase

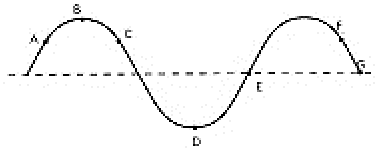
Phase of a wave describes the state of motion as the wave sweeps through an element at a particular position.



In-phase– Two points are said to be in-phase with each other when these two points are at the same position and they both are doing the same thing i.e. both the two points are exhibiting the same behaviour.

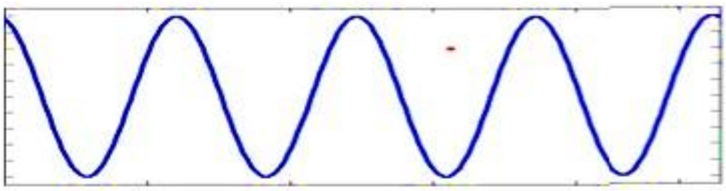
Points C and F are in phase with each other.

Out-of-phase –

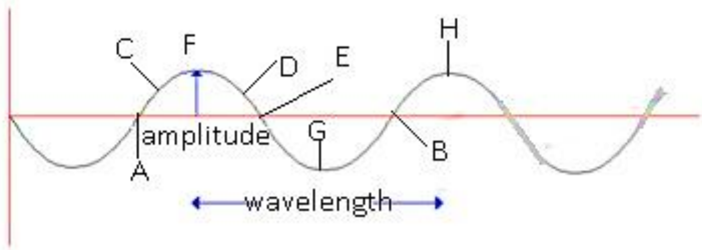


- Two points are said to be out of phase even though they are at the same points but they are doing opposite thing i.e. both the points are exhibiting the different behaviour.
- Out of phase means which is not in phase.

  Points B and D,E and G are out of phase by 1800
- Two waves can be completely in-phase or out of phase with each other. They can be partially in phase or out of phase with each other.
- Observations made from the figure (1).



Figure(1)



- Consider two points A and B on a wave. Their positions as well as their behaviour are same. Therefore points A and B are in phase.
- Consider points A and C on a wave. They are not in phase with each other as their position is not same.
- Similarly the points C and D are not in phase with each other as their positions are same but the behaviour is different. Therefore they are not in phase with each other.
- Consider the points F and G their positions are same but the behaviour is totally opposite. So F and G are out of phase.
- Consider the points F and H; they are in phase with each other as their position is same as well as their behaviour.

Wave Number

Wave number describes the number of wavelengths per unit distance.

Denoted by 'k'.

$y(x,t)= a \sin(kx – \omega t + \varphi)$ assuming $\varphi=0$.

- At initial time t=0:-
  - $y(x,0)=a\sin kx$ (i)

- When x=x+λ then y(x+λ,0)=a sink(x+λ)   (ii)
- When x=x+2λ then y(x+2λ,0)=a sink(x+2λ)

Value of y is equal at all points because all the points' λ, 2λ are in phase with each other. Therefore,

From(i) and (ii) asin kx= a sink(x+λ) =asin(kx+k λ)

This is true if and only if: -k λ =2 πn, where n=1, 2, 3…

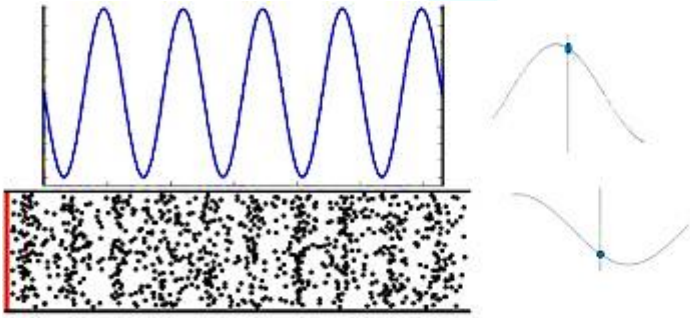
k=(2 π n)/ λ. This is the expression for wave number.

k is also known as propagation constant because it tells about the propagation of the wave.

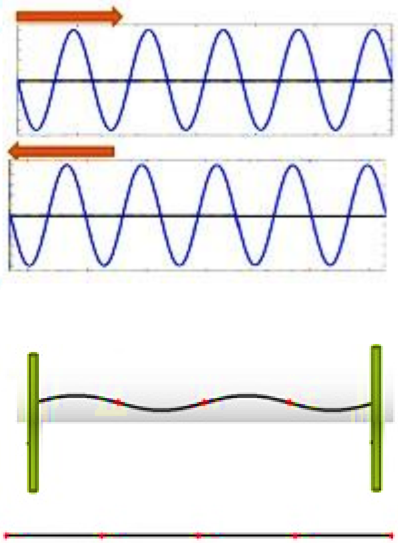
Wave number is an indirect way of describing the propagation of wave.

Travelling Waves

- Travelling waves are those waves which travel from one medium to another.
- They are also known as progressive wave. Because they progress from one point to another.
- Both longitudinal and transverse waves can be travelling wave.
- Wave as a whole moves along one direction.



Standing (Stationary) Waves



- A stationary wave is a wave which is not moving, i.e. it is at rest.
- When two waves with the same frequency, wavelength and amplitude travelling in opposite directions will interfere they produce a standing wave.
- Conditions to have a standing wave:- Two travelling waves can produce a standing wave, if the waves are moving in opposite directions and they have the same amplitude and frequency.
- At certain instances when the peaks of both the waves will overlap. Then both the peaks will add up to form the resultant wave.
- At certain instances when the peak of the one wave combine with the negative of the second wave .Then the net amplitude will become 0.
- As a result a standing wave is produced. In case of stationary wave the wave form does not move.

Explanation:-

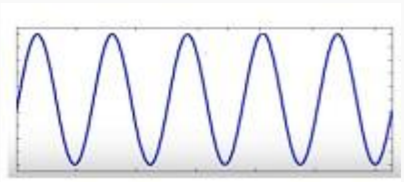
- Consider Its wave in the figure and suppose we have a rigid wall which does not move. When an incident wave hits the rigid wall it reflects back with a phase difference of π.
- Consider IInd wave in the figure, when the reflected wave travels towards the left there is another incident wave which is coming towards right.
- The incident wave is continuously coming come from left to right and the reflected wave will keep continuing from right to left.

- At some instant of time there will be two waves one going towards right and one going towards left as a result these two waves will overlap and form a standing wave.

Mathematically:

- Wave travelling towards left $y_l(x,t) = a \sin(kx - \omega t)$ and towards right $y_r(x,t) = a \sin(kx + \omega t)$
- The principle of superposition gives, for the combined wave
- $y(x, t) = y_l(x, t) + y_r(x, t) = a \sin(kx - \omega t) + a \sin(kx + \omega t)$
- $y(x,t) = (2a \sin kx) \cos \omega t$ (By calculating and simplifying)
- The above equation represents the standing wave expression.
- Amplitude = $2a \sin kx$.
  - The amplitude is dependent on the position of the particle.
  - The $\cos \omega t$ represents the time dependent variation or the phase of the standing wave.

Difference between the travelling wave and stationary wave

Travelling Wave(Progressive Wave)	Stationary Wave (Standing wave)
Waveform moves. Movement of the waveform is always indicated by the movement of the peaks of the wave.	Waveform doesn't move. Peaks don't move.
Wave amplitude is same for all the elements in the medium. Denoted by 'A'.	Wave amplitude is different for different elements. Denoted by asinkx.
Amplitude is not dependent on the position of the elements of the medium.	Amplitude is dependent on the position of the elements of the medium.
$y(x,t)=a\sin(kx - \omega t + \varphi)$	$y(x,t)=2a\sin(kx)\cos(\omega t)$
	

Nodes and Antinodes: system closed at both ends

- System closed at both ends means both the ends are rigid boundaries.
- Whenever there is rigid body there is no displacement at the boundary. This implies at boundary amplitude is always 0. Nodes are formed at boundary.
- Standing waves on a string of length L fixed at both ends have restricted wavelength.
- This means wave will vibrate for certain specific values of wavelength.
- At both ends, nodes will be formed.=>Amplitude=0.
- Expression for node $x = (n\lambda)/2$.This value is true when x is 0 and L.
- When x=L:- $L=(n\lambda)/2 \Rightarrow \lambda=(2L)/n$; n=1,2,3,4,.....
- λ cannot take any value but it can take values which satisfy $\lambda=(2L)/n$ this expression.
- That is why we can say that the standing wave on a string which is tied on both ends has the restricted wavelength.
- As wavelength is restricted therefore wavenumber is also restricted.
- $\nu = v/\lambda$ (relation between wavelength and frequency)

- Corresponding frequencies which a standing wave can have is given as: -ν= (vn)/2Lwhere v= speed of the travelling wave.
- These frequencies are known as natural frequency or modes of oscillations.

Modes of Oscillations:-

- ν= (vn)/2L where v=speed of the travelling wave, L=length of the string, n=any natural number.
- **First Harmonic:-**
  - For n=1, mode of oscillation is known as Fundamental mode.
  - Therefore ν1=v/(2L). This is the lowest possible value of frequency.
  - Therefore ν1is the lowest possible mode of the frequency.
  - 2 nodes at the ends and 1 antinode.
- **Second Harmonic:-**
  - For n=2, ν2=(2v)/ (2L) =v/L
  - This is second harmonic mode of oscillation.
  - 3 nodes at the ends and 2 antinodes.
- **Third Harmonic:-**
  - For n=3,ν3 = (3v)/ (2L).
  - This is third harmonic mode of oscillation.
  - 4 nodes and 3 antinodes.

Problem:- Find the frequency of note emitted (fundamental note) by a string 1m long and stretched by a load of 20 kg, if this string weighs 4.9 g. Given, g = 980 cm s−2?

Answer:-

L = 100 cm T = 20 kg = 20 × 1000 × 980 dyne

m= 4.9/100 = 0.049 g cm-1

Now the frequency of fundamental note produced,

ν= (1/2L) √ (T/m)

ν = 1/ (2x100) √(20x1000x980)/(0.049)

=100Hz

Problem:- A pipe 20 cm long is closed at one end, which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this same source can be in resonance with the pipe, if both ends are open? Speed of sound = 340ms−1?

Answer:-

The frequency of nth mode of vibration of a pipe closed at one end is given by

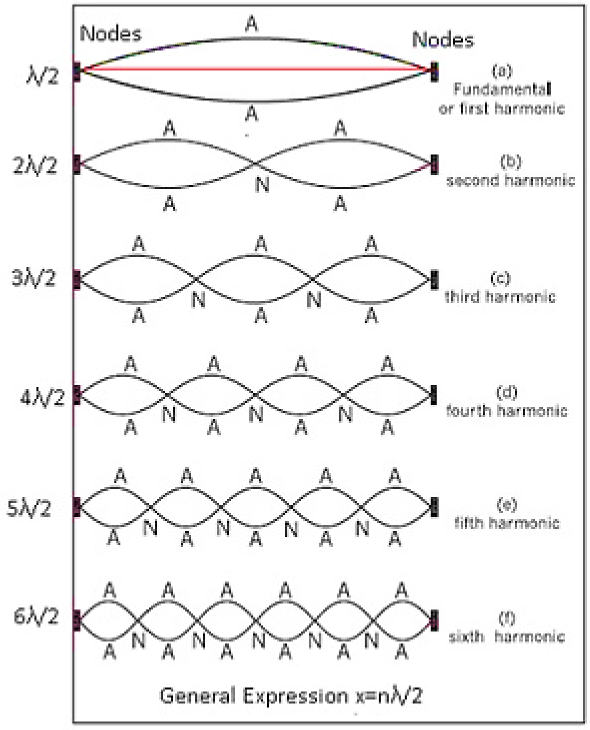
νn=(2n 1)v/4L

v=340ms-1; L=20cm=0.2m;νn=430Hz.

Therefore 430= ((2n-1) x 340)/ (4x0.2)

=>n=1

Therefore, first mode of vibration of the pipe is excited, for open pipe since n must be an integer, the same source cannot be in resonance with the pipe with both ends open.



Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

Beats

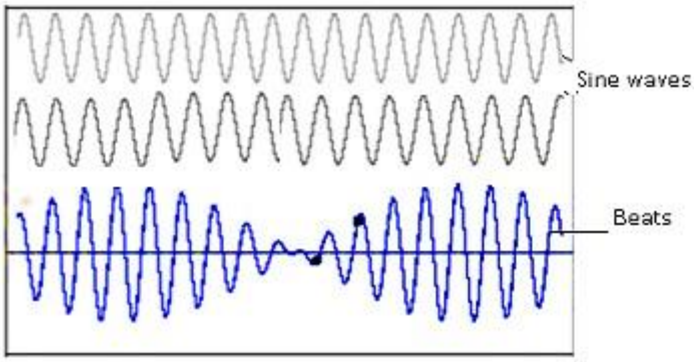
Beats is the phenomenon caused by two sound waves of nearly same frequencies and amplitudes travelling in the same direction.

For example:-

- Tuning of musical instruments like piano, harmonium etc. Before we start playing on these musical instruments they are set against the standard frequency. If it is not set a striking noise will keep on coming till it is set.

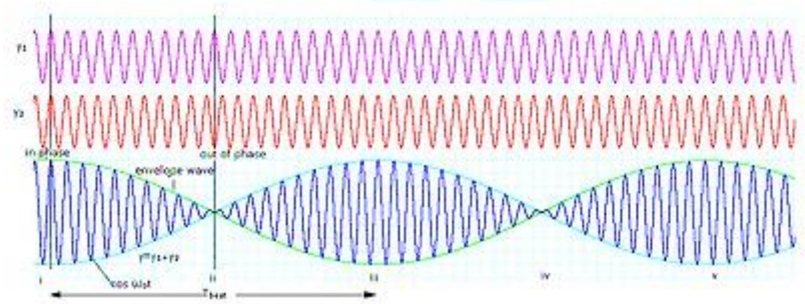
Mathematically

- Consider only the time dependent and not the position dependent part of the wave.
- $s_1 = a \cos \omega_1 t$ and $s_2 = a \cos \omega_2 t$; where amplitude and phase of the waves are same, but the frequencies are varying. Also considering $\omega_1 > \omega_2$.
- When these 2 waves superimpose $s = s_1 + s_2 = a[\cos \omega_1 t + \cos \omega_2 t]$
- By simplifying , $2a (\cos(\omega_1 - \omega_2)/2)t \cos(\omega_1 + \omega_2)/2)t)$
- $\Rightarrow \omega_1 - \omega_2$ is very small as $\omega_1 > \omega_2$.Let $(\omega_1 - \omega_2) = \omega b$
- $\Rightarrow \omega_1 + \omega_2$ is very large. Let $(\omega_1 + \omega_2) = \omega a$
- $s = 2a \cos \omega bt \cos \omega at$
- $\cos\omega at$ will vary rapidly with time and $2a\cos\omega bt$ will change slowly with time.
- Therefore we can say $2a\cos\omega bt$ = constant. As a result $2a\cos\omega bt$ = amplitude as it has small angular variation.



Beat Frequency

- Beat frequency can be defined as the difference in the frequencies of two waves.
- Consider if there is a wave of frequency ω1 and another wave of frequency ω2.Then the beat frequency will be ω1- ω2.
- It is denoted by ω
- Also ω = 2πν
- Therefore ν beat = ν1 – ν2

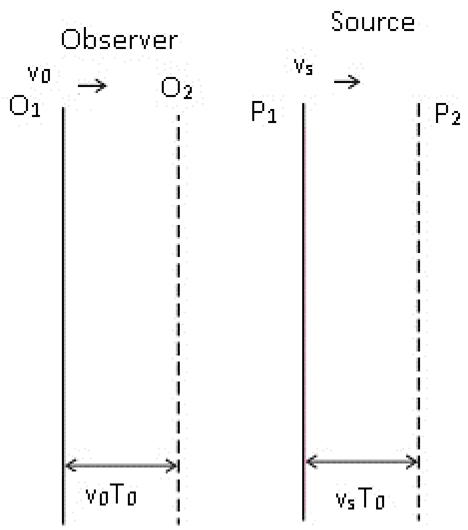


Problem:- Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz?

Answer:- Increase in the tension of a string increases its frequency. If the original frequency of B (νB) were greater than that of A (νA), furtherincrease in νB should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease. This shows thatνB< νA. Since νA – νB = 5 Hz, and νA = 427 Hz, weget νB = 422 Hz.

Doppler's Effect

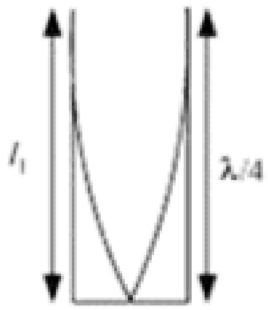
- Doppler Effect is the phenomenon of motion-related frequency change.
- Consider if a truck is coming from very far off location as it approaches near our house, the sound increases and when it passes our house the sound will be maximum. And when it goes away from our house sound decreases.
- This effect is known as Doppler Effect.
- A person who is observing is known as Observer and object from where the sound wave is getting generated it is known as Source.
- When the observer and source come nearer to each other as a result waves get compressed. Therefore wavelength decreases and frequency increases.
- Case 1:- stationary observer and moving source
- Let the source is located at a distance L from the observer.
- At any time t1, the source is at position P1.



- Time taken by the wave to reach observer =L/v where v=speed of the sound wave.
- After some time source moves to position P0 in time T0.
- Distance between P1 and P0 =vsTo where vs is the velocity of the source.
- Let t2be the time taken by the second wave to reach the observer
    - Total time taken by the for the second wave to be sent to the observer = To +(L+vsTo)/v
    - Total time taken by the for the third wave to be sent to the observer=2To +(L+2vsTo)/v
    - Therefore for nth point tn+1 =nTo +(L+nvsTo)/v
    - =>In time tn+1the observer captures n waves.
- Total time taken by the waves to travel Time period T= (tn+1 - t1)/n
- =To +(vsTo)/v =>T=To(1+vs/v)
- Or v= 1/T
- =>v = v0(1+vs/v)-1
- By using binomial Theorem, v= v0 (1- vs/v)
- If the source is moving towards the observer the expression will become v= v0 (1+ vs/v)
- Case 2:- moving observer and stationary source
- As the source is not moving therefore vs is replaced by -v0.
- Therefore v= v0 (1+ v0/v)

The observer O and the source S, both moving respectively with velocities v0 and vs . They are at position O1 and P1 at time t = 0, when the source emits the first crest of a sound, whose velocity is v with respect to the medium. After one period, t = T0, they have moved to O2 and P2,respectively through distances v0 T0 and vs T0, when the source emits the next crest.

Problem:- A metre-long tube opens at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.



Answer:-

Frequency of the turning fork, ν = 340 Hz

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure.

Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation

l1=λ/4 Where,

Length of the pipe,l1=25.5cm=0.255m

Therefore, λ=4l1 =4x0.255 = 1.02m

The speed of sound is given by the relation:

v=νλ =340x1.02 =346.8m/s

Class : 11th Physics
Chapter- 15 : Waves

Waves

Relation b/w wave velocity (v) Wavelength (y) & frequency (v)

$v = \nu\lambda$

Time period (T) $= \dfrac{1}{\text{Frequency}} = \dfrac{1}{\nu}$

Angular Frequency (ω) $= 2\pi\nu$

Wave number, $\nu = \dfrac{1}{\text{wavelength}} = \dfrac{1}{\lambda}$

Beats

Difference in frequencies of two superposing waves,

$\nu_{beat} = \nu_1 \sim \nu_2$

Displacement relation in a plane progressive wave

$y(x, t) = A\sin(-kx + t)\,\omega_0$

$y(x,t) = A\sin 2\pi\left(\dfrac{t}{T} - \dfrac{x}{\lambda}\right)$

Velocity amplitude $v_0 = A\omega^2$

Acceleration Amplitude $a_0 = A\omega^2$

Principle of superposition of waves

When two or more waves are propagating with different displacements then the net displacement of collective wave is given as $Y = y_1 + y_2 + y_3 + \ldots + y_n$

Stationary Waves

Stationary waves

(i) In string: Fundamental Frequency, $\nu = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

(ii) In organ pipe:

(a) Open at both ends,

Fundamental freq. or I harmonic $\nu = \dfrac{v}{2l}$

(b) Closed at one end $\nu = \dfrac{v}{4l}$

(iii)
- In open organ pipe, higher harmonics, both odd & even.
- In closed organ pipe, higher harmonics, odd only.

Dopoler Effect

General Formula, Apparent Frequency

$n^1 = n\left(\dfrac{v \pm v_s \pm v_m}{v \mp v_0 \pm v_m}\right)$

Here, v = speed of sound,

v_0 = observer speed

v_s = source speed

v_m = medium speed

Types

Matter Waves

Waves associated with particles like-electrons, protons, neutrons, atoms, molecules etc.

Mechanical Waves

Require material medium for their propagation

Electromagnetic Waves

Do not require any material medium for their propagation e.g., light waves

Transverse Waves

Individual particles of the medium oscillate perpendicular to the direction of propagation of wave

Speed of transverse wave in a stretched string

$V = \sqrt{\dfrac{T}{m}}$, T = tension & M = mass/length

Longitudinal Waves

Individual particles of the medium oscillate along the direction of propagation of wave.

Speed of a longitudinal wave speed of sound, $V = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{P}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}}$ for air, $\gamma = 7/5$

Properties

Essential properties for propagation
- Elasticity
- Inertia
- Minimum friction

Important Questions

Multiple Choice Questions-

1) Which of the following are mechanical waves
   a) Water waves
   b) Sound waves
   c) Seismic waves
   d) All
2) Which of the following are electromagnetic waves
   a) Light
   b) Radio waves
   c) X rays
   d) All
3) Electromagnetic waves
   a) Requires material medium for their propagation
   b) Do not require material medium for their propagation
   c) Both a and b
   d) None
4) _____ waves can travel through vacuum.
   a) Sound waves
   b) Light waves
   c) Radio waves
   d) Both b and c
5) The speed of electromagnetic waves is
   a) 299,792,458 m/s
   b) 299, 792, 458 km/s
   c) 299, 792, 458 cm/s
   d) None
6) Pressure is given by
   a) F/m
   b) F/V
   c) F/A
   d) FA
7) Sound waves are the
   a) Transverse waves
   b) Longitudinal waves
   c) Both a and b
   d) None
8) Which of the following are progressive waves
   a) Transverse waves
   b) Longitudinal waves
   c) Both a and b
   d) None
9) In case of transvers waves, the direction of motion of particles is
   a) Parallel to the direction of propagation
   b) Perpendicular to the direction of propagation
   c) Normal to the direction of propagation
   d) Both b and c
10) In case of longitudinal waves, the direction of motion of particles is
   a) Parallel to direction of propagation
   b) Perpendicular to direction of propagation
   c) Normal to direction of propagation
   d) None
11) The waves in an ocean are the combination of
   a) Longitudinal waves
   b) Transverse waves
   c) Both a and b
   d) None
12) The maximum displacement of the particle of the wave from its mean or equilibrium position is called as
   a) Phase
   b) Epoch
   c) Distance
   d) Amplitude
13) The minimum distance between two points having the same phase is called as
   a) Frequency of the wave
   b) Amplitude
   c) Wavelength of the wave
   d) None
14) K = 2π/ wavelength, then k is called as
   a) Wave number
   b) Propagation constant
   c) Force constant
   d) Both a and b

15) The SI unit of propagation constant is given by
    a) rad/m
    b) rad
    c) m/rad
    d) rad m

Assertion Reason Questions:

1. ***Directions:***

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false

Assertion: Sine and cosine functions are periodic functions.

Reason: Sinusoidal functions repeats it values after a definite interval of time.

2. ***Directions:***

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.

(c) If assertion is true but reason is false.

(d) If the assertion and reason both are false

Assertion: Simple harmonic motion is a uniform motion.

Reason: Simple harmonic motion is not the projection of uniform circular motion.

Answer Key

MCQ Answers-

1. **Ans:** d) all
2. **Ans:** d) all
3. **Ans:** b) do not require material medium for their propagation
4. **Ans:** d) both b and c
5. **Ans:** a) 299, 792, 458 m/s
6. **Ans:** c) F/A
7. **Ans:** b) longitudinal waves
8. **Ans:** c) both a and b
9. **Ans:** d) both b and c
10. **Ans:** a) parallel to direction of propagation
11. **Ans:** c) both a and b
12. **Ans:** d) amplitude
13. **Ans:** c) wavelength of the wave
14. **Ans:** d) both a and b
15. **Ans:** a) rad/m

Very Short Questions-

1. How is the time period effected, if the amplitude of a simple pendulum is in Creased?
2. Define force constant of a spring.
3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?
4. How is the frequency of oscillation related with the frequency of change in the of K. E and PE of the body in S.H.M.?
5. What is the frequency of total energy of a particle in S.H.M.?
6. How is the length of seconds pendulum related with acceleration due gravity of any planet?
7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.
8. How is the time period of the pendulum effected when pendulum is taken to hills Or in mines?
9. Define angular frequency. Give its S.I. unit.
10. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Very Short Answers-

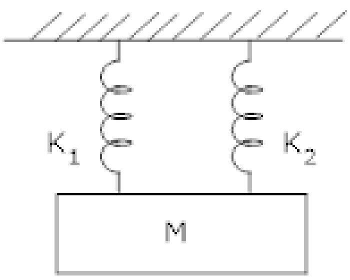
1. **Ans.** No effect on time period when amplitude of pendulum is increased or decreased.
2. **Ans.** The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.
3. **Ans.** Not at the mid-point, between mean and extreme position. it will be at $x = a\sqrt{2}$.
4. **Ans.** P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of S.H.M
5. **Ans.** The frequency of total energy of particle is S.H.M is zero because it retain constant.
6. **Ans.** Length of the seconds pendulum proportional to acceleration due to gravity)
7. **Ans.** Increased

$$T \alpha \frac{1}{\sqrt{g}}$$

8. **Ans.** As T will increase.
9. **Ans.** It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π. $\omega = 2\pi\nu$, S.I. unit is rads s^{-1}
10. **Ans.** No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

Short Questions-

1. A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period 't' of the combined system?



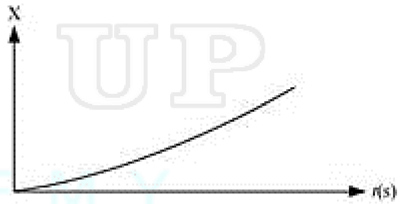
2. Show that the total energy of a body executing SHN is independent of time?
3. A particles moves such that its acceleration 'a' is given by a = -b x where x = displacement from equilibrium position and b is a constant. Find the period of oscillation? 2
4. A particle is S.H.N. is described by the displacement function:

$$x = A\cos(wt + \Phi); w = \frac{2\pi}{T}$$

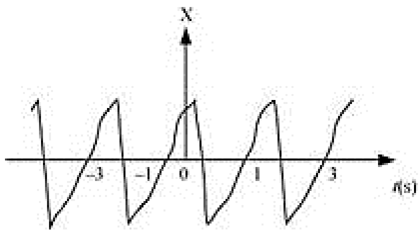
If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm | s, What are its amplitude and phase angle?

5. Determine the time period of a simple pendulum of length = l when mass of bob = m Kg? 3
6. Which of the following examples represent periodic motion?
   (a) A swimmer completing one (return) trip from one bank of a river to the other and back.
   (b) A freely suspended bar magnet displaced from its N-S direction and released.
   (c) A hydrogen molecule rotating about its center of mass.
   (d) An arrow released from a bow.
7. Figure 14.27 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

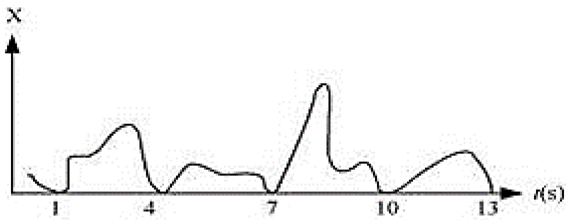
(a)



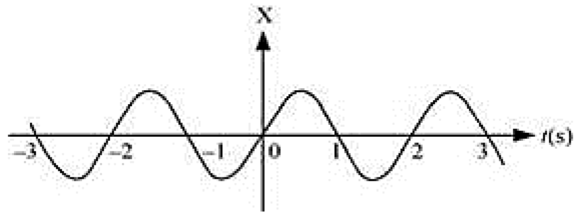
(b)



(c)



(d)



8. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200\, x^2$

(c) $a = -10x$

(d) $a = 100\, x^3$

9. The acceleration due to gravity on the surface of moon is $1.7 ms^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is $9.8\ ms^{-2}$)

10. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Short Answers-

1. **Ans.** If T = Time Period of simple pendulum

m = Mass

k = Spring constant

$$T = 2\pi\sqrt{\frac{m}{k}}$$

then,

or $$k = \frac{4\pi^2 m}{T^2}$$

$$\rightarrow k_1 = \frac{4\pi^2 m}{t_1^2} \text{let } T = t_1$$

For first spring:

$$\rightarrow k_2 = \frac{4\pi^2 m}{t_2^2} \text{let } T = t_2$$

For second spring:

When springs is connected in parallel, effective spring constant, $k = k = k_1 + k_2$

or $$k = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

If t = total time period

$$\frac{4\pi^2 m}{t^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

$$\frac{1}{t^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$$

or $$t^{-2} = t_1^{-2} + t_2^{-2}$$

2. **Ans.** Let y = displacement at any time 't'

a = amplitude

w = Angular frequency

v = velocity,

y = a Sin wt

$$v = \frac{dy}{dt} = \frac{d}{dt}(a \sin wt)$$

So, $v = a\, w \cos wt$

Now, kinetic energy = K. E. $= \frac{1}{2}mv^2$

$$K.E. = \frac{1}{2}mw^2a^2\cos^2 wt \rightarrow 1)$$

Potential energy $= \frac{1}{2}ky^2$

$$P.E. = \frac{1}{2}ka^2\sin^2 wt \rightarrow 2)$$

Adding equation 1) & 2)

Total energy = K.E. + P.E.

$$= \frac{1}{2}mw^2a^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$$

Since $w = \sqrt{\frac{k}{m}} \Rightarrow w^2 m = k^2$

Total energy $= \frac{1}{2}mw^2a^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$

$$= \frac{1}{2}ka^2\cos^2 wt + \frac{1}{2}ka^2\sin^2 wt$$

$$= \frac{1}{2}ka^2(\cos^2 wt + \sin^2 wt)$$

Total energy $= \frac{1}{2}ka$

Thus total mechanical energy is always constant is equal to $\frac{1}{2}ka^2$. The total energy is independent to time. The potential energy oscillates with time and has a maximum value of $\frac{ka^2}{2}$. Similarly, K. E. oscillates with time and has a maximum value of $\frac{ka^2}{2}$. At any instant = constant = $\frac{ka^2}{2}$. The K. E or P.E. oscillates at double the frequency of S.H.M.

3. **Ans.** Given that $a = -b\,x$, Since $a \propto x$ and is directed apposite to x, the particle do moves in S. H. M.

a = b x (in magnitude)

or $$\frac{x}{a} = \frac{1}{b}$$

or $$\frac{\text{Displacement}}{\text{Accleration}} = \frac{1}{b} \to 1)$$

Time period = $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Accleration}}}$

Using equation 1)

$$T = 2\pi\sqrt{\frac{1}{b}}$$

4. **Ans.** Att = 0; x = 1 cm; $w = \pi/s$

t = Time

x = Position

w = Argular frequency

$\therefore$ $$x = A\cos(Wt + \phi)$$

$$1 = A\cos(\pi \times 0 + \phi)$$

$$1 = A\cos\phi$$

Now, $$v = \frac{dx}{dt} = \frac{d}{dt}(A\cos(wt + \phi))$$

Att = 0; $v = \pi$ cm/s; $w = \pi/s$

$$\cancel{\pi} = -A\cancel{\pi}\sin(\pi \times 0 + \phi)$$

$\Rightarrow$ $$-1 = A\sin\phi \to 2)$$

Squaring and adding 1) & 2)

$$A^2\cos^2\phi + A^2\sin^2\phi = 1 + 1$$

$$A^2(\cos^2\phi + \sin^2\phi) = 2$$

$$A^2 = 2$$

$$A = \sqrt{2}\,cm$$

Dividing 2) by 1), we have:

$$\frac{\cancel{A}\sin\phi}{\cancel{A}\cos\phi} = -1$$

$$\tan\phi = -1$$

or $$\phi \equiv \tan^{-1}(-1)$$

$$\phi = \frac{3\pi}{4}$$

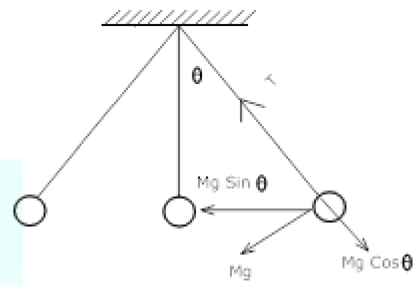
5. **Ans.** It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of B ob

x = Displacement = OB

l = length of simple pendulum



Let the bob is displaced through a small angle θ the forces acting on it:-

1) weight = Mg acting vertically downwards.

2) Tension = T acting upwards.

Divide Mg into its components → Mg Cos θ & Mg Sin θ

T = Mg Cos θ

F = Mg Sin θ

–ve sign shows force is divested towards the ocean positions. If θ = Small,

$$\cong \theta = \frac{Arc\ OB}{l} = \frac{x}{l}$$

Sin θ

$$F = -Mg\frac{x}{l}$$

In S.H.M., vestoring fore,

$$F = -mg\,\theta\, F = -mg\frac{x}{l} \to 1)$$

Also, if k = spring constant

F = –k x

$$\cancel{-}mg\frac{\cancel{x}}{l} = \cancel{-}k\,\cancel{x}\left(\text{equating } F = -mg\frac{x}{l}\right)$$

$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{\cancel{m}\times 1}{\cancel{m}g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

i.e. 1) Time period depends on length of pendulum and 'g' of place where experiment is done.

2) T is independent of amplitude of vibration provided and it is small and also of the mass of bob.

6. **Ans. (b)** and **(c)**

(a) The swimmer"s motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

7. **Ans. (b)** and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

8. **Ans.** (c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$\therefore \qquad a = -\frac{k}{m}x$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation $a = -10\,x$ is written in the above form with $\frac{k}{m} = 10$

Hence, this relation represents SHM.

9. **Ans.** Acceleration due to gravity on the surface of moon, $g' = 1.7\ ms^{-2}$

Acceleration due to gravity on the surface of earth, $g = 9.8\ ms^{-2}$

Time period of a simple pendulum on earth, T = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

l is the length of the pendulum

$$\therefore \quad l = \frac{T^2}{(2\pi)^2}\times g$$

$$= \frac{(3.5)^2}{4\times(3.14)^2}\times 9.8\,m$$

The length of the pendulum remains constant.

On moon"s surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4\times(3.14)^2}\times 9.8}{1.7}} = 8.4\,s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

10. **Ans.** The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration $=\frac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff}=\sqrt{g^2+\left(\frac{v^2}{R}\right)^2}$$

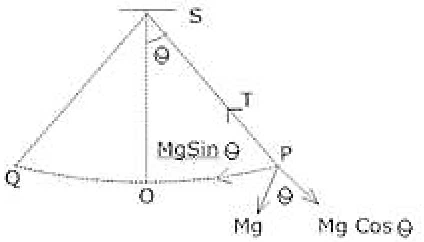
Time period, $T=2\pi\sqrt{\frac{1}{a_{eff}}}$

Where, l is the length of the pendulum

$$\therefore \text{Time period, } T=2\pi\sqrt{\frac{1}{\sqrt{g^2+\frac{v^4}{R^2}}}}$$

Long questions-

1. What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?



2. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

   (a) at the end A,

   (b) at the end B,

   (c) at the mid-point of AB going towards A,

   (d) at 2 cm away from B going towards A,

   (e) at 3 cm away from A going towards B, and

   (f) at 4 cm away from B going towards A.

3. The motion of a particle executing simple harmonic motion is described by the displacement function,

   $x(t)=A\cos(\omega t+\omega)$

   If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s-1. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin $(\omega t+\alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

4. In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

   (a) at the mean position,

   (b) at the maximum stretched position, and

   (c) at the maximum compressed position.

   In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

5. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

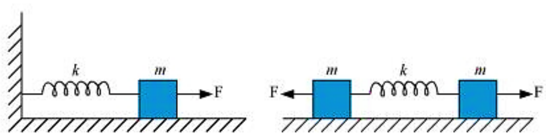
   (a) x = -2 sin (3t + π/3)

   (b) x = cos (π/6 - t)

   (c) x = 3 sin (2πt + π/4)

   (d) x = 2 cos πt

6. Figure 14.30 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force F.

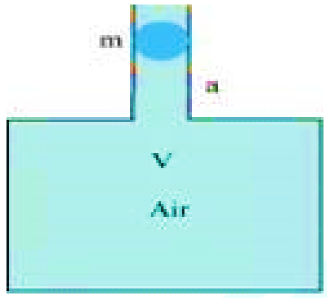


   (a) What is the maximum extension of the spring in the two cases?

   (b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

8. An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal seeFig.14.33.



Long Answers-

1. **Ans.** A simple pendulum is the most common example of the body executing S.H.M, it consist of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support, which is free to oscillate.

Let m = mass of bob

l = length of pendulum

Let O is the equilibrium position, OP = X

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

1) The weight = M g acting vertically downwards.

2) The tension = T in string acting along Ps.

Resolving Mg into 2 components as Mg Cos θ and Mg Sin θ,

Now, T = Mg Cos θ

Restoring force F = - Mg Sin θ

-ve sign shows force is directed towards mean position.

Let θ = Small, so Sin θ ≈ θ = $\frac{Arc(op)}{1} = \frac{x}{1}$

Hence F = - mg θ

$F = -mg\frac{x}{l} \rightarrow 3)$

Now, In S.H.M, F = k x →4) k = Spring constant

Equating equation3) & 4) for F

$-k\,x = -mg\ \frac{x}{l}$

Spring factor $= k = \frac{mg}{l}$

Inertia factor = Mass of bob = m

Now, Time period = T

$= 2\pi\sqrt{\frac{Inertia\ factor}{Spring\ factor}}$

$= 2\pi\sqrt{\frac{m}{mg/l}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

2. **Ans. (a)** Zero, Positive, Positive

(b) Zero, Negative, Negative

(c) Negative, Zero, Zero

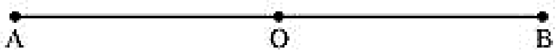
(d) Negative, Negative, Negative

(e) Zero, Positive, Positive

(f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.



A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along AO.

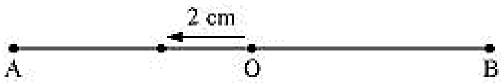
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along B.

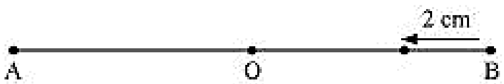
Force is also negative in this case as the particle is directed leftward.

(c)



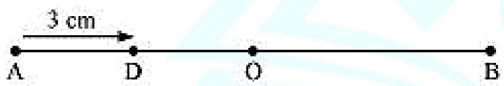
The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)



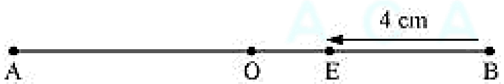
The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle's velocity and acceleration, and the force on it are all negative.

(e)



The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in **(d).**

3. **Ans.** Initially, at $t = 0$:

Displacement, $x = 1$ cm

Initial velocity, $v = \omega$ cm/sec.

Angular frequency, $\omega = \pi$ rad/s^{-1}

It is given that:

$$x(x) = A\cos(\omega t + \phi)$$

$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$

$$A\cos\phi = 1 \quad \text{....}(i)$$

$$\text{Velocity, } v = \frac{dx}{dt}$$

$$\omega = -A\omega\sin(\omega t + \phi)$$

$$1 = A\sin(\omega \times 0 + \phi) = A\sin\phi$$

$$A\sin\phi = 1 \quad \text{....}(ii)$$

Squaring and adding equations (i) and (ii), we get:

$$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$$

$$A^2 = 2$$

$$\therefore \quad A = \sqrt{2}\text{ cm}$$

Dividing equation (ii) by equation (i), we get:

$$\tan\phi = -1$$

$$\therefore \quad \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, \text{.....}$$

SHM is given as:

$$x = B\sin(\omega t + a)$$

Putting the given values in this equation, we get:

$$1 = B\sin[\omega \times 0 + a]$$

$$B\sin a = 1 \quad \text{....}(iii)$$

Velocity, $v = \omega B\cos(\omega t + a)$

Substituting the given values, we get:

$$\pi = \pi B\sin a$$

$$B\sin a = 1 \quad \text{....}(iv)$$

Squaring and adding equations (iii) and (iv), we get:

$$B^2\left[\sin^2 a + \cos^2 a\right] = 1 + 1$$

$$B^2 = 2$$

$$\therefore \quad B = \sqrt{2}\text{ cm}$$

Dividing equation (iii) by equation (iv), we get:

$$\frac{B\sin a}{B\cos a} = \frac{1}{1}$$

$$\tan a = 1 = \tan\frac{\pi}{4}$$

$$a\frac{\pi}{4}, \frac{5\pi}{4}, \text{.....}$$

4. **Ans. (a)** $x = 2\sin 20t$

(b) $x = 2\cos 20t$

(c) $x = -2\cos 20t$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, A = 2.0 cm

Force constant of the spring, k = 1200 Nm^{-1}

Mass, m = 3 kg

Angular frequency of oscillation:

$$\omega=\sqrt{\frac{k}{m}}$$

$$=\sqrt{\frac{1200}{3}}=\sqrt{400}=20\,rads^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin\omega t$

$= 2\sin 20t$

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x=A\sin\left(\omega t+\frac{\pi}{2}\right)$

$$=2\sin\left(20t+\frac{\pi}{2}\right)$$

$= 2\cos 20t$

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x=A\sin\left(\omega t+\frac{3\pi}{2}\right)$

$$=2\sin\left(20t+\frac{\pi}{2}\right)$$

$= -2\cos 20t$

The functions have the same frequency $\left(\frac{20}{2\pi}Hz\right)$ and amplitude (2 cm), but different initial phases $\left(0,\frac{\pi}{2},\frac{3\pi}{2}\right)$.

5. Ans.(a)

$$x=-2\sin\left(3t+\frac{\pi}{3}\right)=+2\cos\left(3t+\frac{\pi}{3}+\frac{\pi}{2}\right)$$

$$=2\cos\left(3t+\frac{5\pi}{6}\right)$$

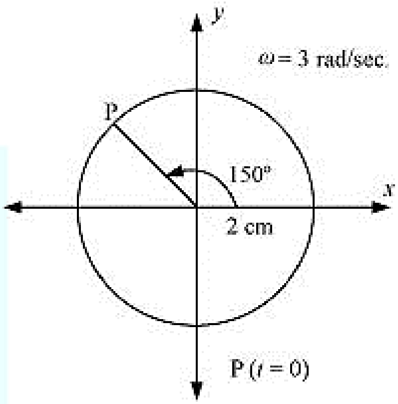
If this equation is compared with the standard SHM equation $x=A\cos\left(\frac{2\pi}{T}t+\phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi=\frac{5\pi}{6}=150°$

Angular velocity, $\omega=\frac{2\pi}{T}=3rad/\text{sec}$.

The motion of the particle can be plotted as shown in the following figure.



(b) $x=\cos\left(\frac{\pi}{6}-t\right)=\cos\left(t-\frac{\pi}{6}\right)$

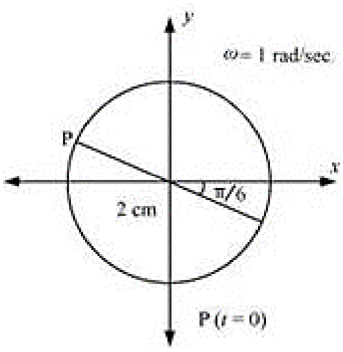
If this equation is compared with the standard SHM equation $x=A\cos\left(\frac{2\pi}{T}t+\phi\right)$, then we get:

Amplitude, A=2

Phase angle, $\phi=\frac{\pi}{6}=30°$

Angular velocity, $\omega=\frac{2\pi}{T}=1\,rad/\text{sec}$.

The motion of the particle can be plotted as shown in the following figure.



(c) $x=3\sin\left(2\pi t+\frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

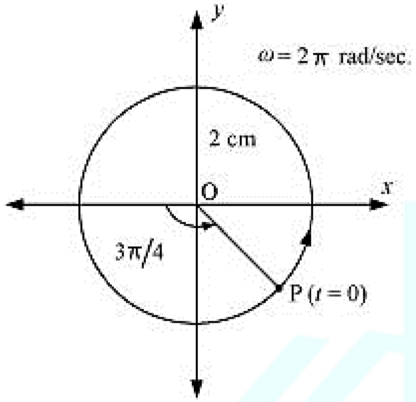
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 3 cm

Phase angle, $\phi = \frac{3\pi}{4} = 135°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3 rad/\text{sec}$.

The motion of the particle can be plotted as shown in the following figure.



(d) $x = 2\cos \pi t$

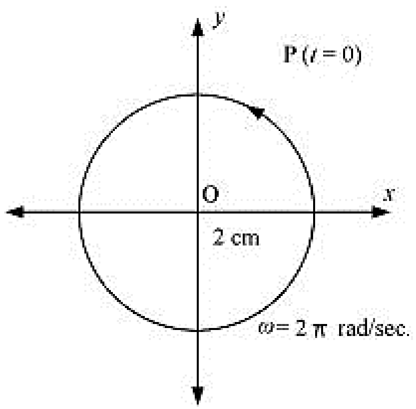
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi$ rad/s

The motion of the particle can be plotted as shown in the following figure.



6. **Ans.(a)** For the one block system:

When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:

$$F = kl$$

Where, k is the spring constant

Hence, the maximum extension produced in the spring, $l = \frac{F}{k}$

For the two block system:

The displacement (x) produced in this case is:

$$x = \frac{1}{2}$$

Net force, $F = +2\ kx\ \ 2k\frac{1}{2}$

$$\therefore \qquad l = \frac{F}{k}$$

(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\frac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore \qquad m\frac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

Where, $\omega^2 = \frac{k}{m}$

ω is angular frequency of the oscillation

$\therefore$ Time period of the oscillation, $T = \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

For the two block system:

$$F = m\frac{d^2x}{dr^2}$$

$$m\frac{d^2x}{dr^2} = -2kr$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dr^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where,

Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

7. **Ans.** Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

F = –(Volume × Density × g)

F =–(A × 2h × ρ × g) = –2 ρ gh

= –k × Displacement in one of the arms (h)

Where,

2h is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, m = Volume of mercury × Density of mercury

$= Al\rho$

$$T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{1}{2g}}$$

$\therefore$ Hence, the mercury column executes simple harmonic motion with time period

$$2\pi\sqrt{\frac{l}{2g}}.$$

8. **Ans.** Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

Volumetric strain $\frac{\textit{Change in volume}}{\textit{Original volume}}$

$$\Rightarrow \qquad \frac{\Delta V}{V} = \frac{ax}{V}$$

Bulk Modulus of air $B = \frac{Stress}{Strain} = \frac{-p}{\frac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball, $F = p \times a$

$$\frac{-Bax}{V}\cdot a$$

$$= \frac{-Ba^2x}{V}$$

In simple harmonic motion, the equation for restoring force is:

$$F = -kx \qquad \text{... }(ii)$$

Where, k is the spring constant

Comparing equations (i) and (ii), we get:

$$= \frac{Ba^2}{V}$$

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

Assertion Reason Answer:

1. (a) If both assertion and reason are true but reason is not the correct explanation of the assertion.
2. (d) If the assertion and reason both are false.

Case Study Questions-

1. A motion that repeats itself at regular intervals of time is called periodic motion. Very often, the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory. The smallest interval of time after which the motion is repeated is called its period. Let us denote the period by the symbol T. Its SI unit is second. The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz). Answer the following. a)
   i. Every oscillatory motion is periodic motion true or false?
      a. True
      b. False
   ii. Circular motion is
      a. Oscillatory motion
      b. Periodic motion
      c. Rotational motion
      d. None of these
   iii. Define period. Give its SI unit and dimensions
   iv. Define frequency of periodic motion. How it is related to time period
   v. What is oscillatory motion
2. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations. We consider the case when the external force is itself fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. The maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.
   i. When a system oscillates with its natural frequency ω, and the oscillations are called
      a. Free oscillations
      b. Forced oscillations
   ii. All free oscillations eventually die out because of
      a. Damping force
      b. electromagnetic force
      c. None of these
   iii. What is free oscillation?
   iv. What is forced oscillations?
   v. What is resonance?

Case Study Answer-

1. Answer

i. (a) True

ii. (b) Periodic motion

iii. The smallest interval of time after which the motion is repeated is called its period. Its SI unit is second and dimensions are [T1].

iv. Reciprocal of Time period (T) gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T i.e. they are inversely proportional to each other. The unit of n is thus s-1 or hertz.

v. Oscillatory motion is type of periodic motion in which body performs periodic to and fro motion about some mean position. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

2. Answer

i. (a) Free oscillations

ii. (b) Damping force

iii. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations.

iv. Forced oscillations are oscillations where external force drives the oscillations with frequency given by external force.

The phenomenon of increase in amplitude when the driving force is close to encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.